Understanding the Structural Characteristics of Convergence Bidding in Nodal Electricity Markets

Mahdi Kohansal, *Member, IEEE*, Ehsan Samani, *Student Member, IEEE*, and Hamed Mohsenian-Rad, *Fellow, IEEE*

***Abstract*—Convergence bidding, a.k.a., virtual bidding, is a market mechanism that is used by several Independent System Operators (ISOs) to increase market efficiency in electricity markets by *closing the gap* between the day-ahead market (DAM) prices and the real-time market (RTM) prices. However, some recent reports by ISOs have questioned whether convergence bids (CBs) act as intended. Motivated by such reports, this paper provides a methodology to identify under what conditions a CB results in price divergence, instead of price convergence. The analysis is done in nodal electricity markets and factors such as *transmission line congestion* are investigated. It is proved that, under some transmission lines congestion configurations, price convergence is guaranteed. In contrast, there are certain transmission lines congestion configurations that can result in price divergence when CBs are submitted at certain nodes. It is also explained how the *aggregate impact* of multiple CBs may lead to decrease (or increase) in the price gap at each bus. Importantly, the analysis in this paper also covers the stochastic case, where we obtain the *probability* of price convergence (or divergence) when we are uncertain about some system parameters.**

***Keywords*: Nodal electricity market, price gap, convergence bidding, transmission line congestion, virtual bidding.**

NOMENCLATURE

Day-Ahead Market:

$\mathbf{p}$	Vector of all bids of any type
$\mathbf{x}$, $\mathbf{y}$	Vectors of physical demand and supply bids
$\mathbf{v}$, $\mathbf{w}$	Vectors of demand and supply CBs
$\mathbf{K}$	Incidence matrix for $\mathbf{p}$ to $\mathbf{x}$
$\mathbf{\Phi}$, $\mathbf{\Psi}$	Incidence matrices for $\mathbf{p}$ and $\mathbf{x}$ to system buses
$\mathbf{V}$, $\mathbf{W}$	Incidence matrices for $\mathbf{v}$ and $\mathbf{w}$ to system buses
$\bar{\mathbf{D}}$	Index matrix for congested transmission lines
$\boldsymbol{\pi}$, $\boldsymbol{\mu}$, λ	Locational, shadow, and reference prices

Real-Time Market:

$\mathbf{z}$	Vector of physical supply bids
$\boldsymbol{l}$	Vector of actual demands at time of operation
$\mathbf{\Theta}$, $\mathbf{\Omega}$	Incidence matrices for $\mathbf{z}$ and $\boldsymbol{l}$ to system buses
$\bar{\mathbf{R}}$	Index matrix for congested transmission lines
$\boldsymbol{\sigma}$, $\boldsymbol{\eta}$, δ	Locational, shadow, and reference prices

Other Parameters:

$\mathbf{c}$	Vector of transmission line capacities
$\mathbf{\Delta}$	Vector of price differences: $\boldsymbol{\pi} - \boldsymbol{\sigma}$
$\mathbf{S}$	Shift factor matrix of the network
α, β	Coefficients of the cost or utility functions
$\mathbf{n}$	Price Difference Sensitivity vector
$\mathbf{\Pi}$	Overall CB Sensitivity Matrix

Abbreviations:

ISO	Independent System Operator
DAM	Day-Ahead Market
RTM	Real-Time Market
CB	Convergence Bid
VB	Virtual Bid
LMP	Locational Marginal Price
FERC	Federal Energy Regulatory Commissions
FTR	Financial Transmission Right
PJM	Pennsylvania, Jersey, Maryland ISO
SCED	Security-Constrained Economic Dispatch
OPF	Optimal Power Flow
KKT	Karush-Kuhn-Tucker

The authors are with the Department of Electrical and Computer Engineering, University of California, Riverside, CA, USA, 92521. This work was supported by the National Science Foundation (NSF) grant 1711944. The corresponding author is H. Mohsenian-Rad, e-mail: hamed@ece.ucr.edu.

I. INTRODUCTION

In a typical two-settlement wholesale electricity market, generation and load entities can participate in both the day-ahead market (DAM) and the real-time market (RTM), cf. [1]–[4]. Ideally, there should be *little or no gap* between the price in the DAM and the prices in the RTM. Otherwise, some generators may practice *market power* and withhold a portion of their capacities to gain more profit from the opportunity to arbitrage between the DAM and RTM [5]–[7].

A. Convergence Bidding

To reduce the price gap, Convergence bids (CBs), a.k.a., Virtual bids (VBs), was introduced to electricity markets and become a part of the Federal Energy Regulatory Commissions (FERC) standard market design [5], [8]. Similar to physical bids, CBs have two types: demand and supply. A demand (supply) CB is a bid to buy (sell) energy in DAM without any obligation to consume (produce) energy. If the CB is cleared in DAM, then the bidder is charged (credited) at the DAM price and credited (charged) at the RTM price. Therefore, the difference between the earning in RTM (DAM) and the cost in DAM (RTM) will be paid to the bidder. The process of clearing CBs and the related payment is outlined in Fig. 1.

From FERC's and ISOs' perspective, if participants make profit through CBs, *it must automatically help closing the price gap* [5]: if DAM price tends to be higher than RTM price,

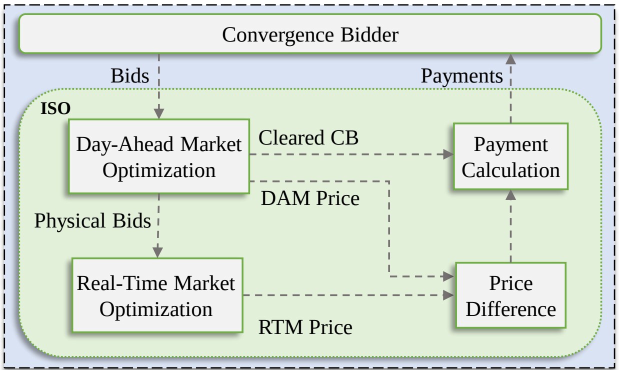

Fig. 1. Outline of the process of clearing convergence bids.

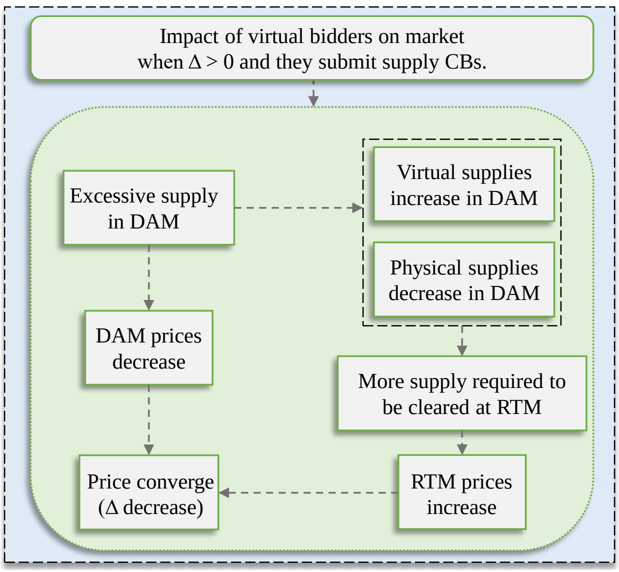

Fig. 2. The expected impact of profitable supply CBs on price convergence.

convergence bidders will seek to profit by placing supply CBs which decrease the DAM price by increasing supply in the DAM. This increase in supply CB and decrease in DAM price could also reduce the amount of physical supply committed and scheduled in the DAM, which would tend to increase RTM price. Fig. 2 outlines how a profitable CB is expected to improve price convergence between DAM and RTM. In this scenario, supply CBs could help improve price convergence by reducing DAM price and increasing RTM price. A similar argument can be made for placing a demand CB when DAM price tends to be lower than RTM price [9].

B. The Issue

There are recent ISO reports raising concerns about CBs, questioning whether they perform as expected. In fact, it is generally difficult for ISOs to analyze how CBs may have affected price convergence and market efficiency [10]–[12]. For example, here is a related quote from the California ISO 2015 Annual Report on Market Issues and Performance [10]: "*However, the degree to which convergence bidding has actually increased market efficiency has not been assessed. In some cases, virtual bidding may be profitable for some market participants without increasing market efficiency significantly or even decreasing market efficiency.*" Here is another quote from the PJM 2015 Report on Virtual Transactions in Energy Markets [11]: "*In considering when and to what degree virtual trading offers benefits to PJM markets, it is important to account for these distinctions before definitively concluding that the generally accepted principles of market efficiency as demonstrated by trading in other financial and commodity marketplaces hold equally well to PJMs energy markets.*" The above PJM report later added the following note: "*The multiple facets of virtual transactions must be understood in order to understand how market rules can be further enhanced to maximize the usefulness of virtual transactions.*"

The above quotes exemplify the current state of uncertainty and debate about the impact of CBs in electricity market industry. This paper seeks to identify and explain some of the root causes for such observations by ISOs.

C. Related Literature

Despite the fact that CBs are widely adopted by ISOs, there is limited literature on addressing the issues related to CBs in electricity markets. The common approach so far has been to use *historical market data* from different ISOs to conduct *statistical analysis* on market prices in long-term [13]–[16].

As for the few studies that take a rather analytical approach to CBs, so far, most of them have focused on cases where the CBs are somewhat *abused*, either by a market player, e.g., when submitted strategically in conjunction with Financial Transmission Rights (FTR)s [17]–[20], or by an adversary, e.g., in a cyber-physical attack [21]. In contrast, in this paper, the focus is on investigating CBs when they are used as intended, yet they may demonstrate counter-intuitive results.

There are studies on selecting CBs to *maximize the profit* for the market participant, e.g., by using optimization or learning techniques, c.f. [22]–[24]. The common assumption is that the market participant does *not* affect the prices in the market, because these studies are *not* concerned with any such impact.

There are a few recent studies that have pointed out the complexities around CBs in electricity markets and the fact that CBs in electricity markets cannot be evaluated in the same way that they are assessed in other markets [10]–[12], [25]. However, so far, no prior study has provided any analytical method to explain such complexities and their root causes.

D. Summary of Contributions

Motivated by these recent observations, in this paper, we study the conditions under which a CB (or a group of CBs) results in price convergence or price divergence in a nodal electricity market. The analysis in this paper is *not* data-driven; it is instead based on looking at the basic formulation of CBs in nodal electricity markets in order to study how they impact on the DAM and RTM prices. The structural analysis in this paper is in fact *from the view point of the ISO*, because the ISO is concern about the overall impact of CBs and efficiency of market. This is one of the main differences between this study and the majority of the existing studies in the literature that often look at the concept of convergence bidding from the view

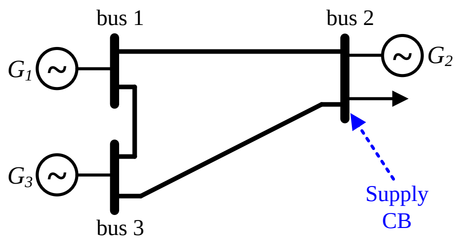

Fig. 3. The simple power network that is studied in this section.

point of the market participant. Thus, our goal is to answer the following simple yet fundamental question: does convergence bidding in electricity markets always act as intended and result in converging the prices of the DAM and RTM, just like in a typical commodity or financial market? Our results are along the line with the recent ISO reports as quoted in Section I.B. The contributions in this paper are summarized as follows:

- This paper investigates the structural characteristics of CBs in nodal electricity markets, considering network topology, location of CBs, nodal prices, and specially the congestion status of transmission lines. Note that, transmission line congestion is one of the main differences between electricity markets and other commodity and financial markets. We start off with three insightful representative test cases, to show that, depending on the extent and location of transmission line congestion, the outcome of a CB at a bus in a nodal electricity market can create price converge or price divergence at that bus.
- A sensitivity analysis based on a closed-form model, is conducted to explain *how* the prices in the DAM and RTM are affected by CBs; and subsequently *why* and under *what* exact underlying topological and grid operational conditions, placing CBs result in price divergence.
- Insightful sufficient conditions and their engineering implications are also discussed. In particular, it is proved that if the power system does not experience any transmission line congestion at DAM or RTM, then any profitable CB always helps the system reduce the price gap between DAM and RTM. More importantly, it is shown that the price convergence by profitable CBs is also almost guaranteed if the set of congested transmission lines in RTM is a subset of or equal to that in DAM.
- In order to address uncertainty, most importantly in the amount of load and renewable generation, a scenario-based stochastic analysis is proposed in this paper. The probability distribution of the rate of convergence between the DAM and RTM prices under the random scenarios is calculated and the results are investigated.
- Finally, a framework is provided to explain how the overall, i.e., aggregate impact of multiple CBs may lead to price convergence or divergence at each bus. This framework is general, and can be used in both deterministic and stochastic studies of CBs in nodal electricity markets.

The conference version of this paper was published in [26].

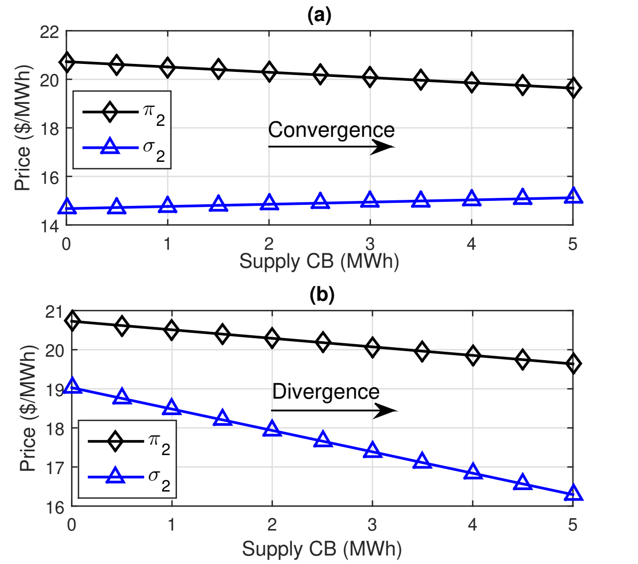

Fig. 4. The DAM price π and the RTM price σ at Bus 2 versus the cleared energy of a supply CB at the same bus: (a) Example 1, (b) Example 2.

II. Motivation Via Examples

In this section, we present three toy examples to demonstrate the fundamental concepts that we seek to investigate in this paper. All three examples are based on the three-bus network in Fig. 3. Generators G_1 and G_3 participate in both the DAM and RTM, while generator G_2 participates only in the RTM. In all cases, a supply CB is placed at bus 2. Cost functions for all generators is in form of $0.5\alpha_i \mathbf{x}_i^2 + \beta_i \mathbf{x}_i$. The price components of the supply bids submitted to the DAM are $\alpha_1 = 0.3$, $\beta_1 = 3$, $\alpha_3 = 0.8$, and $\beta_3 = 8$. The price components of the supply bids submitted to the RTM are $\alpha_1 = 1.8$, $\beta_1 = 10$, $\alpha_2 = 1.7$, $\beta_2 = 5$, $\alpha_3 = 0.1$, and $\beta_3 = 14$. The reactance for all transmission lines is 0.1 Ω and their resistance is negligible. In Examples 1 and 2, the demand entity at Bus 2 submits a self-schedule demand bid at 75 MWh to the DAM and its actual demand in the RTM turns out to be 90 MWh. In Example 3, we use the wind generation data from [27] and the load data from [28] to generate 150 scenarios for the net load.

Example 1 - No Congestion: Suppose all transmission lines have infinite capacity; thus, no transmission line can be congested. Accordingly, LMP is the same at all buses. If no CB is submitted to the market, i.e., when the CB is zero, then the cleared market prices in the DAM and RTM are obtained as $\boldsymbol{\pi}_1 = \boldsymbol{\pi}_2 = \boldsymbol{\pi}_3 = \$20.72$ and $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2 = \boldsymbol{\sigma}_3 = \$14.67$. Consider the diagrams in Fig. 4(a). As we increase the size of the supply CB at bus 2, the prices in the DAM increase while the prices in the RTM decrease; thus, resulting in *convergence* between the market prices. This is what is intended for a CB.

Example 2 - Congestion: Next, suppose the capacity of the transmission line between buses 1 and 2 is 47 MW. In the absence of CBs, no line is congested in the DAM and $\boldsymbol{\pi}_1 = \boldsymbol{\pi}_2 = \boldsymbol{\pi}_3 = \$20.72$. However, the transmission line between buses 1 and 2 is congested at the RTM. Accordingly, the RTM LMPs will be $\boldsymbol{\sigma}_1 = \$10.29$, $\boldsymbol{\sigma}_2 = \$19.03$, and $\boldsymbol{\sigma}_3 = \$14.66$. Consider the diagrams in Fig. 4(b). As we increase the size of the supply CB at bus 2, the prices in both the DAM and

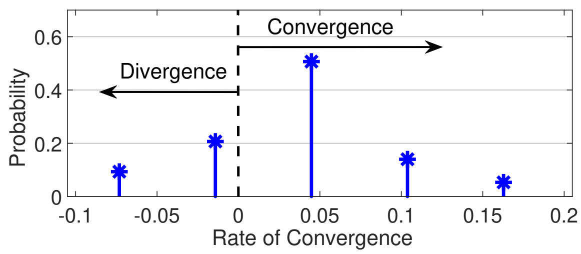

Fig. 5. The probability distribution of the rate of convergence between the DAM and RTM prices under the random scenarios in Example 3.

RTM decrease; however the rate of decreasing is higher for the prices in the DAM. Thus results in *divergence* in the market prices. This is the opposite of what is intended for a CB.

Example 3 - Stochastic Parameters: The observations that we made in Examples 1 and 2 are fundamental. In particular, even if the parameters of the system are stochastic, we can still observe both price convergence and price divergence. To see this, suppose generator G_2 at bus 2 is replaced with a wind farm and a load, which together form a *random net load* that follows a scenario-based probability distribution. The minimum, maximum and average of the net load is 2.05 MWh, 94.11 MWh, and 63.44 MWh, respectively. The average net load is used in the DAM and the actual scenarios are used in the RTM. The capacity of the line between buses 1 and 2 is 40 MW. Suppose the supply CB at bus 2 is fixed at 0.1 MWh. Fig. 5 shows the probability distribution for the *rate of convergence* between the DAM and RTM prices, where a negative value indicates price divergence. The rate of convergence is the price gap *before* and *after* the presence of the CB. In this example, *price convergence* occurs at 70% probability and *price divergence* occurs at 30% probability.

All the above numerical results, as well as the case studies in Section V, are obtained by solving the corresponding market optimization problems using CPLEX in MATLAB. The simulations are done on a PC with Intel Xeon CPU @3.80GHz and 16 GB RAM. Each case study may require solving multiple market optimization problems. For instance, in Example 3, there are 150 random scenarios. For each scenario, there are four market optimization problems, i.e., for the DAM and the RTM; and also for before and after placing the CB. Thus, a total of $600 = 150 \times 4$ market optimization problem were solved in order to obtain the results in this example. For each optimization problem, the number of iterations are less than 15 and the average total CPU time is less than 400 milliseconds.

III. Sensitivity Analysis of Day-Ahead and Real-Time Market Prices to Convergence Bids

The toy examples in Section II suggest that there might be some structural characteristics in nodal electricity markets that affect the impact of CBs on such markets. The first step to understand these characteristics is to analyze the sensitivity of the DAM and RTM prices with respect to the CBs.

A. System Model to Clear Convergence Bids

In this paper, an offer-based nodal electricity market is considered for the DAM and RTM models, which is cleared by Security-Constrained Economic Dispatch (SCED). Since our goal is to investigate *how* CBs affect the price gap, other aspects such as start-up and shut-down costs are not considered. Thus, there is no binary variable in the market model; accordingly, the SCED problem is converted to DC Optimal Power Flow (OPF). Here, we mean to show that, even in a simplified core market clearing model, price divergence can indeed happen. In the market clearing process, the supply and demand bids represent the cost and utility functions for the supply and demand entities, respectively. These functions take quadratic forms, i.e., $0.5\alpha_i \mathbf{x}_i^2 + \beta_i \mathbf{x}_i$. Note that, α_i is greater than for supply bids and less than zero for demand bids. Each bid comprises coefficients α_i and β_i as well as the minimum and maximum range for generation or consumption.

Based on the submitted bids, ISO solves the below OPF problem which is formulated in matrix form [9], [14], [29]:

$$\begin{aligned} \textbf{min} \quad & 0.5\,\mathbf{p}^T \mathbf{A} \mathbf{p} + \mathbf{b}^T \mathbf{p} && \text{(1a)} \\ \textbf{s.t.} \quad & \mathbf{1}^{\mathrm{T}}\,\mathbf{p} = 0 \qquad : \lambda && \text{(1b)} \\ & -\mathbf{c} \le \mathbf{S}\,\mathbf{\Phi}\,\mathbf{p} \le \mathbf{c} \quad : \boldsymbol{\mu}^-, \boldsymbol{\mu}^+ && \text{(1c)} \\ & \mathbf{p}^{\min} \le \mathbf{p} \le \mathbf{p}^{\max} && \text{(1d)} \end{aligned}$$

where the optimization variables are cleared energy of physical and virtual supplies and loads in the system, as follow:

$$\mathbf{p} \triangleq \begin{bmatrix} \mathbf{x} & \mathbf{y} & \mathbf{v} & \mathbf{w} \end{bmatrix}^{\mathrm{T}}. \tag{2}$$

In (1), we maximize the social welfare of the system (i.e., minimize the negative social welfare). The objective function comprises the cost functions of generators minus the utility functions of loads. In (1), $\mathbf{A}$ is a matrix of the diagonal quadratic coefficients in the DAM bids. It comprises α and $-\alpha$ for all supply and demand bids in the DAM, respectively. Note that, both physical and convergence bids are taken into consideration, as shown in the below equation:

$$\mathbf{A} = \begin{bmatrix} \alpha_i & .. & .. & .. & .. \\ .. & -\alpha_i & .. & .. & .. \\ .. & .. & .. & .. & .. \\ .. & .. & .. & \alpha_i & .. \\ .. & .. & .. & .. & -\alpha_i \end{bmatrix} \begin{matrix} \rightarrow \text{supply} \\ \rightarrow \text{demand} \\ \\ \rightarrow \text{supply} \\ \rightarrow \text{demand} \end{matrix} \tag{3}$$

Also, $\mathbf{b}$ is the vector of all linear coefficients in the DAM bids:

$$\mathbf{b} = \begin{bmatrix} \beta_i \\ -\beta_i \\ .. \\ .. \\ \beta_i \\ -\beta_i \end{bmatrix} \begin{matrix} \rightarrow \text{supply} \\ \rightarrow \text{demand} \\ \\ \\ \rightarrow \text{supply} \\ \rightarrow \text{demand} \end{matrix} \tag{4}$$

Constraint (1b) guarantees power balance between generation and demand. Constraint (1c) represents power flow equations and enforces transmission line capacities. In this constraint, $\mathbf{\Phi}$ is the binary vector, connecting the generator and load indices to the system nodes. Thus, $\mathbf{\Phi}\,\mathbf{p}$ is the power injection at nodes. Also, $\mathbf{S}$ is the shift factor matrix of the network. Constrain (1d) is to clear all bids within their submitted power range. The Lagrange multipliers in (1b) and (1c) provide the reference and shadow prices, respectively, to obtain the LMPs as:

$$\boldsymbol{\pi} = \lambda \mathbf{1} - \mathbf{S}^T \boldsymbol{\mu} \tag{5}$$

where $\boldsymbol{\mu} = \boldsymbol{\mu}^{+} - \boldsymbol{\mu}^{-}$. Note that, since the shift factors for all transmission lines with respect to the reference bus are always zero, for the reference Bus i, we have $\boldsymbol{\pi}_i = \lambda$.

The RTM market clearing optimization problem in presence of convergence bids can also be formulated as [9], [14], [29]:

$$\begin{aligned} \textbf{min} \quad & 0.5\, \mathbf{z}^T \mathbf{C}\, \mathbf{z} + \mathbf{d}^T \mathbf{z} && \text{(6a)} \\ \textbf{s.t.} \quad & \mathbf{1}^T \mathbf{z} + \mathbf{1}^T \mathbf{x} = \mathbf{1}^T \boldsymbol{l} \qquad : \delta && \text{(6b)} \\ & -\mathbf{c} \le \mathbf{S}(\boldsymbol{\Psi}\, \mathbf{x} + \boldsymbol{\Theta}\, \mathbf{z} - \boldsymbol{\Omega}\, \boldsymbol{l}) \le \mathbf{c} \quad : \boldsymbol{\eta}^{-}, \boldsymbol{\eta}^{+} && \text{(6c)} \\ & \mathbf{z}^{\min} \le \mathbf{z} \le \mathbf{z}^{\max} && \text{(6d)} \end{aligned}$$

where the optimization variables are the entries of vector $\mathbf{z}$.

Despite their structural similarities, there are two key differences between the DAM optimization problem in (1) and the RTM optimization problem in (6). First, as in practice, the RTM optimization problem in (6) is based on only physical bids but *not* convergence bids [9], [29], [30]. Second, due to the nature of the RTM, the physical demand bids are *not* allowed, instead, the constant loads are replaced, which are obtained from the ISO forecast algorithms, c.f. [31]. Therefore, the power balance constraint in (6b) includes only physical supply bids on its left-hand side and the forecasted load on its right-hand side. Also, note that, the physical supply bids cleared in the DAM appear as constant in the RTM.

The RTM LMPs are obtained similar to the DAM LMPs as

$$\boldsymbol{\sigma} = \delta \mathbf{1} - \mathbf{S}^T \boldsymbol{\eta}. \tag{7}$$

Although CBs do not appear in the RTM clearing process, because they *do* affect the cleared physical supply bids in the DAM, they indirectly have impact on the LMPs of the RTM.

Note that, the focus in this paper is on the following:

$$\boldsymbol{\Delta} \triangleq \boldsymbol{\pi} - \boldsymbol{\sigma}, \tag{8}$$

which is the difference between the LMPs in the DAM and the RTM. We are concerned with whether such difference *decreases* or *increases* at each bus under different CB scenarios.

B. Closed-Form Sensitivity Analysis

Given the mechanism of clearing CBs in Section III-A, next, we present a theorem to show how the cleared energy of a CB can affect the price gap at the bus where the CB is placed.

Theorem 1. *Consider a CB at Bus i. Without loss of generality, suppose Bus i is the reference bus and the CB is a profitable supply bid, whose cleared energy bid is denoted by* $\boldsymbol{v}_i$. *(a) The price gap* $\boldsymbol{\Delta}_i = \boldsymbol{\pi}_i - \boldsymbol{\sigma}_i$ *at Bus i is a piecewise linear function of the cleared CB* $\boldsymbol{v}_i$. *(b) The slope of such function, i.e., the right-sided partial derivative, is obtained as*

$$\begin{aligned} \frac{\partial \boldsymbol{\Delta}_i}{\partial \boldsymbol{v}_i} = \frac{\partial \boldsymbol{\pi}_i}{\partial \boldsymbol{v}_i} - \frac{\partial \boldsymbol{\sigma}_i}{\partial \boldsymbol{v}_i} &= \frac{-1}{\boldsymbol{1}^T \boldsymbol{h}} - \frac{1}{\boldsymbol{1}^T \boldsymbol{e}} \frac{1}{\boldsymbol{1}^T \boldsymbol{h}} (\boldsymbol{1}^T \hat{\boldsymbol{K}} \boldsymbol{h} - \boldsymbol{r} \hat{\boldsymbol{K}} \boldsymbol{h}) \\ &= -\frac{1}{\boldsymbol{1}^T \boldsymbol{h}} \frac{1}{\boldsymbol{1}^T \boldsymbol{e}} (\boldsymbol{1}^T \boldsymbol{e} + \boldsymbol{1}^T \hat{\boldsymbol{K}} \boldsymbol{h} - \boldsymbol{r} \hat{\boldsymbol{K}} \boldsymbol{h}), \end{aligned} \tag{9}$$

where

$$\boldsymbol{h} \triangleq \boldsymbol{\Lambda} \boldsymbol{1} - \boldsymbol{\Lambda} \boldsymbol{X}^T (\boldsymbol{X} \boldsymbol{\Lambda} \boldsymbol{X}^T)^{-1} \boldsymbol{X} \boldsymbol{\Lambda} \boldsymbol{1}, \tag{10}$$

$$\boldsymbol{e} \triangleq \boldsymbol{\Gamma} \boldsymbol{1} - \boldsymbol{\Gamma} \boldsymbol{Y}^T (\boldsymbol{Y} \boldsymbol{\Gamma} \boldsymbol{Y}^T)^{-1} \boldsymbol{Y} \boldsymbol{\Gamma} \boldsymbol{1}, \tag{11}$$

$$\boldsymbol{r} \triangleq \boldsymbol{1}^T \boldsymbol{\Gamma} \boldsymbol{Y} (\boldsymbol{Y} \boldsymbol{\Gamma} \boldsymbol{Y}^T)^{-1} \bar{\boldsymbol{R}} \boldsymbol{S} \hat{\boldsymbol{\Psi}}, \tag{12}$$

and $\boldsymbol{\Lambda}$, $\boldsymbol{\Gamma}$, $\hat{\boldsymbol{K}}$, $\boldsymbol{X}$, $\boldsymbol{Y}$, $\bar{\boldsymbol{R}}$, *and* $\hat{\boldsymbol{\Psi}}$ *are constant matrices that depend on cleared bids and admittance and congestion status of lines.*

The proof of Theorem 1 is given in the appendix A. This theorem can be rephrased also to explain the impact of placing profitable demand CBs by replacing $\mathbf{v}_i$ with $-\mathbf{w}_i$.

The expression in equation (20) can be used to explain how the difference between the DAM and RTM prices would change as a function of $\mathbf{v}_i$. In fact, if $\boldsymbol{\Delta}_i > 0$ in the absence of the CB, then a supply CB at Bus i reduces the price gap at that bus ***if and only if*** $\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i < 0$. For instance, in Example 1 in section II, if the supply CB is within the range $[0, 5]$ MWh, then $\boldsymbol{\Delta}_2 = 6.05$ and $\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i = -0.31 < 0$; Therefore, placing a supply CB at that bus closes the price gap. In contrast, in Example 2, we have $\boldsymbol{\Delta}_2 = 1.69$ and $\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i = +0.33 > 0$, which results in price divergence.

In summary, the impact of a CB on the LMP at the bus where the CB is placed depends on the coefficient shown in equation (9), which may enforce *convergence* (desirable) or *divergence* (undesirable) between the DAM and RTM prices. Therefore, compared to the impact of CBs in commodity and financial markets, the impact of CBs in nodal electricity markets is *significantly more complicated* that can sometimes create counter intuitive results. Such complex issues are the root causes for the concerns that are raised by multiple ISOs on CBs performance, as we mentioned in Section I.B.

Next, we will further investigate the results in Example 2 in Section II to understand under what operational conditions placing a CB results in price convergence or divergence.

C. Further Analysis on Theorem 1

Recall from Section I that some ISOs, such as the California ISO, have clearly indicated their assumption/expection that if a CB participant makes profit off of submitting a CB, then it must help closing the price gap. Mathematically speaking, this can be expressed as one of the following two cases:

- If $\boldsymbol{\Delta}_i > 0$, then the supply CB at Bus i is profitable. Accordingly, ISOs expect that $\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i \le 0$, such that the CB helps reducing the price gap.
- If $\boldsymbol{\Delta}_i < 0$, then the demand CB at Bus i is profitable. Accordingly, ISOs expect that $\partial \boldsymbol{\Delta}_i / \partial \mathbf{w}_i = -\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i \ge 0$, such that the CB helps reducing the price gap.

Consequently, ISOs always presume that $\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i$ is less than zero. This idea is based on a financial common sense that increasing (decreasing) a supply CB leads to deficit (surplus) demand in DAM and surplus (deficit) demand in RTM, which in turn decreases (increases) the DAM price and increases (decreases) the RTM price. However, such seemly common sense does *not* always hold in nodal electricity markets. For example, as we saw in Example 2 in Section II, $\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_i > 0$.

Corollary 1. *For any Bus i, if the DAM price* $\boldsymbol{\pi}_i$ *is initially higher than the RTM price* $\boldsymbol{\sigma}_i$, *i.e.,* $\boldsymbol{\Delta}_i > 0$ *, then a supply CB at Bus i results in price divergence* ***if and only if:***

$$\boldsymbol{r} \hat{\boldsymbol{K}} \boldsymbol{h} \ge \boldsymbol{1}^T \boldsymbol{e} + \boldsymbol{1}^T \hat{\boldsymbol{K}} \boldsymbol{h}. \tag{13}$$

Proof. Since $\mathbf{\Lambda}$ is a diagonal positive semidefinite matrix:

$$\mathbf{1}^T\mathbf{h} = \|\mathbf{\Lambda}^{0.5}\mathbf{1} - \mathbf{\Lambda}^{0.5}\mathbf{X}^T(\mathbf{X}\mathbf{\Lambda}\mathbf{X}^T)^{-1}\mathbf{X}\mathbf{\Lambda}\mathbf{1}\|_2^2 \geq 0 \tag{14}$$

Similarly, we can prove that $\mathbf{1}^T\mathbf{e} \geq 0$. From these, and also due to equations (9) and (13), we conclude that $\partial\mathbf{\Delta}_i/\partial\mathbf{v}_i > 0$. With this in mind, and since $\mathbf{\Delta}_i > 0$, if a supply CB is placed at Bus i, then the DAM and RTM prices diverge at Bus i. $\square$

The above Corollary can be used to indicate whether or not placing a CB at a bus results in price convergence at that bus. Interestingly, from (14) and (37) we always have $\partial\boldsymbol{\pi}_i/\partial\mathbf{v}_i \leq 0$, even though we may not always have $\partial\mathbf{\Delta}_i/\partial\mathbf{v}_i \leq 0$. In other words, what the ISOs expect is partly true; that is, increasing a supply (demand) CB at a bus decreases (increases) the DAM price at that bus. However, the impact of CBs on RTM prices, i.e., the sign of $\partial\boldsymbol{\sigma}_i/\partial\mathbf{v}_i$, may vary depending on how $\mathbf{1}^T\hat{\mathbf{K}}\mathbf{h}$ and $\mathbf{r}\hat{\mathbf{K}}\mathbf{h}$ may stand compared to each other.

Corollary 2. *If neither DAM nor RTM experience congestion, then placing a profitable CB at a bus is guaranteed to result in convergence between the DAM and RTM prices at that bus.*

Proof. Since the power grid is not congested in both markets, we have $\bar{\mathbf{D}} = \bar{\mathbf{R}} = \mathbf{0}$. From this, together with the definition of $\mathbf{X}$ and $\mathbf{Y}$ in (35) and (40), we have $\mathbf{X} = \mathbf{Y} = \mathbf{0}$. By substituting these terms in (10), (11), and (12), we conclude that

$$\mathbf{h} = \mathbf{\Lambda}\mathbf{1}, \qquad \mathbf{e} = \mathbf{\Gamma}\mathbf{1}, \qquad \mathbf{r} = \mathbf{0}. \tag{15}$$

By substituting the above terms in (13), we have

$$\mathbf{1}^T\mathbf{e} + \mathbf{1}^T\hat{\mathbf{K}}\mathbf{h} = \mathbf{1}^T\mathbf{\Gamma}\mathbf{1} + \mathbf{1}^T\hat{\mathbf{K}}\mathbf{\Lambda}\mathbf{1} \geq \mathbf{r}\hat{\mathbf{K}}\mathbf{h} = 0. \tag{16}$$

where the inequality is due to $\mathbf{\Lambda}$ and $\mathbf{\Gamma}$ being diagonal positive semi-definite matrices and $\hat{\mathbf{K}}$ comprising basis vectors. $\square$

The above Corollary explains the price convergence in Example 1. Note that, if the grid is not congested in either market, then the electricity market turns into a typical two-settlement financial market, in which CBs always reduce the price gap and enhance market efficiency. As we see next, transmission line congestion is a key factor to determine whether a CB can create price convergence or divergence.

Corollary 3. *Suppose all marginal, i.e., price-maker, bids in the DAM are of type physical supply. Also, suppose the set of all congested transmission lines in the RTM is a subset of or equal to the set of all congested transmission lines in the DAM. In this case, the CBs are always help the market efficiency by reducing the price gap between DAM and RTM.*

Proof. Since the set of congested transmission lines in RTM is a subset of that in DAM, we can construct matrix $\bar{\mathbf{R}}$ using a subset of the rows in matrix $\bar{\mathbf{D}}$, i.e., $\bar{\mathbf{R}} = \mathbf{\Upsilon}\bar{\mathbf{D}}$. Also, since all marginal bids in the DAM are assumed to be of type physical supply, we have $\hat{\mathbf{K}} = \mathbf{I}$ and $\hat{\mathbf{\Psi}} = \hat{\mathbf{\Phi}}_{-i}$. Therefore, we have

$$\mathbf{1}^T\hat{\mathbf{K}}\mathbf{h} = \mathbf{1}^T\mathbf{h} \tag{17}$$

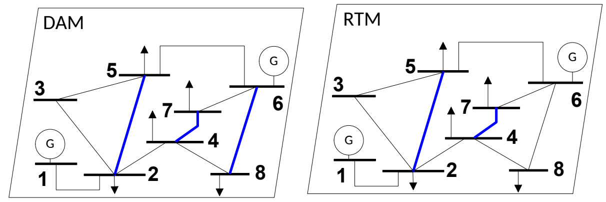

Fig. 6. An example for a transmission line congestion configuration that satisfies the conditions in Corollary 3. Congested lines are shown in black.

and

$$\begin{aligned}\mathbf{r}\hat{\mathbf{K}}\mathbf{h} &= \mathbf{r}\mathbf{h}\\ &= (\mathbf{1}^T\mathbf{\Gamma}\mathbf{Y}(\mathbf{Y}\mathbf{\Gamma}\mathbf{Y}^T)^{-1}\mathbf{\Upsilon}\bar{\mathbf{D}}\mathbf{S}\hat{\mathbf{\Phi}}_{-i})\\ &\quad \times (\mathbf{\Lambda}\mathbf{1} - \mathbf{\Lambda}\mathbf{X}^T(\mathbf{X}\mathbf{\Lambda}\mathbf{X}^T)^{-1}\mathbf{X}\mathbf{\Lambda}\mathbf{1})\\ &= \mathbf{1}^T\mathbf{\Gamma}\mathbf{Y}(\mathbf{Y}\mathbf{\Gamma}\mathbf{Y}^T)^{-1}\mathbf{\Upsilon}\\ &\quad \times (\mathbf{X}\mathbf{\Lambda}\mathbf{1} - \mathbf{X}\mathbf{\Lambda}\mathbf{X}^T(\mathbf{X}\mathbf{\Lambda}\mathbf{X}^T)^{-1}\mathbf{X}\mathbf{\Lambda}\mathbf{1}) = 0.\end{aligned} \tag{18}$$

After substituting (17) and (18) in (13), we have:

$$\mathbf{1}^T\mathbf{e} + \mathbf{1}^T\hat{\mathbf{K}}\mathbf{h} = \mathbf{1}^T\mathbf{e} + \mathbf{1}^T\mathbf{h} \geq \mathbf{r}\hat{\mathbf{K}}\mathbf{h} = 0, \tag{19}$$

where the inequality is due to $\mathbf{1}^T\mathbf{h} \geq 0$ and $\mathbf{1}^T\mathbf{e} \geq 0$. $\square$

Fig. 6 shows an example of a transmission line congestion configuration that satisfies the condition in Corollary 3. There are three congested transmission lines in the DAM and two congested transmission lines in the RTM. Importantly, the set of congested transmission lines in the RTM is a subset of the set of congested transmission lines in the DAM. Thus, from Corollary 3, profitable CBs are guaranteed to help close the price gap between the DAM and RTM in the situation in this example. Note that, the condition in Corollary 3 does *not* hold in Example 2 in Section II; therefore, Corollary 3 sheds lights on why we observed price divergence (instead of price convergence) between the DAM and RTM in Example 2.

Interestingly, the conditions in Corollary 3 often *do* hold in practice, at least in the current market conditions. For instance, the current demand bids are almost solely price-taker [31]. Also, the total cleared demand and supply CBs are currently only a small portion ($\sim$12%) of the cleared physical supply bids [10]. Therefore, most marginal bids are currently of type physical supply. This may explain why the ISOs have *not yet* encountered *severe undesirable divergence* in practice, even though they have already observed *some* traces of such results that have raised concerns, see Section I. However, many ISOs, such as in California, have reported plans to increase the limits on CBs, e.g., see [9]. If that happens, it may eventually lead to conditions where the use of CBs can create major price divergence, which may lead to severe market efficiency issues.

To summarize, Fig. 7 shows the sufficient conditions; explained in Corollaries 1, 2, and 3; to cause price convergence or price divergence when a CB is cleared at a bus. The conditions dependent highly on transmission line congestion configuration in both DAM and RTM. Accordingly, as a first step, a market operator may use the sufficient conditions in Corollary 2 and Corollary 3 to see if a CB improves the price gap in the market by simply checking the transmission lines congestion configuration of DAM and RTM. If not, the

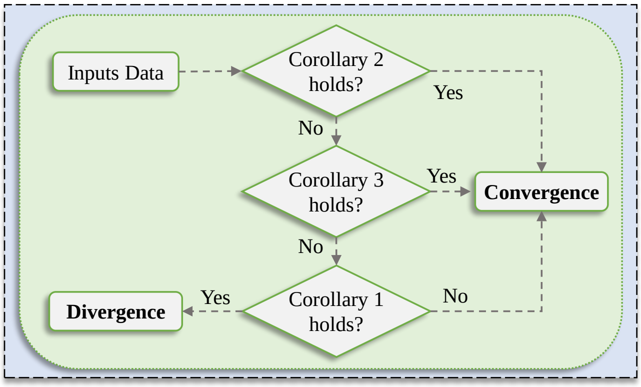

Fig. 7. Summary of the Corollaries 1, 2, and 3 to guarantee price convergence or price divergence when a CB is placed in the electricity market.

market operator may then use the condition in Corollary 1 to check whether a CB may cause price convergence or even price divergence between the DAM and RTM.

IV. Extended Analysis

The analysis in Section III provided us with a set of tools to investigate the impact of placing a CB on creating price convergence/divergence at the bus where the CB is placed. These tools can be used to further expand our analysis to also address the more general cases, as we discuss in this section.

A. Overall Impact Analysis

In practice, several CBs can be placed at different buses. Placing a CB at a bus influences the prices not only at the bus where it is placed but also at *other buses*. The collective impact of the CBs at different buses determines the overall convergence or divergence properties at each bus.

Theorem 2. *Consider a CB at bus i. Without loss of generality, suppose the CB is a profitable supply bid ($\boldsymbol{\Delta}_i > 0$), whose cleared energy bid is denoted by $\boldsymbol{v}_i$. The bus price gap vector is a piecewise linear function of the cleared CB $\boldsymbol{v}_i$ and the slops of such vector function, denoted by $\boldsymbol{n}_i$, are obtained as*

$$\boldsymbol{n}_i \triangleq \frac{\partial \boldsymbol{\Delta}}{\partial \boldsymbol{v}_i} = \frac{\partial \boldsymbol{\pi}}{\partial \boldsymbol{v}_i} - \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{v}_i}, \tag{20}$$

where

$$\frac{\partial \boldsymbol{\pi}}{\partial \boldsymbol{v}_i} = \begin{bmatrix} -\boldsymbol{1}^T \\ \bar{\boldsymbol{D}}\boldsymbol{S} \end{bmatrix}^T \boldsymbol{H}^{-1} \begin{bmatrix} 1 \\ \boldsymbol{0} \end{bmatrix} \tag{21}$$

$$\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{v}_i} = \begin{bmatrix} \boldsymbol{1}^T \\ -\bar{\boldsymbol{R}}\boldsymbol{S} \end{bmatrix}^T \boldsymbol{E}^{-1} \begin{bmatrix} \boldsymbol{1}^T \\ \boldsymbol{R}\boldsymbol{S}\hat{\boldsymbol{\Psi}} \end{bmatrix} \hat{\boldsymbol{K}}\boldsymbol{\Lambda} \begin{bmatrix} \boldsymbol{1}^T \\ -\boldsymbol{X} \end{bmatrix}^T \boldsymbol{H}^{-1} \begin{bmatrix} 1 \\ \boldsymbol{0} \end{bmatrix} \tag{22}$$

and

$$\boldsymbol{H} \triangleq \begin{bmatrix} \boldsymbol{1}^T \\ \boldsymbol{X} \end{bmatrix} \boldsymbol{\Lambda} \begin{bmatrix} \boldsymbol{1}^T \\ -\boldsymbol{X} \end{bmatrix}^T, \quad \boldsymbol{E} \triangleq \begin{bmatrix} \boldsymbol{1}^T \\ \boldsymbol{Y} \end{bmatrix} \boldsymbol{\Gamma} \begin{bmatrix} \boldsymbol{1}^T \\ -\boldsymbol{Y} \end{bmatrix}^T. \tag{23}$$

The proof of Theorem 2 is given in Appendix B. This theorem can be used to explain the impact of a CB on the difference between the DAM and RTM prices at *all* buses. Specifically, vector $\mathbf{n}_i$ expresses the sensitivity of the price gap vector to a cleared CB submitted at Bus i in the market. The subscript is added to the sensitivity vectors to indicate the bus at which the CB is submitted. For instance, row j of vector $\mathbf{n}_i$, denoted by $\mathbf{n}_i^j$, refers to the sensitivity of the price gap at Bus j once a CB is placed at Bus i. Note that, Theorem 2 is a generalization of Theorem 1 in Section III.

First, consider Example 1 in Section II. From the analysis in Theorem 2, one can show that placing the supply CB at Bus 2 results in the following price gap sensitivity vector:

$$\mathbf{n}_2 = \begin{bmatrix} -0.31 & -0.31 & -0.31 \end{bmatrix}^T. \tag{24}$$

Note that, since no transmission line is congested in Example 1, neither in DAM nor RTM, the CB has an identical impact on LMPs at all buses. In fact, in Example 1, placing a CB at *any* bus can result in reducing the price gap at *all* buses.

Next, consider Example 2 in Section II, where we observed price divergence. From Theorem 2, placing a supply CB at Bus 2 results in the following price gap sensitivity vector:

$$\mathbf{n}_2 = \begin{bmatrix} -0.95 & +0.33 & -0.31 \end{bmatrix}^T. \tag{25}$$

Since $\boldsymbol{\Delta} > 0$, only a negative entry in matrix $\mathbf{n}_2$ indicates price convergence at the bus corresponding to that entry. A *positive* entry indicates price *divergence*. Therefore, from (25), placing a CB at Bus 2 in Example 2 results in decreasing the price gap at buses 1 and 3, but increasing the price gap at Bus 2.

The overall CB sensitivity matrix for *all* CBs is obtained as

$$\mathbf{\Pi} = [\partial \boldsymbol{\Delta}_i / \partial \mathbf{v}_j]_{i,j} \tag{26}$$

This matrix, together with the incidence matrices $\mathbf{V}$ and $\mathbf{W}$, can be used to analyze the overall impact of all CBs at each bus under various network operating condition. For instance, for small identical deviations of all CBs, whether of type supply or demand, the entries of the vector $\mathbf{\Pi}(\mathbf{V}-\mathbf{W})$ indicate whether the DAM and RTM prices converge or diverge at each bus. In this case, if $\boldsymbol{\Delta} > 0$, then any negative (positive) entry indicates price convergence (divergence) in the corresponding bus.

CBs can cause price convergence at some buses, and price divergence at some other buses. The final *overall* outcome of the market is a balance between the convergence and divergence forces caused by different cleared CBs. It is natural for ISOs to want to know how and why such balance is shaped. They may also want to know which CBs are creating the divergence forces to possibly block them. The above analysis provides ISOs with proper tools to address all these challenges.

B. Stochastic Analysis

Recall from Section II that the observations that we made in the deterministic cases in Examples 1 and 2 can be made also in the stochastic case in Example 3. Similarly, the analysis in Sections III and IV are also applicable to the case where certain parameters of the power grid or market are random.

Suppose the randomness in load and generation levels are captured using S random scenarios and the probability for each random scenario s is ϕ^s, where $s = 1, \ldots, S$. Under each scenario s, the price gap in (8) is obtained as

$$\boldsymbol{\Delta}^s \triangleq \boldsymbol{\pi} - \boldsymbol{\sigma}^s. \tag{27}$$

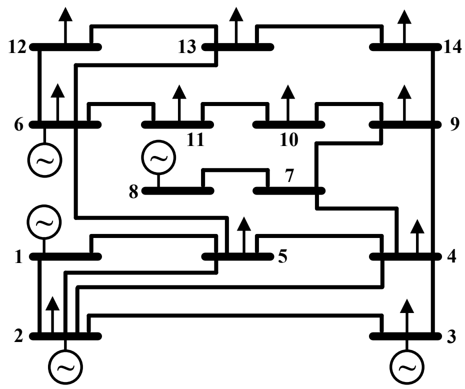

Fig. 8. The IEEE 14-bus standard system that is studied in Section V.

TABLE I
THE PRICE DATA OF IEEE 14-BUS SYSTEM WITHOUT CB.

Bus	$\boldsymbol{\pi}_i$ (\$)	$\boldsymbol{\sigma}_i$ (\$)	$\boldsymbol{\Delta}_i$ (\$)
1	38.76	28.82	+ 9.94
2	41.43	30.52	+10.91
3	42.93	35.45	+ 7.48
4	44.21	39.64	+ 4.57
5	45.08	22.56	+22.52
6	44.79	28.35	+16.44
7	44.37	36.55	+ 7.82
8	44.37	36.55	+ 7.82
9	44.45	34.93	+ 9.52
10	44.51	33.81	+10.70
11	44.64	31.15	+13.49
12	44.76	28.87	+15.89
13	44.74	29.27	+15.47
14	44.58	32.47	+12.11

Note that, superscript s appears only for the RTM price $\boldsymbol{\sigma}$, because, in practice, the uncertainty is most considerable in the real-time market. Nevertheless, the analysis would be the same if we also add superscript s also to the DAM price $\boldsymbol{\pi}$.

Next, we apply the analysis in Theorem 1, and obtain the change in the price gap at bus i under random scenario s as

$$\partial \boldsymbol{\Delta}_i^s / \mathbf{v}_i, \tag{28}$$

using mathematical expressions similar to (7)-(10), where in each case the notations are updated according to realization of the random parameters under random scenario s.

As in Example 3, the key concern here is to figure out the probability for the DAM and RTM prices to converge (or diverge). The probability of price convergence is obtained as

$$\sum_{s=1}^{S} \phi^s \mathbb{I}\left(\partial \boldsymbol{\Delta}_i^s / \partial \mathbf{v}_i > 0,\right), \tag{29}$$

where $\mathbb{I}(\cdot)$ is the indicator function, which is 1 if the inequality holds or 0 otherwise. The probability of price divergence is

$$\sum_{s=1}^{S} \phi^s \mathbb{I}\left(\partial \boldsymbol{\Delta}_i^s / \partial \mathbf{v}_i < 0,\right). \tag{30}$$

A similar analysis can also be done for the overall impact assessment in Section IV.A; where for each random scenario s, we obtain the overall CB sensitivity matrix for all CBs as

$$\boldsymbol{\Pi}^s = \left[\partial \boldsymbol{\Delta}_i^s / \partial \mathbf{v}_j\right]_{i,j}. \tag{31}$$

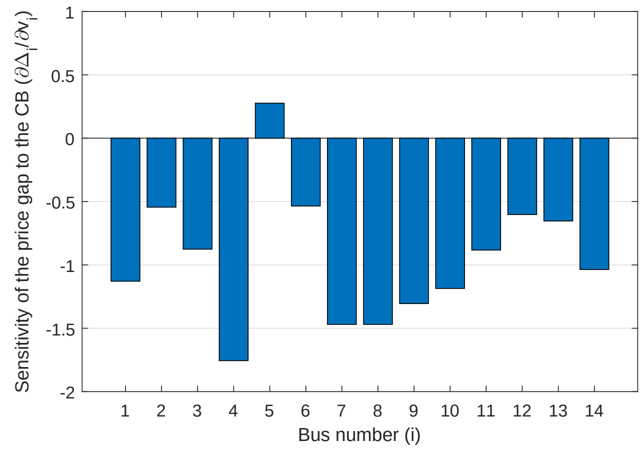

Fig. 9. Price difference sensitivity at different buses to a CB at the same bus.

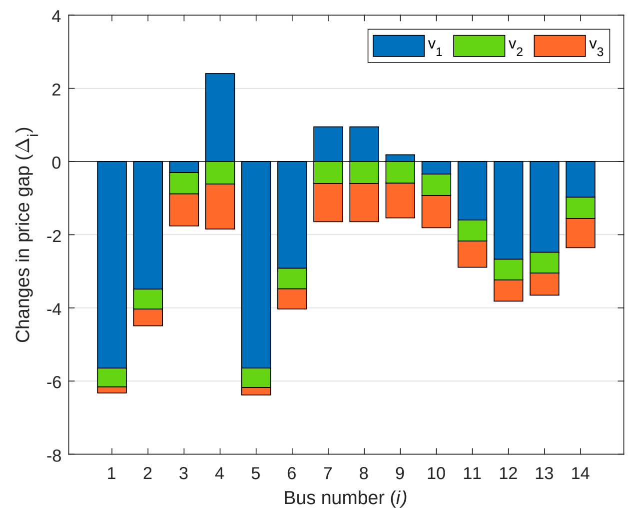

Fig. 10. The changes in price difference at different buses caused by placing supply CBs at buses 1, 2, and 3 ($\mathbf{v}_1$= 5 MWh, $\mathbf{v}_2$= 1 MWh, $\mathbf{v}_3$= 1 MWh).

The probability of price convergence and the probability of price divergence can then can calculated at each bus using similar formulations as in (29) and (30), respectively.

V. ADDITIONAL CASE STUDIES

This section provides additional case studies based on the IEEE 14-Bus test system, as shown in Fig. 8, c.f. [32]. Only active power of loads and generators is considered. The transmission lines resistances are neglected. Suppose generators G_1, G_2 and G_3 participate in the DAM and their supply bids are $\alpha_1 = 0.4$, $\beta_1 = 14$, $\alpha_2 = 0.5$, $\beta_2 = 12$, $\alpha_3 = 0.3$, and $\beta_3 = 17$. Generators G_6 and G_8 participate in the RTM and their supply offers are $\alpha_6 = 0.9$, $\beta_6 = 15$, $\alpha_8 = 0.8$, and $\beta_8 = 18$. It is assumed that the capacity of transmission lines between buses 1 and 5 and buses 4 and 5 are 31 MW and 23 MW, respectively. Furthermore, it is assumed that 80% of the loads is cleared in the DAM by submitting self-schedule demand bids. The remaining 20% of the loads are cleared in the RTM. In the absence of CBs, the LMPs of the DAM and RTM and the price differences are as in Table I. In this case, the line between buses 1 and 5 is congested in the DAM and the line between buses 4 and 5 is congested in the RTM.

As shown in Table I, the DAM price $\boldsymbol{\pi}$ is greater than the RTM price $\boldsymbol{\sigma}$ in *all* buses. Therefore, submitting supply CBs are always profitable, thus justified, for an independent market participant. Fig. 9 shows the sensitivity of the price gap at each bus to the supply CB submitted at that bus. We can see that, the sensitivity of the price difference is positive at Bus 5, i.e., $\partial \boldsymbol{\Delta}_5/\partial \mathbf{v}_5 > 0$, but negative at other buses, i.e., $\partial \boldsymbol{\Delta}_i/\partial \mathbf{v}_i < 0$, for all $i \neq 5$. At Bus 5, the sensitivity of DAM and RTM prices to a CB at Bus 5 are -0.73 and -1.0 respectively; thus the overall sensitivity of price gap to the CB is $+0.27 > 0$. Therefore, as we place a supply CB at Bus 5, the prices in both the DAM and RTM decrease; however the rate of decreasing is higher for the price in the DAM. Therefore, placing a supply CB at Bus 5 causes divergence across the DAM and the RTM prices at Bus 5. In fact, the condition for price divergence in Corollary 1 holds at this bus. This is an interesting observation, as it confirms that the analysis in this paper can be applied to larger networks with several buses and transmission lines.

Next, suppose three supply CBs are placed at buses 1, 2 and 3. The supply CBs at buses 1, 2, and 3 are 5 MWh, 1 MWh, and 1 MWh, respectively, i.e., $\mathbf{v}_1 = 5,\ \mathbf{v}_2 = 1,\ \mathbf{v}_3 = 1$. Fig. 10 shows the impact of each CB on the price difference at buses 1 to 14. The change in price gap caused by each CB is distinguished using a different color. The overall change in the price gap at each bus is the summation of all three changes. For example, consider the bars at Bus 1. The largest change in price is caused by $\mathbf{v}_1$. The second largest change in price is caused by $\mathbf{v}_2$. The smallest change in price is caused by $\mathbf{v}_3$. However, all changes are on the negative side. That means, all the three CBs result in price convergence at Bus 1.

An interesting observation is that the supply CB at Bus 1 increases the price gap at Bus 4 and also at Bus 7, 8 and 9; such increase is only *partially compensated* by the supply CB at Bus 2 and Bus 3 for the price gap at bus 4. Thus, the overall impact of the CBs is to create *price divergence* at Bus 4. In contrast, the increase in price gap at Bus 7, 8 and 9 caused by the CB at Bus 1 is *fully compensated* by the supply CB at Bus 2 and Bus 3. The overall impact of the CBs is to create *price convergence* at Bus 7. From Fig. 10, the price divergence that is caused by a CB at a bus can be compensated by the price convergence that is caused by a CB at another bus. In such cases, the *overall impact* of all CBs could be to decrease or increase the price gap. Of course, if the overall impact of CBs is to create price divergence at a bus, then the ISO may choose to remove those CBs that are counter productive and cause an increase (instead of a decrease) in the the price gap.

Last but not least, we study the stochastic case by considering $S = 100$ random scenarios for two uncertain net loads at buses 4 and 13. The minimum, maximum and average net load at bus 4 is 14.12 MWh, 52.96 MWh, and 38.24 MWh, respectively. These numbers at bus 13 are 3.98 MWh, 14.95 MWh, and 10.8 MWh, respectively. For each random scenario s, we obtain the diagonal entries of matrix $\mathbf{\Pi}^s$ in (31), i.e., the price sensitivity at each bus when the CB is placed at the same bus. We also calculate the probability of price convergence by using the expression in (29). At bus 5, the probability of price convergence when a CB is placed at this bus is only 40%. This may suggest that the ISO should not allow placing a CB at this bus. At all other buses, price convergence is guaranteed.

VI. Conclusions

Based upon intuitive case studies and mathematical analysis, this paper explained how and why the structural characteristics of the power grid can affect the performance of CBs in nodal electricity markets. First, a fundamental sensitivity analysis was done to explain how the prices in the DAM and RTM are affected by CBs. Closed-form sensitivity models were obtained with respect to factors such as transmission line congestion configuration. Accordingly, it was investigated *under what conditions* and *to what extent* the outcome of CBs would be price convergence (desirable) or price divergence (undesirable). It was proved that, if no transmission line is congested, then any profitable CB helps the system by decreasing the price gap between DAM and RTM. It was also proved that, if the set of congested transmission lines in RTM is a subset of that in DAM, then the price convergence is still guaranteed by profitable CBs. For the stochastic case, the *probability* of price convergence (or divergence) was obtained when we are uncertain about some system parameters. Finally, a methodology was presented to explain how the *aggregate impact* of multiple CBs may increase or decrease the price gap between DAM and RTM at each bus. The results in this paper can enhance our understanding of CBs in nodal electricity markets. They can also help ISOs explain some of their recent observations, such as what we quoted in Section I.

Appendix

A. Proof of Theorem 1

Since Bus i is taken as the reference bus, the price gap at Bus i is obtained as $\boldsymbol{\Delta}_i = \lambda - \delta$. Let $\mathbf{v}_{-i}$ denote the set of all supply CBs other than $\mathbf{v}_i$. We can now define:

$$\mathbf{p}_{-i} \triangleq \begin{bmatrix} \mathbf{x} & \mathbf{y} & \mathbf{v}_{-i} & \mathbf{w} \end{bmatrix}^{\mathrm{T}} \tag{32}$$

as the optimal solution of the problem in (1) other than $\mathbf{v}_i$. We also define $\mathbf{A}_{-i}$, $\mathbf{b}_{-i}$, $\mathbf{p}^{\min}_{-i}$, $\mathbf{p}^{\max}_{-i}$, and $\mathbf{\Phi}_{-i}$ by removing row i and/or column i from $\mathbf{A}$, $\mathbf{b}$, $\mathbf{p}^{\min}$, $\mathbf{p}^{\max}$, and $\mathbf{\Phi}$.

Suppose we decompose vector $\mathbf{p}_{-i}$ into vector $\bar{\mathbf{p}}_{-i}$ for entries that are *binding* by one or both of the two constraints in (1d) and vector $\hat{\mathbf{p}}_{-i}$ for entries that are *not* binding by any of these two constraints. Similarly, we define $\bar{\mathbf{A}}_{-i}$, $\hat{\mathbf{A}}_{-i}$, $\bar{\mathbf{b}}_{-i}$, $\hat{\mathbf{b}}_{-i}$, $\bar{\mathbf{p}}^{\min}_{-i}$, $\hat{\mathbf{p}}^{\min}_{-i}$, $\bar{\mathbf{p}}^{\max}_{-i}$, $\hat{\mathbf{p}}^{\max}_{-i}$, $\hat{\mathbf{\Phi}}_{-i}$, and $\bar{\mathbf{\Phi}}_{-i}$. We also decompose vector $\boldsymbol{\mu}$ into vector $\bar{\boldsymbol{\mu}}$ for the Lagrange multipliers corresponding to the *binding* constraints in (1c). Suppose $\bar{\mathbf{D}}$ denotes a row-reduced identity matrix, i.e., an identity matrix with the same size of matrix $\mathbf{S}$ after we eliminate its rows that correspond to the non-binding transmission line capacity constraints. We also define $\hat{\boldsymbol{\mu}}$ as the Lagrange multipliers that are *not* binding by any of the transmission line capacity constraints. We can show that the convex optimization problem in (1) is *equivalent* to the following problem [33], [34]:

$$\min_{\hat{\mathbf{p}}_{-i}} \quad 0.5\, \hat{\mathbf{p}}_{-i}^T\, \hat{\mathbf{A}}_{-i}\, \hat{\mathbf{p}}_{-i} + \hat{\mathbf{b}}_{-i}^T\, \hat{\mathbf{p}}_{-i} \tag{33a}$$

$$\text{s.t.} \quad \mathbf{1}^T \hat{\mathbf{p}}_{-i} + \mathbf{1}^T \bar{\mathbf{p}}_{-i} + \mathbf{v}_i = 0 \qquad : \lambda \tag{33b}$$

$$\bar{\mathbf{D}}\mathbf{S}\left(\hat{\mathbf{\Phi}}_{-i}\, \hat{\mathbf{p}}_{-i} + \bar{\mathbf{\Phi}}_{-i}\, \bar{\mathbf{p}}_{-i}\right) = \bar{\mathbf{D}}\mathbf{c} \qquad : \bar{\boldsymbol{\mu}}, \tag{33c}$$

where $\mathbf{v}_i$ and $\bar{\mathbf{p}}_{-i}$ are fixed at their optimal values but $\hat{\mathbf{p}}_{-i}$ is variable. The objective function in (31a) includes the terms that depend on $\hat{\mathbf{p}}_{-i}$. Because Bus i is the reference bus, $\mathbf{S\Phi p} = \mathbf{S\Phi}_{-i}\mathbf{p}_{-i}$. Also, only those line capacity constraints that are binding at the optimal solution of problem (1) are used here.

Because (33) is a convex quadratic program, it can be solved by equivalently solving its KKT conditions [33], [34], which is a system of linear equations over $\hat{\mathbf{p}}_{-i}$, λ and $\bar{\boldsymbol{\mu}}$:

$$\mathbf{\Lambda}^{-1}\hat{\mathbf{p}}_{-i} + \hat{\mathbf{b}}_{-i} = \begin{bmatrix}\mathbf{1}^T \\ -\mathbf{X}\end{bmatrix}^T \begin{bmatrix}\lambda \\ \bar{\boldsymbol{\mu}}\end{bmatrix} \tag{34a}$$

$$\begin{bmatrix}\mathbf{1}^T \\ \mathbf{X}\end{bmatrix} \hat{\mathbf{p}}_{-i} = \mathbf{n} - \begin{bmatrix}1 \\ \mathbf{0}\end{bmatrix} \mathbf{v}_i. \tag{34b}$$

where

$$\mathbf{X} \triangleq \bar{\mathbf{D}}\mathbf{S}\hat{\mathbf{\Phi}}_{-i}, \quad \mathbf{\Lambda} \triangleq \hat{\mathbf{A}}_{-i}^{-1}, \tag{35}$$

$$\mathbf{n} \triangleq \begin{bmatrix} -\mathbf{1}^T\bar{\mathbf{p}}_{-i} \\ \bar{\mathbf{D}}\mathbf{c} - \bar{\mathbf{D}}\mathbf{S}\,\bar{\mathbf{\Phi}}_{-i}\,\bar{\mathbf{p}}_{-i}\end{bmatrix}. \tag{36}$$

The coefficients in (34) do not change as long as the binding constraints do not change at the solution of problem (1). If a binding constraint becomes unbinding or vice versa, then some or all matrices $\mathbf{\Lambda}$, $\hat{\mathbf{b}}_{-i}$, $\mathbf{X}$, and $\mathbf{n}$ may change, but keeping the relationship between variables, i.e., λ and $\mathbf{v}_i$, linear. Thus, the overall relationship is piecewise linear. From (34), we have:

$$\partial\lambda/\partial\mathbf{v}_i = -1/\mathbf{1}^T\mathbf{h}. \tag{37}$$

where $\mathbf{h}$ is defined in (10) in Section III.B.

The analysis of the RTM prices is similar. We can first show that problem (6) is *equivalent* to the following problem:

$$\min_{\hat{\mathbf{z}}} \quad 0.5\,\hat{\mathbf{z}}^T\hat{\mathbf{C}}\,\hat{\mathbf{z}} + \hat{\mathbf{d}}^T\hat{\mathbf{z}} \tag{38a}$$

$$\text{s.t.} \quad \mathbf{1}^T\hat{\mathbf{z}} + \mathbf{1}^T\bar{\mathbf{z}} + \mathbf{1}^T\hat{\mathbf{x}} + \mathbf{1}^T\bar{\mathbf{x}} = \mathbf{1}^T l \qquad : \delta \tag{38b}$$

$$\bar{\mathbf{R}}\mathbf{S}(\bar{\mathbf{\Psi}}\,\bar{\mathbf{x}} + \hat{\mathbf{\Psi}}\,\hat{\mathbf{x}} + \bar{\mathbf{\Theta}}\,\bar{\mathbf{z}} + \hat{\mathbf{\Theta}}\,\hat{\mathbf{z}} - \mathbf{\Omega}\,l) = \bar{\mathbf{R}}\mathbf{c} \quad : \bar{\boldsymbol{\eta}}, \tag{38c}$$

where $\hat{\mathbf{x}} = \hat{\mathbf{K}}\hat{\mathbf{p}}_{-i}$ and $\bar{\mathbf{x}} = \bar{\mathbf{K}}\bar{\mathbf{p}}_{-i}$. Again, since (38) is a convex quadratic program, it can be solved by equivalently solving its KKT optimality conditions, which in this case are a system of linear equations over $\hat{\mathbf{z}}$, δ and $\bar{\boldsymbol{\eta}}$. The linear coefficient of δ as a function of $\mathbf{v}_i$ is obtained as

$$\frac{\partial\delta}{\partial\mathbf{v}_i} = \frac{\partial\delta}{\partial\hat{\mathbf{p}}_{-i}} \cdot \frac{\partial\hat{\mathbf{p}}_{-i}}{\partial\mathbf{v}_i} = \frac{1}{\mathbf{1}^T\mathbf{e}}\frac{1}{\mathbf{1}^T\mathbf{h}}(\mathbf{1}^T\hat{\mathbf{K}}\mathbf{h} - \mathbf{r}\hat{\mathbf{K}}\mathbf{h}) \tag{39}$$

where

$$\mathbf{Y} \triangleq \bar{\mathbf{R}}\mathbf{S}\hat{\mathbf{\Theta}}, \quad \mathbf{\Gamma} = \hat{\mathbf{C}}^{-1}, \tag{40}$$

and $\mathbf{e}$ and $\mathbf{r}$ are as in (11) and (12). Since λ and δ are both piecewise linear function of $\mathbf{v}_i$, their difference, i.e., $\mathbf{\Delta}_i$ is also a piecewise linear function of $\mathbf{v}_i$. The slope of such function is derived as in (9) by subtracting (39) from (37). ■

B. Proof of Theorem 2

From equation (5), the linear coefficient of DAM price function $\boldsymbol{\pi}$ with respect to CB $\mathbf{v}_i$ can be obtained as

$$\frac{\partial\boldsymbol{\pi}}{\partial\mathbf{v}_i} = \frac{\partial\lambda}{\partial\mathbf{v}_i}\mathbf{1} - (\bar{\mathbf{D}}\mathbf{S})^T\frac{\partial\bar{\boldsymbol{\mu}}}{\partial\mathbf{v}_i}. \tag{41}$$

Then, by replacing λ and $\bar{\boldsymbol{\mu}}$ from the KKT conditions of the DAM optimization in (34), we can achieve $\partial\boldsymbol{\pi}/\partial\mathbf{v}_i$. Similarly, at RTM the below equation can be derived from (7):

$$\frac{\partial\boldsymbol{\sigma}}{\partial\mathbf{v}_i} = \left(\frac{\partial\delta}{\partial\hat{\mathbf{p}}_{-i}}\mathbf{1} - (\bar{\mathbf{R}}\mathbf{S})^T\frac{\partial\bar{\boldsymbol{\eta}}}{\partial\hat{\mathbf{p}}_{-i}}\right)\frac{\partial\hat{\mathbf{p}}_{-i}}{\partial\mathbf{v}_i}. \tag{42}$$

Finally, $\partial\boldsymbol{\sigma}/\partial\mathbf{v}_i$ can be obtained as shown in equation (22) by using equation (42) and replacing $\partial\hat{\mathbf{p}}_{-i}/\partial\mathbf{v}_i$ and $\partial\delta/\partial\hat{\mathbf{p}}_{-i}$ from KKT conditions of DAM and RTM, respectively. ■

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Mahdi Kohansal (S'14-M'18) received his M.Sc. degree in electrical engineering from Amirkabir University of Technology, Tehran, Iran, in 2013 and his Ph.D. degree in electrical and computer engineering from the University of California, Riverside, CA, USA, in 2018. He is currently a data scientist focusing on electricity markets at Advanced Microgrid Solutions (AMS), San Francisco, CA, USA. His Research interests include the application of mathematical optimization in power systems, electricity markets, and smart power grids.

Ehsan Samani (S'16) received the B.Sc. and M.Sc. degrees in electrical engineering from the University of Tehran, Tehran, Iran, in 2016 and 2018, respectively. He is working toward the Ph.D. degree in electrical engineering with the University of California, Riverside, CA, USA. His research interests include electricity market, application of mathematical optimization in power systems, and smart grids. Ehsan is currently working on optimization models and data analysis of U.S. electricity markets specially CAISO.

Hamed Mohsenian-Rad (S'04-M'09-SM'14-F'20) received the Ph.D. degree in electrical and computer engineering from the University of British Columbia, Vancouver, BC, Canada, in 2008. He is currently an Associate Professor of electrical engineering with the University of California, Riverside, CA, USA. His research interests include modeling, data analysis, and optimization of power systems and smart grids. He was the recipient of the National Science Foundation CAREER Award, the Best Paper Award from the IEEE Power and Energy Society General Meeting, and the Best Paper Award from the IEEE Int. Conference on Smart Grid Communications. He is an Editor of the IEEE TRANSACTIONS ON SMART GRID and the IEEE POWER ENGINEERING LETTERS.