Event Location Identification in Distribution Networks Using Waveform Measurement Units

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Abstract—A new method is proposed to identify the location of events in distribution networks using data from waveform measurement units (WMUs). When an event occurs, WMUs provide GPS-synchronized measurements of voltage and current waveforms in time-domain that are captured during the event. Given such data, the proposed method identifies the bus number where the event occurred. Here, an event is defined rather broadly as any major change in the voltage or current waveforms. An event may have various causes, such as capacitor bank switching, load switching, a minor fault, etc. The first step in the proposed method is to characterize the oscillatory modes of the captured waveform; namely their frequency, damping rate, magnitude, and angle. The next step is to model the underlying circuit at the dominant mode of the event. The final step is to locate the cause of the event based on certain forward and backward calculations on the obtained circuit model. As few as only two WMUs, one at the beginning of the feeder and one at the end of the feeder, are sufficient to identify the location of the event. The performance of the developed method is verified on the IEEE 33-bus test system. The results verify the accuracy and robustness of the proposed method in identifying the location of events in distribution networks.

Keywords: Event location identification, power distribution, waveform measurement unit, WMU, synchronous waveforms, synchrowaveforms, modal analysis, data-driven method.

I. INTRODUCTION

Distribution networks are continuously exposed to events that cause sudden changes in network conditions [1]. Events in distribution systems can be categorized into two groups: power quality (PQ) events and faults. The most common PQ events are capacitor switching and voltage sag which plague the operation of sensitive electronic loads. The most common fault events are low impedance and high impedance faults which cause interruption in service. It is crucial for utilities to accurately identify the source of events in order to identify incipient faults, take corrective actions, update models, etc.

In the literature, several methods have been proposed to locate the source of events in power distribution systems. Several methods, such as those that are often used in protective-relays, work based on estimating the impedance or the distance between the event location and the sensor location, e.g., see [2]. There are also methods that work based on estimating the arrival time and velocity of the traveling waves that are generated by events, e.g., see [3]. However, both of the above two families of methods are primarily intended for major events, such as faults. That is, they are often incapable of identifying the locations of PQ events. Moreover, they sometimes have the drawback that they identify multiple locations for the event.
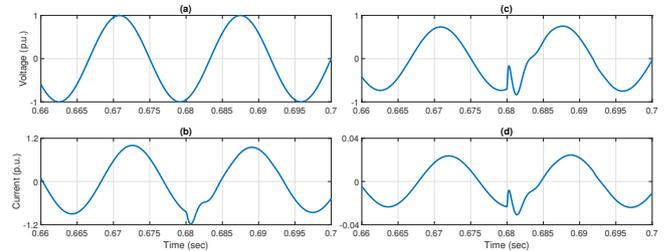
Fig. 1. Examples of synchronized voltage and current waveform measurements that are captured by two WMUs on a distribution feeder: (a)-(b) measurements from WMU 1 that is installed at the beginning of the feeder; (c)-(d) measurements from WMU 2 that is installed at the end of the feeder.

Another class of methods that is emerging only recently works based on data from distribution-level phasor measurement units (D-PMUs), a.k.a., micro-PMUs [4], [5]. Examples of methods that use D-PMU data are presented in [6]–[10]. Some of these methods are still primarily concerned with faults; however, in general the use of D-PMU data has resulted in major advances in identifying the locations of both PQ events and faults in power distribution systems, e.g., in [7]. Nevertheless, the use of D-PMU in this context imposes an inherent limitation that the associated method can be applied only to those events that have considerable impact on the steady state voltage or current; and therefore on the voltage and current phasors that are measured by D-PMUs.

In this paper, we take a different approach and propose to use *synchronized voltage and current waveform measurements* to identify the location of events in power distribution systems. Such measurements are provided by a new generation of sensors, called *waveform measurement units* (WMUs). WMUs provide GPS-synchronized measurements in time-domain [11]. They capture the voltage and current waveforms during various events, both faults and PQ events. The reporting rate of a typical WMU is 256 samples per cycle [12], which is much higher than that of a D-PMU which at most reports two samples per cycle [5]. Data from WMUs have been used recently to investigate harmonic addition or cancellation, to monitor power lines, or to detect sub-synchronous resonance [11], [13].

Fig. 1 shows examples of voltage and current waveforms that are captured by two WMUs, denoted by WMU 1 and WMU 2, during a transient PQ event on the IEEE 33-bus distribution test system. WMU 1 is installed at the substation at the beginning of the feeder. The impact of the event at this location is visible only in the current waveform. WMU 2 is installed at the end of the feeder. The impact of the event is visible in both the current and voltage waveforms.

The question that we seek to answer in this paper is: *can we use synchronized waveform measurements, such as those in Fig. 1, and identify the bus number where the event has occurred?* We show that the answer is 'Yes'. Our proposal
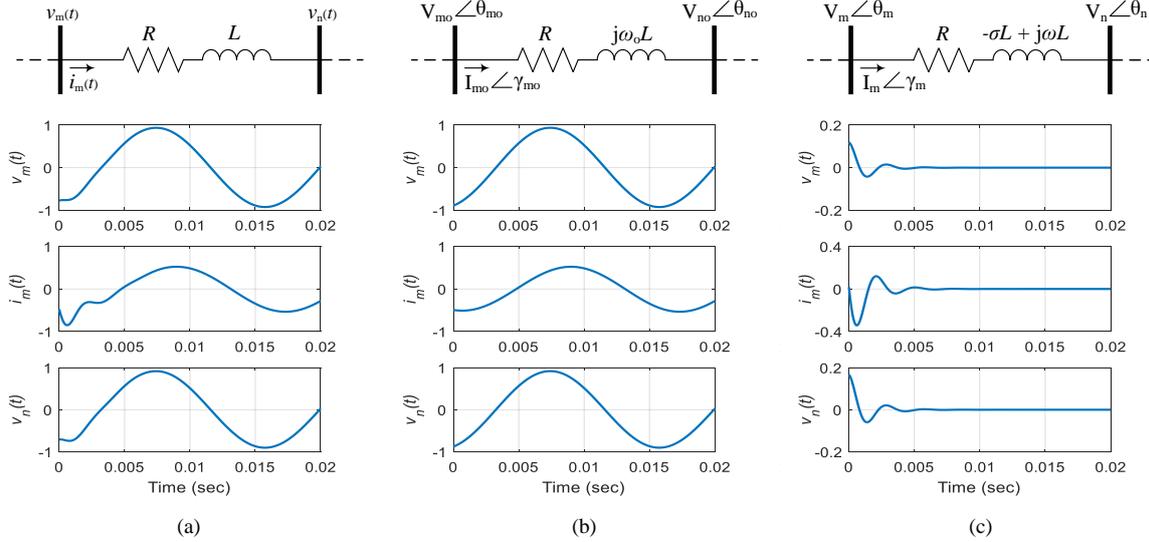
Fig. 2. Analysis of the network immediately after the event occurs: (a) the original circuit model and waveform measurements; (b) the circuit model and waveform measurements under the *fundamental mode*; (c) the circuit model and waveform measurements under the *event mode*.

is to model and investigate the underlying circuit of the feeder at the dominant event mode. Our method takes a hybrid data-driven and model-based approach; which results in an accurate and robust algorithm to identify the location of events, specially PQ events, in power distribution systems.

II. SYSTEM MODEL

Consider any line segment on the understudy distribution feeder, as shown in Fig. 2(a). It connects bus m to bus n. The resistance and inductance of the line segment are denoted by R and L, respectively. Suppose an event occurs at time $t = 0$ at some unknown bus somewhere on the distribution system. Let $v_m(t)$ and $v_n(t)$ denote the voltage waveforms at bus m and bus n, respectively, that are captured immediately after the event; and $i_{mn}(t)$ denotes the corresponding current waveform on the line segment between the two buses.

Without loss of generality we assume that each captured voltage or current waveform has two components; a *sinusoidal fundamental component* and a *damping event component*. If the event introduces multiple modes then we can simply take the dominant component and the rest of the analysis remains the same. The above three waveforms can be expressed as:

$$v_m(t) = V_{m\circ} \cos(\omega_\circ t + \theta_{m\circ}) + V_m e^{-\sigma t} \cos(\omega t + \theta_m),$$
$$v_n(t) = V_{n\circ} \cos(\omega_\circ t + \theta_{n\circ}) + V_n e^{-\sigma t} \cos(\omega t + \theta_n),$$
$$i_m(t) = I_{m\circ} \cos(\omega_\circ t + \gamma_{m\circ}) + I_m e^{-\sigma t} \cos(\omega t + \gamma_m),$$
(1)

where $V_{m\circ}$ and V_m, $V_{n\circ}$ and V_n, and $I_{m\circ}$ and I_m denote the magnitudes of the fundamental component and the event component, respectively; $\theta_{m\circ}$ and θ_m, $\theta_{n\circ}$ and θ_n, and $\gamma_{m\circ}$ and γ_m denote the phase angles of the fundamental component and the event component, respectively; $\omega_\circ = 2\pi f_\circ$ is the rotational frequency of the fundamental component whose frequency is $f_\circ$; and finally $-\sigma$ and $\omega = 2\pi f$ are the damping rate and rotational frequency of the event component with frequency f. The event characteristics can also be shown in complex format as $-\sigma + j\omega$, where $j = \sqrt{-1}$.

It bears mentioning that the frequency of the event mode can take a wide range of values, depending on the type of the event. For instance, in a capacitor bank switching event, the frequency of the event mode is *much larger* than the fundamental frequency; and can represent a harmonic or an inter-harmonic. In contrast, in a sub-synchronized resonance event, the frequency of the event mode is *much smaller* than the fundamental frequency. All such various cases can be addressed by the methodology that is presented in this paper.

Next, due to superposition, we can decompose the circuit model in Fig. 2(a) into *two* models; one at the fundamental mode $\pm j\omega_\circ$ as shown in Fig. 2(b); and one at the event mode $-\sigma \pm j\omega$ as shown in Fig. 2(c). Accordingly, we can write the Kirchhoff's Voltage Law (KVL) for each circuit. For the model of the fundamental mode in Fig. 2(b), we have:

$$V_{m\circ} \angle \theta_{m\circ} - V_{n\circ} \angle \theta_{n\circ} = Z_\circ I_{m\circ} \angle \gamma_{m\circ}, \qquad (2)$$

where

$$Z_\circ = R + j\omega_\circ L \qquad (3)$$

is the impedance of the line at fundamental mode. Similarly, for the model of the event mode in Fig. 2(c), we have:

$$V_m \angle \theta_m - V_n \angle \theta_n = Z I_m \angle \gamma_m, \qquad (4)$$

where

$$Z = R - \sigma L + j\omega L \qquad (5)$$

is the impedance of the line at event mode. Notice the difference between (3) and (5) and that the damping rate of the event mode appears as a resistive term in (5).

For the rest of this paper, we focus on the system model under the event mode, i.e., the setup in Fig. 2(c). Specifically, we construct the network model based on the *most dominant event mode* that is present in the captured waveforms.

III. EVENT LOCATION IDENTIFICATION METHOD

A. Methodology

Consider a distribution feeder that consists of n buses, as shown in Fig. 3. Suppose two WMUs are installed on the feeder, one at the beginning and one at the end of the feeder.

1) Step 1: Forward Nodal Voltage Calculation: Suppose an event occurs at unknown bus k. We define an event as a change in resistance, inductance, or capacitance at this bus. The event creates distortion in the voltage and current waveforms across the distribution feeder. In principle, any distortion creates some new modes, in addition to the fundamental mode. As in Section II, we consider the most dominant event mode in the center of our analysis, i.e., the event mode with the largest magnitude, with damping rate σ and rotational frequency ω.

WMU 1 in Fig. 3 measures the voltage and current waveforms at the beginning of the feeder. We refer to the measurements at WMU 1 as the *upstream* measurements, denoted by subscript u. Let V_u and I_u denote the voltage and current phasors at the *event mode* that are extracted from the voltage waveform and current waveform at WMU 1. We can calculate the nodal voltages at the *event mode* along the feeder as follows:

$$
\begin{aligned}
V_1^f &= V_u, \\
V_2^f &= V_1^f - (I_u)Z_1, \\
&\vdots \\
V_n^f &= V_{n-1}^f - (I_u - I_2^f - \cdots - I_{n-1}^f)Z_{n-1},
\end{aligned} \tag{6}
$$

where superscript f stands for *forward calculation*; V_i^f indicates the voltage at bus i; I_i^f indicates the current drawn at bus i; Z_i denotes the impedance of line i. We shall again emphasize that all the notations in (6) are associated with the event mode. From (5) in Section II, for each line segment i with resistance R_i and inductance L_i, we can write:

$$
Z_i = R_i - \sigma L_i + j\omega L_i, \tag{7}
$$

We assume that loads have constant resistance and inductance, which can be obtained from pseudo-measurements. If there is a lateral in the network, the lateral is replaced with its equivalent resistance and inductance, which can again be obtained from pseudo-measurements. This is a rational assumption because the pseudo-measurements are always available and more importantly, our method is robust against the error in pseudo-measurements. Under this assumption, for load i with resistance and inductance equal to R_i^d and L_i^d, respectively, the admittance of the load at the *event mode* is

$$
Y_i = (Z_i^d)^{-1} \quad , \quad Z_i^d = R_i^d - \sigma L_i^d + j\omega L_i^d, \tag{8}
$$

where R_i^d and L_i^d denote the resistance and inductance at bus i and they are determined based on the pseudo-measurements and system voltage. Note that, the admittance of load at bus k, where the event happens, is *unknown*, because the change in the elements at this bus is unknown.

The current injection at bus i under the event mode is the product of the nodal voltage and the admittance at bus i, both under the event mode, as shown below:

$$
I_i = Y_i V_i^f. \tag{9}
$$

By substituting (9) into (6), we can sequentially obtain $V_1^f, \ldots, V_k^f$, i.e., the voltages for buses 1 to k. However, since the change in the admittance is not known at event bus k, we cannot obtain $V_{k+1}^f, \ldots, V_n^f$, i.e., the voltages for buses $k+1$ to n. To be more precise, we can make such calculations
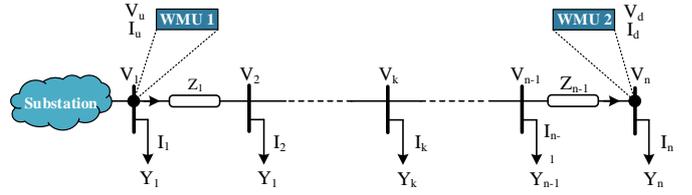
Fig. 3. The network model for a distribution feeder with two WMUs.

at buses $k+1$ to n; but the calculations will not be correct. Therefore, we can break down the outcome of the forward nodal voltage calculation into two parts, as shown below:

$$
\underbrace{\{V_1^f, \cdots, \boxed{V_k^f}}_{\text{correct}}, \underbrace{V_{k+1}^f, \cdots, V_n^f\}}_{\text{incorrect}} \tag{10}
$$

Importantly, since we do *not* know which bus is the event bus, we also do *not* know at what bus our forward nodal voltage calculations switch from being correct to being incorrect.

2) Step 2: Backward Nodal Voltage Calculation: We repeat the same analysis, but this time we do it backward. We start from WMU 2 at bus n and calculate the nodal voltages background all the way to bus 1, still at the event mode:

$$
\begin{aligned}
V_n^b &= V_d, \\
V_{n-1}^b &= V_n^b + (I_d)Z_{n-1}, \\
&\vdots \\
V_1^b &= V_2^b + (I_2^b + \cdots + I_{n-1}^b + I_d)Z_1,
\end{aligned} \tag{11}
$$

where superscript b stands for *backward calculation*. By substituting (9) into (11), we can sequentially obtain $V_k^b, \ldots, V_n^f$, i.e., the voltages for buses k to n. However, since the change in the admittance is not known at event bus k, we cannot obtain $V_1^b, \ldots, V_{k-1}^b$, i.e., the voltages for buses 1 to $k-1$. Thus, we can break down the outcome of the backward nodal voltage calculation into two parts, as shown below:

$$
\underbrace{\{V_1^b, \cdots, V_{k-1}^b}_{\text{incorrect}}, \underbrace{\boxed{V_k^b}, \cdots, V_n^b\}}_{\text{correct}} \tag{12}
$$

Importantly, since we do *not* know which bus is the event bus, we also do *not* know at what bus our backward nodal voltage calculations switch from being correct to being incorrect.

3) Step 3: Minimizing Discrepancy: By comparing (10) and (12), we can see that *either* the forward nodal voltage calculation *or* the backward nodal voltage calculation is incorrect at every bus; *except* for the event bus k, see the circles in (10) and (12). In other words, even though we do *not* know which bus is the event bus, we *do* know that the forward and backward nodal voltage calculations are going to have discrepancy, except for event bus k. We can use this fact to identify the event bus. Let us define a *discrepancy index* at the event mode as follows:

$$
\Psi_i = |V_i^f - V_i^b|, \ \forall i = 1, \cdots, n, \tag{13}
$$

where $|.|$ returns the magnitude of phasor $V_i^f - V_i^b$. We can now identify the event bus as follows:

$$
k^\star = \arg\min_i \Psi_i. \tag{14}
$$
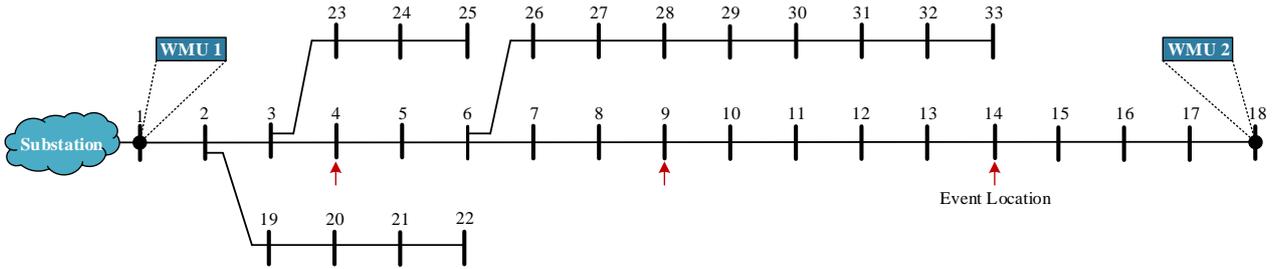
Fig. 4. The IEEE 33-bus distribution system test equipped with two WMUs. The location of the possible events is indicated with red arrows.

Algorithm 1 Event Location Identification

Input: WMU measurements and pseudo-measurements
Output: The location of an event
  1: Obtain the dominant event mode $-\sigma + j\omega$.
  2: Model the circuit at the dominant event mode.
  3: Obtain $\{V_i^f\}_{i=1}^n$ using (6) and (9).
  4: Obtain $\{V_i^b\}_{i=1}^n$ using (11) and (9).
  5: Obtain $\{\Psi_i\}_{i=1}^n$ using (13).
  6: Obtain the event bus $k^\star$ using (14).

Putting it simply, the event bus is the bus where the discrepancy between the forward nodal voltage calculation and the backward nodal voltage calculation is minimized. It is worth to mention that, in our methodology, we assumed that the events occur at buses. If an event occurs on a line, e.g., a fault, then our method can identify the event line between the two buses with the smallest discrepancies.

B. Algorithm

A step by step procedure to identify the location of event is outlined in Algorithm 1. At first, we extract the characteristics of oscillatory modes of the captured waveform measurements including frequency, damping rate, magnitude, and angle from the installed WMUs on the distribution feeder. This can be done using any modal analysis, c.f. [14], [15]. Next, we go through the forward and backward calculations under the event mode that we explained in the previous sub-section. The voltage and current waveform measurements that are needed to run Algorithm 1 can come from as few as only two WMUs that are deployed at the beginning and at the end of the feeder. It bears mentioning that if the event occurs on a lateral, then our method is still able to identify the bus at the beginning of the lateral as the event bus, indicating that the event has occurred somewhere along the lateral. In order to identify the true location of the event, it is necessary to use a WMU at the end of the lateral.

C. Characteristics and Comparison with Phasor Analysis

It is insightful to compare the key characteristics of the method in this paper, which works based on data from WMUs, with those of the method in [7], which works based on data from D-PMUs. *First*, the method here uses the *transient* measurements; while the method in [7] inherently uses the *steady state* measurements. As a result, the method here is applicable to more events, because it does not require a change in the steady state measurement in order to identify

the event location. *Second*, the method here does *not* require constructing an equivalent circuit based on compensation theorem for the distribution system, which is a necessity for the method in [7]. This key advantage is the direct result of not doing the analysis at the fundamental mode in this paper. *Third*, the method in this paper can be used for identify the location of sub-cycle events. This is of course a direct result of using the waveform measurements, as opposed to the fundamental phasor measurement. *Fourth*, it should be noted that the analysis in this paper uses both frequency and damping rate of the event mode; which we extract from the waveform measurements.

IV. CASE STUDY

Consider the IEEE 33-bus test system in Fig. 4. Suppose two WMUs are deployed in the network, one at the beginning and one at the end of the feeder. Each WMU measures both the voltage and current waveforms. Without loss of generality, all loads are modeled with constant resistance and inductance obtained from pseudo-measurements. The waveform data is generated in PSCAD [16]. The event that is studied here is the switching on action of a 400 kVAR capacitor bank. The event is assumed to be happened at bus 4, 9, or 14, as marked on Fig. 4. The switching event occurs at time 0.68 second.

If the capacitor is at bus 9 then the waveforms are measured as in Fig. 1, see Section I. Using modal analysis, c.f. [14], the dominant event mode in this case is obtained as $-630 + j2\pi \times 566$. The voltage and current phasors under such event mode are obtained at WMU 1 as $V_u \approx 0.00$ and $I_u = 0.316 \angle 85.2°$; and at WMU 2 as $V_d = 0.823 \angle -6.9°$ and $I_d \approx 0.00$.

A. Accuracy and Robustness

The measurements at the event mode are used to obtain the discrepancy index according to (13). The results are shown in Fig. 5 for three different locations for the event bus. We can see that in all cases, the discrepancy index reaches its minimum at the event bus, i.e., at bus 4 in Fig. 5(a), bus 9 in Fig. 5(b), and bus 14 in Fig. 5(c). This confirms the accuracy of the proposed event location identification method.

Another factor to check is robustness. This can be checked by comparing the lowest discrepancy index with the *second lowest* discrepancy index. The larger the differences is between these two discrepancy indexes, the more robust in the performance. We can see that the differences are considerable in all three cases; thus, confirming the robustness of the method.
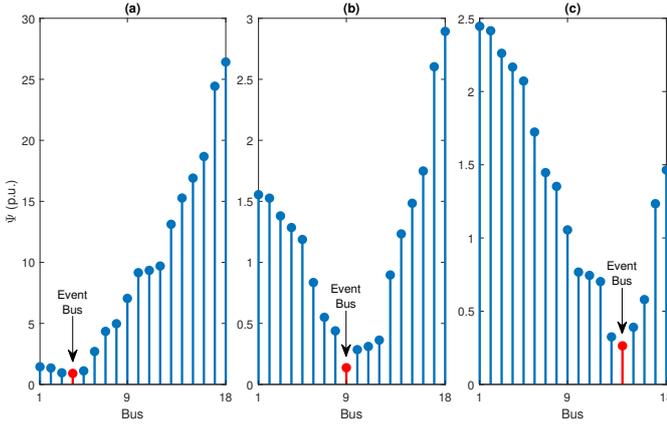
Fig. 5. Discrepancy index when the event occurs at 3 different locations: (a) bus 4; (b) bus 9; and (c) bus 14. The minimum discrepancy is shown in red and the event bus is marked with an arrow.
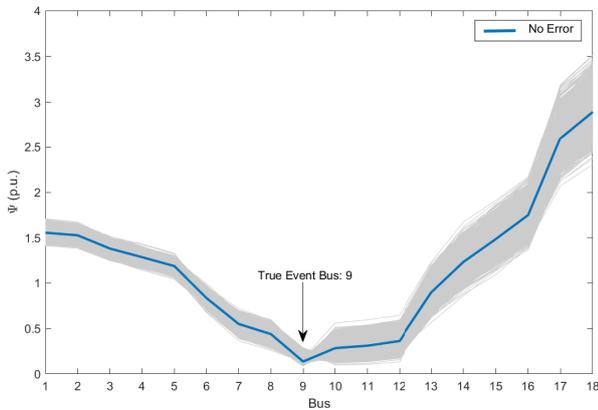
Fig. 6. Discrepancy index under 10,000 random error scenarios.

B. Error in Pseudo-Measurements and Parameters

Next, we investigate accuracy and robustness in presence of error in network parameters and pseudo-measurements. We use Monte Carlo simulation to generate 10,000 different test scenarios. It is assumed that the error in network parameters has a Gaussian distribution with zero mean and standard deviation of 10%. Also, the error in pseudo-measurements has a Gaussian distribution with zero mean and standard deviation of 30%.

The discrepancy curves under these random scenarios are shown in Fig. 6, in which the blue curve shows the discrepancy without the presence of error. As we can see, certain scenarios may deviate the minimum of the discrepancy curve away from the true event bus, such as to neighboring bus 10. The summary of the results is given in Table I. Despite the errors in pseudo-measurements and network parameters, the correct event bus is identified in 96% of the scenarios; thus, further confirming the robustness of the proposed method.

V. CONCLUSIONS

This paper developed a novel method to use data from WMUs to identify the location of events, in particular PQ events, in distribution networks. The method is based on modeling the underlying power distribution circuit at the dominant mode of the event, which is often a complex mode comprising both oscillations and damping. The location of

TABLE I
ACCURACY IN EVENT LOCATION IDENTIFICATION

Correct Bus	Neighboring Bus	Other Bus
95.78 %	4.22 %	0.00 %

the source of the event is identified by using the forward and backward voltage calculations on the obtained circuit model. The proposed method was applied to the IEEE 33-bus test system. The results confirmed the accuracy of the method in identifying the correct location of the event. They also showed that the performance is robust against error in network parameters and pseudo-measurements. The proposed method can be adopted in practice at low cost, because it requires as few as only two WMUs to identify the location of an event.

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