Linear Distribution System State Estimation Using Synchrophasor Data and Pseudo-Measurement

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***Abstract*—State estimation is often a challenging task in distribution systems due to deploying a limited number of measurement devices. Moreover, the integration of precise distribution-level phasor measurement unites, a.k.a micro-PMUs, along with inaccurate pseudo-measurements in state estimation introduces another challenge. These issues might decrease the efficiency of traditional standard weighted least squares (WLS) for distribution system state estimation. This paper proposes a novel linear distribution system state estimation with taken different types of measurements, including micro-PMU measurements and line current measurements, into account. To involve pseudo-measurements into the linear state estimation, a linearization method based on the Taylor's approximation is adopted to reformulate pseudo-measurement functions in a linear form. The results obtained via numerical simulations show that the proposed linear state estimation method performs similarly to the standard nonlinear WLS estimator. In addition, the sensitivity analyses results show that our method has a better performance compared to WLS once a limited number of highly precise micro-PMUs are accompanied with inaccurate pseudo-measurements in state estimation.**

***Keywords*: State estimation, distribution system, micro-PMU, line current sensor, pseudo-measurement, linearization.**

I. Introduction

Real-time monitoring plays a key role in the effective management and control of distribution system applications, e.g., [1]. Distribution System State Estimation (DSSE) is a cornerstone tool in advance distribution system monitoring. It uses the measurements to best approximate the states of the system which best fit the available measurements. Over the past few decades, transmission system state estimation (TSSE) has been well developed. However, there exist some fundamental differences between transmission system and distribution system, which makes TSSE methods unsuitable for DSSE. Typically, transmission systems have sufficient measurement redundancy that makes them beyond the observability. Such assumption may not be applied to distribution systems, because they are either unobservable beyond the substation or monitored through a few measurement units installed across the feeder. In this regard, it is required to make use of some historical data of nodal loading conditions, so-called pseudo-measurements, to run DSSE. Recently, there is a growing interest among electric utilities to deploy distribution-level phasor measurement units, a.k.a., micro-PMUs, for different applications, c.f., [2]–[4]. Of interest application in this paper, micro-PMUs could substantially enhance the DSSE results by providing the direct voltage phasor measurements. However, due to limited number of potential deployed micro-PMUs in distribution system, it is not possible to run a full PMU-based state estimation as proposed for transmission level [5]. Therefore, the integration of highly precise micro-PMU data along with less accurate pseudo-measurements is introduced as another challenge in DSSE.

Among the existing DSSE techniques, the weighted least squares (WLS) technique is the most widely used method. In WLS, either bus voltages [6] or branch currents [7] can be selected as the state variables, and the measurement functions are expressed based on them. Typically, the measurements are nonlinear function of state variables, which necessitates utilizing some numerically iterative methods, e.g., Newton-Raphson, to deal with the measurements nonlinearity [8, Ch. 2]. The main challenge of WLS method is instability issue of the iterative solution, which may either fail to convergence or become sensitive to initial point. In particular, once the use of some highly accurate mciro-PMUs measurement, having very large weights in the WLS objective, are accompanied with inaccurate pseudo-measurement, having very small weights in WLS objective, the significant variation among the weights may causes issues in iterative solution.

Several methods have been developed to overcome the issues with the standard nonlinear WLS method. In [9], the authors proposed the semidefinite programming (SDP) technique for DSSE. This method indeed relaxes nonconvex WLS problem into a solvable convex problem by the semidefinite relaxation. For large networks, SDP method may be computationally complex and reaches to local optimum solution due to the nonconvex rank-one constraint relaxation. Another alternative for standard nonlinear WLS is linear WLS state estimator, which is highly of interest for the integration of synchropahsor data in DSSE. In [10], the authors developed a DSSE by linearizing load injection measurements and using the current injections as the measurements. This method requires a load flow pre-analysis to calculate the current measurements. In [11], authors proposed Bayesian linear state estimator (BLSE) based on micro-PMU data as well as pseudo-measurements. The method use a fully linear approximation of the power flow equations, but it should be solved in an iterative manner.

In this paper, we develop an linear DSSE (LDSSE) with considering different types of measurements, including, micro-PMUs and line current sensors. To involve pseudo-measurements into LDSSE, a linearization method based on the Taylor's approximation is adopted to reformulate pseudo-measurement functions in a linear form. Numerical results based on Monte Carlo simulations show that LDSSE of-

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fers a similar performance as the traditional nonlinear WLS state estimator, even LDSSE outperforms WLS under certain conditions. We also evaluated the LDSSE algorithm performance with respect to phasor measurements accuracy, pseudo-measurements uncertainty, and number of deployed micro-PMUs. The results show that the LDSSE is preferable for the network with limited number of precise micro-PMUs accompanied with inaccurate pseudo-measurements.

II. Linear State Estimation Methodology

A. Weighted Least Squares

Let define the measurement equations as follow

$$z = h(x) + e, \tag{1}$$

where x and z denote the state vector and measurement vector, respectively. Also, $h(\cdot)$ denotes vector of functions that describes the relationship between measurements and the state variables. In addition, e denotes vector of measurement errors, typically assumed to have a Gaussian distribution with zero mean and covariance matrix R. If the errors are independent, R is a diagonal matrix with σ_i^2 values, where σ_i is the standard deviation of the error associated with measurement i. Given W as the R^{-1}, the state vector can be obtained by solving the following weighted least squares (WLS) optimization problem

$$\textbf{minimize } J(x) = \sum_{i \in \mathcal{Z}} \parallel W_i \, z_i - h_i(x) \parallel^2, \tag{2}$$

where $||.||^2$ denotes norm-2 and $\mathcal{Z}$ is set of measurements. If $h_i(\cdot)$ is a nonlinear measurement function, then the optimization problem in (2) can be solved iteratively, e.g., Newton-Raphson method [8, Ch. 2], by setting the derivative of J with respect to x to zero

$$\frac{\partial J(x)}{\partial x} = H^T(x) W [z - h(x)] = 0, \tag{3}$$

where $H(x) = \frac{\partial h(x)}{\partial x}$ and $(.)^T$ denote the Jacobian matrix and transpose operator, respectively. The iterative methods are prone to divergence when there exists significant variation between measurement errors, e.g., errors in micro-PMUs data versus errors in pseudo-measurements.

If all the measurement functions are linear, i.e., $z = hx$, then there is a closed-form solution for optimization problem in (2) as:

$$\hat{x} = [h^T W \, h]^{-1} h^T W z, \tag{4}$$

The non-iterative state estimation addresses the convergence challenge mentioned above once the $h^T W \, h$ is well-condition matrix [12, Ch. 2], which holds in most of situation in distribution system.

B. Measurement Functions

Let define nodal voltage vector in a n-bus distribution system as $V = e + jf$, where $e = [e_1, \cdots, e_n]$ and $f = [f_1, \cdots, f_n]$ vectors denote real part and an imaginary part of the nodal voltages, respectively. Here, we define the nodal voltages in rectangular coordinate system as the system state vector; hence, we will consider $x = [e \quad f]$ as the state vector in the rest of the paper. In this regard, we should consider the measurements as well as measurement functions in rectangular form. The measurement functions for three sets of measurements available for DSSE are described as follows.

Nodal Voltage: Recently developed distribution-level synchrophasor, a.k.a micro-PMUs, are able to measure the nodal voltage phasor. Therefore, PMU technology opens-up the possibility of directly measuring system state, which could enhance the quality of state estimation. The measurement function for the micro-PMU voltage measurement can be stated as follows:

$$\underbrace{\begin{bmatrix} u_i & 0_{n\times 1} \\ 0_{n\times 1} & u_i \end{bmatrix}}_{h_i^V} x = \underbrace{\begin{bmatrix} \mathrm{Re}\{V_i\} \\ \mathrm{Im}\{V_i\} \end{bmatrix}}_{z_i^V}, \tag{5}$$

where $u_i = [0, \cdots, 1, \cdots, 0]$ is $1 \times n$ unitary vector whose element i^{th} is equal to 1 and other elements are zero; and Re$\{.\}$ and Im$\{.\}$ denote the real part and imaginary part.

Line Current: Line current sensors are inexpensive and can be installed very quickly in distribution systems; hence, utilities are highly of interest to integrate line current sensors in their system. A typical line current sensor can provide phasor current either of the following two types of measurements:

1) Magnitude and Phase Angle
2) Magnitude and Relative Phase Angle to Electric Field

The first type of measurements are available in those line current sensors equipped with GPS devices; and phase angle is measured relative to a GPS-synchronized reference signal. The second type of measurements are available practically in any line current sensor even if no GPS device is available. In fact, almost all commercially available line current sensors, whether electric or optic, are capable of measuring electric field (e-field) around the line. Relative phase angle to e-filed could be an good estimation of current phasor angle because, first, the e-field around a conductor is *almost in-phase* with the voltage of the conductor; second, the voltage of the conductor at any point on a distribution feeder are relatively close to each other. Therefore, we can assume that the line current phasor measurements are available regardless the technology used for measuring the phase angle. Without loss of generality, by considering short line models, the measurement function for the line current sensor measurement can be stated as follows:

$$I_{ij} = (V_i - V_j) Y_{ij}, \tag{6}$$

which by separting the real part and imagiary part in (6), we can write

$$\begin{aligned} \mathrm{Re}\{I_{ij}\} &= (e_i - e_j) G_{ij} - (f_i - f_j) B_{ij} \\ \mathrm{Im}\{I_{ij}\} &= (e_i - e_j) B_{ij} + (f_i - f_j) G_{ij}, \end{aligned} \tag{7}$$

where G_{ij} and B_{ij} are the real part and imaginary part of Y_{ij} which is the admittance of line ij. Hence, the line measurement matrix is written as follows:

$$\underbrace{\begin{bmatrix} G_f[ij] & -B_f[ij] \\ B_f[ij] & G_f[ij] \end{bmatrix}}_{h_{ij}^I} x = \underbrace{\begin{bmatrix} \mathrm{Re}\{I_{ij}\} \\ \mathrm{Im}\{I_{ij}\} \end{bmatrix}}_{z_{ij}^I} \tag{8}$$

where $G_f[ij] = [0, \cdots, G_{ij}, \cdots, -G_{ij}, \cdots, 0]$ is a $1 \times n$ vector whose i^{th} element is G_{ij} and j^{th} element is $-G_{ij}$; and $B_f[ij] = [0, \cdots, B_{ij}, \cdots, -B_{ij}, \cdots, 0]$ is a $1 \times n$ vector whose i^{th} element is B_{ij} and j^{th} element is $-B_{ij}$.

Load Measurement: In distribution system, the power load injections are not measured directly; therefore, load measurements are often obtained through pseudo-measurements. Pseudo-measurements are typically calculated using short-term load forecasting by smart meters kWh data or historical data information regarding the customer usage. The power injection equation can be written as follows:

$$V_i\, I_i^* = S_i \tag{9}$$

where I_i is the current injection at node i and $(.)^*$ is conjugate operator; S_i is apparent power injection at node i which is assumed to be obtained from pseudo-measurement. Measurement function (9) is nonlinear due to the multiplication of phasors V_i and I_i, so it is not compatible for the linear form of state estimation in (4). There are some methods to make this equation linear. For example in [10], the authors proposed to substitute this equation with the current injection equation as a linear function of state variables and calculate the current measurement injections from pseudo-measurements. In this paper, we use the method based on Taylor expansion and engineering approximation [13] to make this equation linear. To this end, a linear approximation is developed on complex numbers as apposed to on real numbers as in the conventional load flow formulations. Suppose we express the voltage at each bus i in per unit and in relationship with the voltage at the reference bus at the substation. That is, suppose we write

$$V_i = 1 - \Delta V_i. \tag{10}$$

By applying the Taylor series around zero and neglecting the high order terms in the Taylor series, we can write [13]:

$$\frac{1}{V_i} = \frac{1}{1 - \Delta V_i} \simeq 1 + \Delta V_i = 2 - V_i. \tag{11}$$

Experimental results have shown that the error associated with this approximation is very minor, e.g., see [14, Ch. 4]. In particular, for a typical operation range where $|\Delta V_i| \leq 0.1$, i.e. $0.9 \leq |V_i| \leq 1.1$, the error is less than 1%. Next, we substitute the approximation in (11) into (9) and rewrite constraint (9) in a reordered form as

$$I_i^* = S_i(2 - V_i). \tag{12}$$

By writing I_i based on the nodal voltages, (12) can be re-write as follows:

$$\sum_{j \in \mathcal{N}_i} (V_i - V_j) Y_{ij} = S_i^*(2 - V_i^*). \tag{13}$$

where notation $\mathcal{N}_i$ is the set of the nodes that are connected to node i. By rearranging (13) and separating the imaginary part and real part, we would have:

$$\begin{aligned} &\sum_{j \in \mathcal{N}_i} [(e_i - e_j)G_{ij} - (f_i - f_j)B_{ij}] + e_i P_i - f_i Q_i = 2P_i \\ &\sum_{j \in \mathcal{N}_i} [(e_i - e_j)B_{ij} + (f_i - f_j)G_{ij}] - e_i Q_i - f_i P_i = -2Q_i \end{aligned} \tag{14}$$

we can write this equation in matrix form as follows:

$$\underbrace{\begin{bmatrix} G_{bus}[i] + u_i P_i & -B_{bus}[i] - u_i Q_i \\ B_{bus}[i] - u_i Q_i & G_{bus}[i] - u_i P_i \end{bmatrix}}_{h_i^S} x = 2 \underbrace{\begin{bmatrix} P_i \\ -Q_i \end{bmatrix}}_{z_i^S} \tag{15}$$

where, $G_{bus}[i]$ is a $1 \times n$ vector whose i^{th} element is $\sum_{j \in \mathcal{N}_i} G_{ij}$ and j^{th} element is $-G_{ij}$; $B_{bus}[i]$ is a $1 \times n$ vector whose i^{th} element is $\sum_{j \in \mathcal{N}_i} B_{ij}$ and j^{th} element is $-B_{ij}$;

All in all, by considering the measurement functions in (5), (8), and (15) for all the measurements, we can have the system measurement function as follows:

$$\underbrace{\begin{bmatrix} h^V \\ h^I \\ h^S \end{bmatrix}}_{h} x = \underbrace{\begin{bmatrix} z^V \\ z^I \\ z^S \end{bmatrix}}_{z} \tag{16}$$

By substituting z and h obtained from (16) into (4), the linear state estimation is solved and the state variables are estimated.

III. CASE STUDIES

This section demonstrates the effectiveness of the proposed LDSSE method by applying it to the IEEE 33 bus test system. The single line diagram of the feeder is shown in Fig. 1, and the relevant technical data can be found in [15]. Unless stated otherwise, we assume that the voltage phasor and line current phasor are respectively measured by the micro-PMUs and line current sensors whose locations are shown in Fig.1. We also assume that the net power load of the feeder is measured at the feeder-head using a standard substation SCADA system. Otherwise, one can install another line current sensor at feeder-head. Pseudo-measurements are calculated for all the buses based on the total feeder net power and load allocation according to the low-voltage transformers capacities [16].

A. Base Case

Here, we assume that micro-PMUs provide the voltage phasor measurements with accuracy 0.1% ($\sigma_V = 0.1\%$); line current sensors provide the current phasor measurements with accuracy 1% ($\sigma_I = 1\%$); and pseudo-measurements provide the active and reactive power injection measurements with accuracy 25% ($\sigma_S = 25\%$). We examine the performance of LDSSE method through different measurement scenarios generated by Monte Carlo method [17].

Fig. 2 shows the error box results for estimated voltage magnitude of all buses in the test system. As can be seen, the mean value of estimated voltages (boxes center) are on the blue line which shows the correct voltage magnitude of buses. In addition, the 25^{th} and 75^{th} percentiles are close to the true value, which indicates that in most of the scenarios, the estimated voltages are so closed to the true value. Moreover, the maximum errors and outliers (pluses) are not much far away from the true value, which makes the proposed LDSSE method reliable for voltage magnitude estimation. The error box results for estimated voltage angles are also shown in Fig. 3. Although some outliers show large deviation of estimated voltage angle from the true value, the majority of the estimated

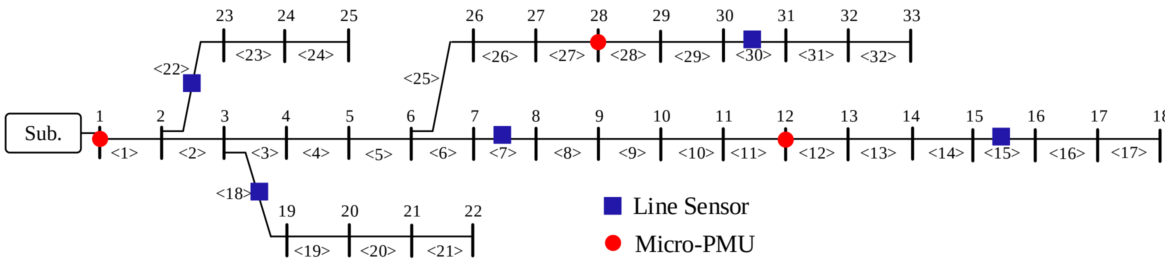

Fig. 1. The IEEE 33 bus test system that is used in our case studies. Three micro-PMUs are deployed on buses 1, 12, 28; and five line current sensors are deployed on line segments <7>, <15>, <18>, <22>, and <30>.

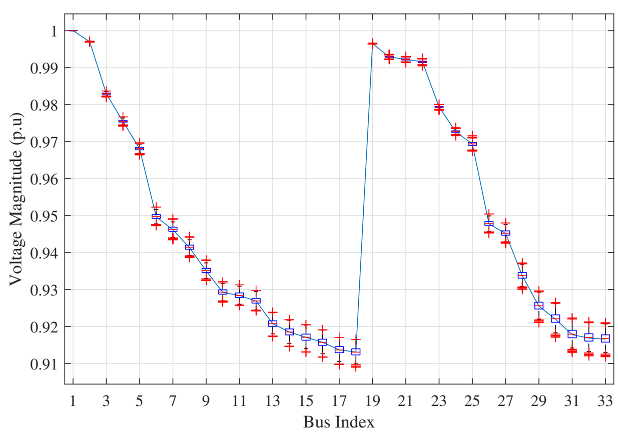

Fig. 2. Accuracy of voltages magnitude estimated by LDSSE in IEEE 33 bus test system.

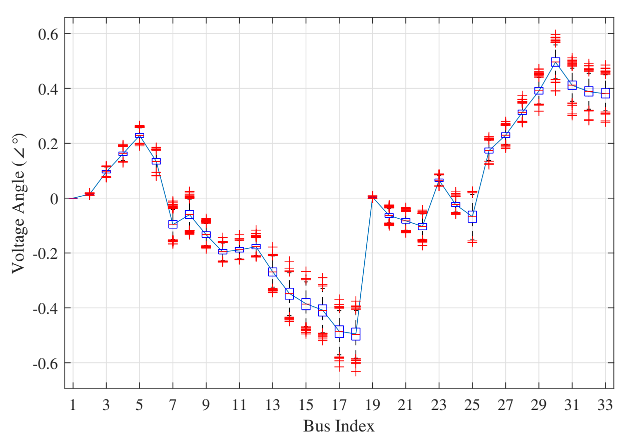

Fig. 3. Accuracy of voltages angle estimated by LDSSE in IEEE 33 bus test system.

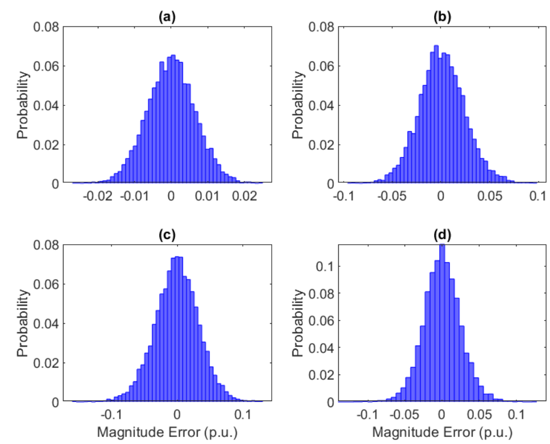

Fig. 4. Probability distribution of errors for estimated voltages magnitude at buses (a) 3, (b) 11, (c) 19, and (d) 33.

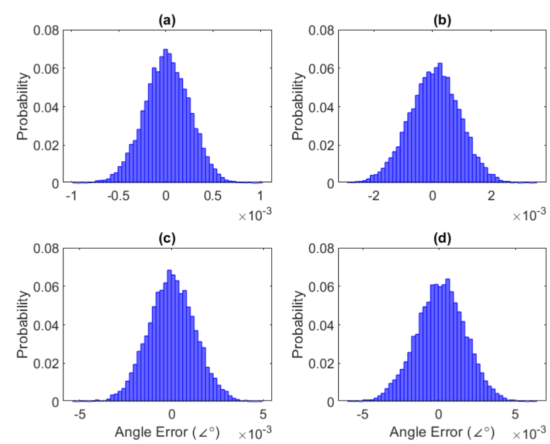

Fig. 5. Probability distribution of errors for estimated voltages angle at buses (a) 3, (b) 11, (c) 19, and (d) 33.

voltage angles (boxes) are very close to the true value. Therefore, the results statistically demonstrate the effectiveness of LDSSE for voltage angle estimation as well. The probability distributions of estimated voltages magnitude and estimated voltages angle for a few selected buses are shown in Fig. 4 and Fig. 5, respectively. As expected, probability density functions associated with the errors follow the Gaussian distribution.

B. Comparison: LDSSE vs. WLS

In this section, we compare the performance of our proposed method with standard nonlinear WLS state estimation method. In order to have a fair comparison, we applied both methods to the same practical test scenarios, where both methods have access to the measurements and pseudo-measurements as in Section III-A. WLS method has not an closed form solution and it should be solved through heuristic or iterative methods with the choice of an initial point. Here, we employ the Newton-Raphson iterative method [8, Ch. 2] with the flat initial point, where all the buses have voltage magnitude 1 p.u. and voltage angle $0°$, to solve nonlinear WLS in (2).

Fig. 6 shows the expected error for estimated voltages in both methods. The results demonstrate that LDSSE and WLS methods have relatively similar performance, even LDSSE

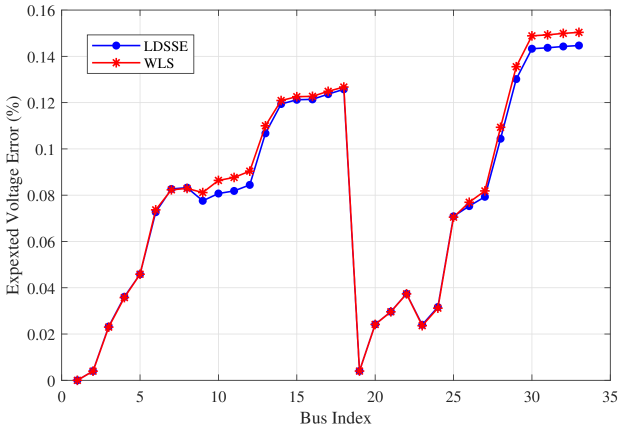

Fig. 6. The expected errors for the estimated nodal voltages phasor obtained from LDSSE and nonlinear WLS.

shows a better performance for some buses. In this regard, we can conclude that the linearized pseudo-measurement functions proposed in (15) does not deteriorate the accuracy of state estimation method and guarantees relatively the same accuracy as WLS method. The reason could be due to the compensation effect of least squares state estimation which makes up for the error caused by the Taylor's approximation in (11).

C. Analysis of Sensitivity and Robustness

In practice, the measurements and pseudo-measurement may have a wide range of accuracy. Here, we examine the robustness of the proposed LDSSE method against any given level of inaccuracy in comparison with WLS method performance. We use Mont Carlo approach to generate different scenarios for the given level of measurements and pseudo-measurements error.

1) Pseudo-Measurement Accuracy: In practice, the utility's knowledge about pseudo-measurements are not precise. The range of uncertainty associated with pseudo-measurements may vary significantly.

If pseudo-measurements are obtained from smart meters, then their error is limited to 10%. For pseudo-measurements that are calculated based on short-term load forecasting or historical data, e.g., based on a load allocation according to the low-voltage transformers capacities, the error can increase to 50%. The results of both LDSSE and WLS methods accuracy versus different levels of error in pseudo-measurements are shown in Fig 7(a). This figure shows the mean value error associated with the estimated voltages of all the buses. As shown, once pseudo-measurements are accurate, i.e., the error is limited to 10%, the WLS has a better performance. However, by increasing the inaccuracy in the pseudo-measurements, the LDSSE outperforms WLS. For accurate pseudo-measurements, the linearization error caused by Taylor's approximation is comparable with the pseudo-measurement errors, therefore the results are affected by such approximation in LDSSE method and lower efficiency is achieved in comparison with WLS. However, by increasing the pseudo-measurement inaccuracy, in one hand, the linearization approximation error in LDSSE is much smaller respect to the pseudo-measurements error, so such approximation does not affect the solution quality; on the other hand, because the pseudo-measurements are far away from their true value, it would be much probable for WLS to reach a local optimum.

2) Micro-PMU Measurement Accuracy: In principle, two sources of error can be considered in the context of using micro-PMUs for voltage phasor measurements: the error in the micro-PMU device itself; as well as the error in the instrumentation channel. The latter is associated with the errors due to the PTs, control cables, and burden at the input of the micro-PMU. Based on various field experience and given the fact that micro-PMUs have very high precision with typical accuracy at 0.01% in magnitude and 0.003° in angle [18]; it is only the error in the instrumentation channel that is of concern in practice and should be considered. Typically, PTs used for micro-PMUs are very precise such that they can guarantee the accuracy of 0.1%. However, the micro-PMUs may serve with the typical PT measurement which have the accuracy up to 1%. The results of both state estimation methods accuracy versus different levels of error in micro-PMU measurements are shown in Fig 7(b). As can be seen, for the precise micro-PMU voltage measurements, the LDSSE has a better performance compared to WLS method; however, as the measurements error increases, this superiority decreases such that for error further than 0.3% ($\sigma_V \geq 0.3\%$), the WLS outperforms the LDSSE. That is because, for the accurate micro-PMUs, the weight of voltage measurements in least squares objective is much larger than those of pseudo-measurements, therefore the LDSSE approximation error in pseudo-measurement function is masked. However, by increasing the error in voltage measurements, the pseudo-measurements impact on least squares objective function increases, whereby decreasing the performance of LDSSE due to linearization approximation.

3) Line Sensor Accuracy: As mentioned in Section II-B, the line current phasors can be measured by line current sensors, either directly and precisely if the sensor is equipped with GPS, or indirectly and approximately if the sensor is not equipped with GPS; which in that case, it measures the relative phase angle by measuring e-field. Based on the different types of sensors that are available, the error in current magnitude is assumed to be 1% to 3%. The results of both state estimation methods accuracy versus different levels of error in line current sensors are shown in Fig 7(c). As expected, by increasing the error level in line current sensor, the performance of both LDSSE and WLS state estimators deteriorates; however, the LDSSE maintain the superiority over WLS for whole range of error in line current sensor.

4) Number of Micro-PMUs: Micro-PMUs can considerably enhance the performance of state estimation due to directly measuring the state variables (voltage phasors) and providing highly accurate measurements. However, the usage of large number of micro-PMUs has not economic justification for distribution systems. The results of both state estimation methods accuracy versus different number of deployed micro-PMUs are shown in Fig 8. For each micro-PMUs number scenario, micro-PMUs are deployed evenly across the feeders. As expected, by increasing the number of micro-PMUs the accuracy of both LDSSE and WLS state estimators enhances.

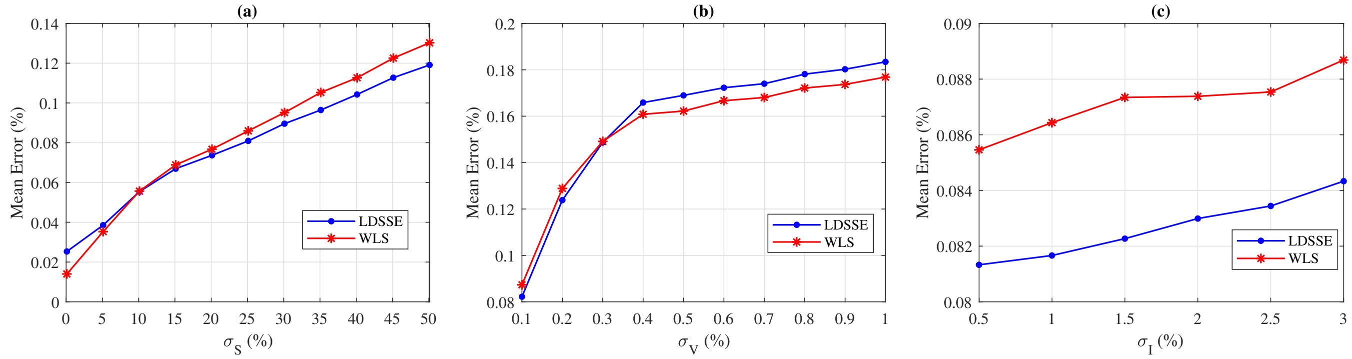

Fig. 7. Performance comparison of both LDSSE and WLS methods against: (a) pseudo-measurement accuracy; (b) micro-PMU measurement accuracy; (c) line current measurement accuracy.

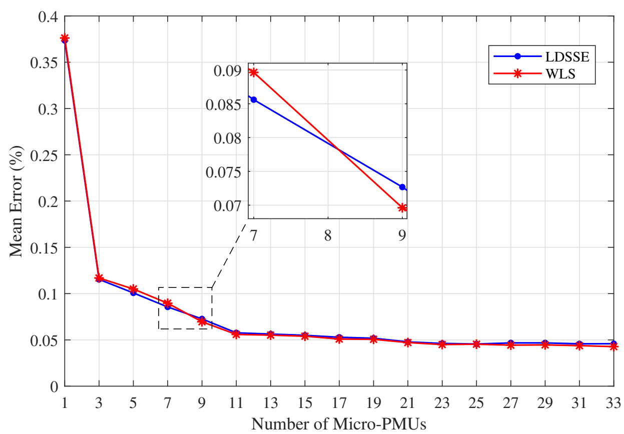

Fig. 8. Performance comparison of both LDSSE and WLS methods against number of deployed micro-PMUs in the test feeder.

The comparison of results shows similar accuracy and sensitivity of both methods respect to the number of micro-PMU deployment. However, once the number of deployed micro-PMUs are less that 9, the LDSSE demonstrates relatively better performance. That is because, by deploying more micro-PMUs in the system, the number of nodal voltages which directly measured increases, so the chance of WLS to reach the global optimum solution increases.

IV. Conclusion

This paper proposed a novel linear state estimation method with considering different types of measurements, including synchrophasor measurements and pseudo-measurements. The numerical results demonstrates the effectiveness of the proposed method for estimation of both voltage magnitude and angle. Moreover, we evaluated the performance of the proposed linear state estimation method in comparison with the standard nonlinear weighted least squares (WLS) method with respect to pseudo-measurements, micro-PMUs, and line current sensors accuracy as well as the number of deployed micro-PMUs. The results show that our method guarantees the same performance compare to the WLS method, and even better performance under certain conditions. Based on the results, our method shows superiority respect to WLS method, once limited number of highly accurate micro-PMU are accompanied with inaccurate pseudo-measurements in state estimation. All in all, the proposed method could be a reliable alternative for the traditional standard WLS.

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