Exercises

2.1 Consider the measurement of RMS current in Example 2.1. Suppose the accuracy of the ammeter that is connected to the secondary side of the CT is ±0.5% rdg. What is the possible range of the true current at the primary side of the CT under the following two different scenarios?
(a) The CT is ideal.
(b) The CT is *not* ideal, and its accuracy is ±1%.

2.2 The secondary side of a 400:5 CT is connected to an ammeter with burden of 1.47 W and 0.92 VARs, a meter protection relay with a burden of 0.82 W and 0.80 VARs, and a wiring with a burden of 3.60 W and 0.01 VARs. All these elements are connected in series. For all elements, the burden is given at a 5 A–60 Hz secondary current.
(a) How much is the total burden?
(b) Based on Table 2.2, select the burden designation class for this CT.

2.3 The relationship between current I and magnetic field strength B that is measured by a non-contact current sensor in Figure 2.36(a) is expressed as $B = \mu_0 I \cos(\delta)/2\pi r$. Suppose the same sensor is installed on the ground [138], underneath a *balanced* three-phase transmission line in two configurations, as shown in Figures 2.36(b) and (c). Let B_1 and B_2 denote the strength of the magnetic field that is measured by the sensor in each case.
(a) Obtain an expression for B_1 and an expression for B_2.
(b) Express the height of the conductor h as a function of B_1, B_2, and d.

2.4 The working principle of an MOCT is shown in Figure 2.37. The angle of rotation in the polarization is denoted by $\beta = \nu B d$, where ν is a constant and d is the length of the optic tube [79]. If current $i(t)$ in the power cable that creates the magnetic field is a sinusoidal wave, then magnetic field $B(t)$ is also

Table 2.2 IEEE C57.13 Standard burdens for CTs with 5 A secondary winding [137]

Burden Designation	Resistance (Ω)	Inductance (mH)
B-0.1	0.09	0.116
B-0.2	0.18	0.232
B-0.5	0.45	0.580
B-0.9	0.81	1.040
B-1.8	1.62	2.080

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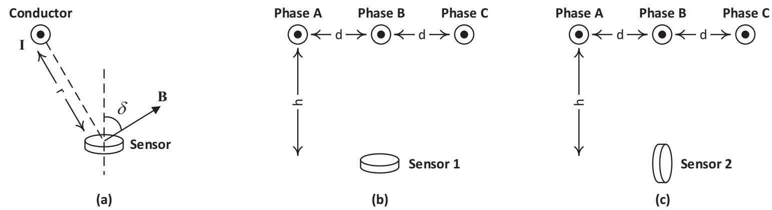

Figure 2.36 Measuring magnetic field strength of conductors in Exercise 2.3.

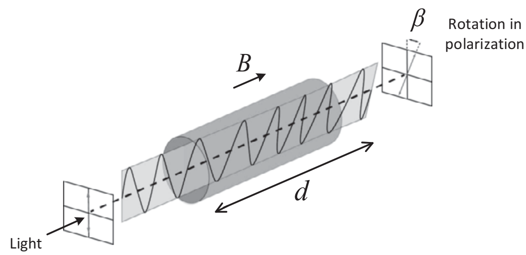

Figure 2.37 The relationship between magnetic field strength B and the amount of rotation in polarization of light that goes through an optic tube; see Exercise 2.4.

a sinusoidal wave; accordingly, $\beta(t)$ is also a sinusoidal wave. Express the RMS value of $\beta(t)$ in terms of the RMS value of $i(t)$.

2.5 The relationship between the magnetic-field-induced voltage $v_s(t)$ in a Rogowski coil in a non-contact current sensor and the current $i(t)$ that flows through the conductor is expressed as follows:

$$v_s(t) = -\frac{AN\mu_0}{l}\frac{di(t)}{dt}, \tag{2.36}$$

where N is the number of turns in the Rogowski coil, A is the area of each turn, l is the length of the winding, and μ_0 is a constant. Suppose $i(t)$ is purely sinusoidal. Express the RMS value of $v_s(t)$ in terms of the RMS value of $i(t)$.

2.6 In Example 2.3, suppose an anti-aliasing filter is designed to filter out any measurement with a frequency above 6 Hz, instead of above 4 Hz.
(a) Is this sufficient to prevent the aliasing scenario in Figure 2.5?
(b) What frequencies may still alias in presence of this anti-aliasing filter?

2.7 From the measurements in Figure 2.6, we have $\Delta V = 285.8 - 285.6 = 0.2$ V and $\Delta I = 145.5 - 136.6 = 8.9$ A. How much is the increase in apparent power load on the one phase that is shown in this figure?

2.8 File `E2-8.csv` contains voltage measurements over a period of 12 hours. The reporting rate of the measurements is *almost* one reading per minute. The first few readings are shown in Figure 2.38. Suppose τ denotes the time difference (in seconds) between any two consecutive readings of voltage.
(a) Calculate τ for all the measurements in the file.
(b) Plot the histogram for τ. Explain your observation.
(c) Is there any *missing* measurement? How many?

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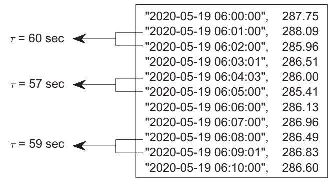

Figure 2.38 Some of the measurements in Exercise 2.8 and examples for calculating τ.

2.9 Suppose there is a momentary voltage spike event, possibly due to a lightning strike, that affects only one cycle. The RMS value of the affected voltage cycle is 145 V. The RMS value of all other cycles is 120 V. Suppose a measurement window of length 4 is used by the voltmeter with *weights* 0.4, 0.3, 0.2, and 0.1, where the highest weight is given to the most recent cycle. The reporting rate of the voltmeter is one RMS value per cycle.

(a) How much is the *peak* of the reported RMS values?

(b) How many of the reported RMS values are affected by the event?

2.10 Two voltage swell events at a distribution feeder are marked with numbers 1 and 2 in Figure 2.39(a). The current on the *same* feeder and the voltage on a *neighboring* feeder are shown in Figures 2.39(b) and (c), respectively. Specify whether each event is a local event or a nonlocal event.

2.11 Consider two neighboring power distribution feeders, and suppose we cross-examine all RMS voltage events above 0.5% rated magnitude. The results based on one month of minute-by-minute RMS voltage measurements are shown in Figure 2.40(a). Each point indicates one event. The location on the x-axis indicates the *change* in voltage on Feeder 1. The location on the y-axis indicates the *change* in voltage on Feeder 2. Suppose we *classify* the event points into eight groups, as shown in Figure 2.40(b). The number of events in each of these eight groups is 107, 102, 67, 0, 106, 104, 65, and 0, respectively.

(a) What percentage of the voltage sag events that are detected on Feeder 1 are caused by the loads and equipment on Feeder 1?

(b) What percentage of the voltage sag events that are detected on Feeder 2 are caused by the loads and equipment on Feeder 2? Compare the results with Part (a).

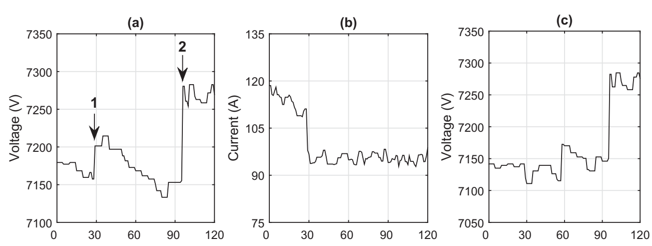

Figure 2.39 Minutely RMS voltage and current measurements in Exercise 2.10.

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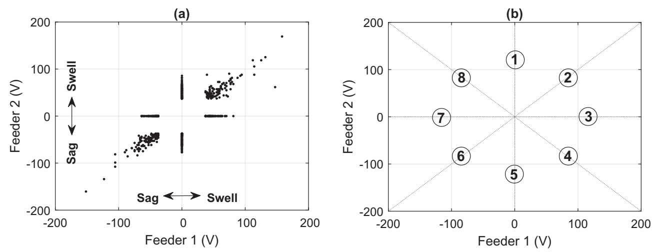

Figure 2.40 Cross-examining voltage events between two feeders in Exercise 2.11 [139].

2.12 File `E2-12.csv` contains the voltage and current measurements, averaged across three phases, at the AC side of a utility-scale PV inverter over a period of one week at one reading per five minutes.

(a) Plot voltage versus current in a scatter plot.

(b) Repeat Part (a) for the measurements between 8 AM and 6 PM. That is, *exclude* the measurements that are obtained *before* 8 AM or *after* 6 PM.

2.13 Consider $n \gg 1$ noise-contaminated samples from signal $x(t)$. Suppose the fixed component of the samples, i.e., their average, is removed. We define:

$$\mathbf{Y} = \begin{bmatrix} x(0) & x(1) & \cdots & x(l) \\ x(1) & x(2) & \cdots & x(l+1) \\ \vdots & \vdots & & \vdots \\ x(n-l-1) & x(n-l) & \cdots & x(n-1) \end{bmatrix}, \tag{2.37}$$

where $l = \lfloor n/2 \rfloor$. The Singular Value Decomposition (SVD) of this matrix is obtained as $\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$, where $\boldsymbol{\Sigma}$ is a diagonal matrix containing the singular values of matrix $\mathbf{Y}$, denoted by $\varsigma_1, \ldots, \varsigma_{l+1}$. The diagonal entries in $\boldsymbol{\Sigma}$ are sorted in a descending order, i.e., $\varsigma_1 \geq \ldots \geq \varsigma_{l+1}$. A simple noise reduction method works based on setting all singular values that are smaller than ϵ percentage of ς_1 to zero, and then reconstructing samples $x(0), \ldots, x(n)$ from the revised matrix $\mathbf{Y}$. Apply this noise reduction method to the RMS voltage measurements in file `E2-13.csv`. Try $\epsilon = 1\%$ and $\epsilon = 5\%$.

2.14 Consider the *single-mode* oscillation in Figure 2.41. Starting from time $t = 0$, the signal takes the following form: $x(t) = Ae^{\sigma t}\cos(wt + \varphi)$. Obtain the amplitude A, phase angle φ, frequency w, and damping factor σ. You can estimate the frequency by examining the oscillation interval; such as from one positive peak to another positive peak. Other parameters can be estimated similarly by examining various aspects of the signal in the figure.

2.15 In practice, it is common to quantify the damping factor of each oscillatory mode in terms of its *damping ratio*, in percentage, which is obtained as

$$\zeta_i = \frac{-\sigma_i}{\sqrt{\sigma_i^2 + w_i^2}} \times 100\%. \tag{2.38}$$

Obtain the damping ratio for all the oscillatory modes in Table 2.1.

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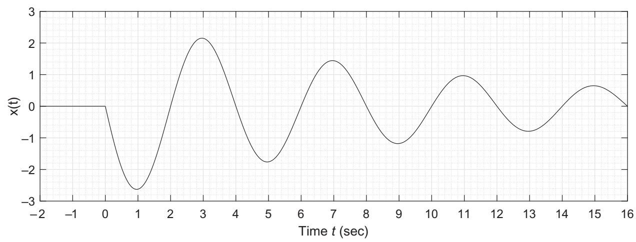

Figure 2.41 Single-mode oscillation in Exercise 2.14. Oscillation starts at time $t = 0$.

2.16 File `E2-16.csv` contains the voltage measurements for the transient oscillations in Figure 2.18. We want to go through the steps to obtain the modes that are shown in Table 2.1. Recall that in Example 2.16 we have $m = 101$.

(a) Obtain vector $\mathbf{\Psi}$ and matrix $\mathbf{X}$.

(b) Solve the LS problem in (2.12) to obtain $a_1, \ldots, a_m$.

(c) Solve the characteristics polynomial in (2.15) to obtain $z_1, \ldots, z_m$.

(d) Use (2.17) to obtain $w_1, \ldots, w_m$ and $\sigma_1, \ldots, \sigma_m$.

(e) Obtain vector $\mathbf{\Phi}$ and matrix $\mathbf{Z}$.

(f) Use (2.19) to obtain $R_1, \ldots, R_m$.

(g) Use (2.20) to obtain $A_1, \ldots, A_m$ and $\varphi_1, \ldots, \varphi_m$.

2.17 Repeat Exercise 2.16, but this time set $m = 201$. Present the oscillation modes in a table similar to Table 2.1.

2.18 The accuracy of the Prony method for any given number of modes m can be evaluated by calculating the *root mean square error* (RMSE) between the original measurements and the Prony estimation of the measurements:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{\tau=1}^{n}\left(x(\tau) - \sum_{i=1}^{m} A_i e^{\sigma_i \tau}\cos(w_i \tau + \varphi_i)\right)^2}. \quad (2.39)$$

(a) Obtain the RMSE for the results in Exercise 2.16.

(b) Obtain the RMSE for the results in Exercise 2.17.

2.19 File `E2-19.csv` contains a voltage profile at a 120 V single-phase load. We want to identify the events in this voltage profile by using the Min-Max method, with a threshold of 2 V. The detection window size is 10.

(a) Use non-overlapping detection windows. List the events that you detect.

(b) Use detection windows with 50% overlap. List the events that you detect.

2.20 Repeat Exercise 2.19, but use the MAD method with $\gamma = 1.4826$ and $\zeta = 3$.

2.21 File `E2-21.csv` contains the minute-by-minute measurements for phase voltages V_A and V_B and line-to-line voltage V_{AB} for a duration of one hour. Plot θ, i.e., the angle between the voltage phasors at Phase A and Phase B, while it changes during the hour; see Eq. (2.27).

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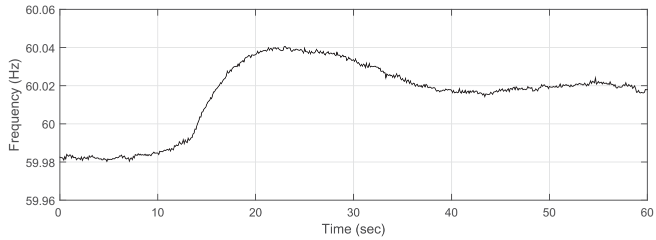

Figure 2.42 Measurements during a system-wide frequency event in Exercise 2.26 [133].

2.22 File `E2-22.csv` contains two sets of second-by-second three-phase voltage measurements that are taken at two close-by locations on the same circuit. The phases for the first set of measurements are labeled as A, B, and C. The phases for the second set of measurements are labeled as a, b, and c.

(a) Obtain the correlation coefficients across all phases, i.e., between Phase A and Phase a, between Phase A and Phase b, between Phase A and Phase c, etc. A total of *nine* correlation coefficients should be calculated.

(b) Solve the phase identification problem. That is, take phases A, B, and C as reference and identify the phase connectivity for Phases a, b, and c.

2.23 File `E2-23.csv` contains minute-by-minute three-phase voltage measurements at a load location over a period of one week.

(a) Plot the histogram of the PU during this week.

(b) Does PU exceed 1% at any time?

2.24 File `E2-24.csv` contains the frequency measurements for duration of one day at one reading per second. Nominal frequency is 60 Hz.

(a) Plot the histogram for these measurements at 0.01 Hz resolution.

(b) What percentage of the measurements are outside the 60 ± 0.036 range?

(c) Plot the frequency during the event in which it drops below 59.95 Hz. How long does the frequency stay below $60 - 0.036$ Hz during this event?

2.25 Based on their FRC values, the system frequency in which North American interconnection is likely to be *least* affected by losing a 500 MW generator?

2.26 Consider the frequency measurements in Figure 2.42.

(a) Suppose the measurements are done in California. Explain the reason for the surge in frequency during the 10th and the 20th seconds. In particular, estimate the amount of change (in MW) of generation or consumption.

(b) Suppose the measurements are done in Texas. Repeat Part (a).

2.27 Consider an energy storage unit with 1.5 MWh energy rating and 250 kW power rating. Suppose this storage unit is used to regulate frequency. The storage unit is discharged whenever the frequency drops below $60 - 0.036 = 59.964$ Hz; see Figure 2.43(a). The storage unit is charged whenever the frequency exceeds above $60 + 0.036 = 60.036$ Hz; see Figure 2.43(b). Suppose the storage

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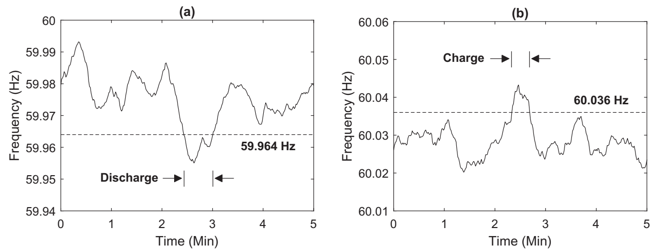

Figure 2.43 Operation of the energy storage unit in response to changes in frequency in Exercise 2.27: (a) charge the storage unit when the frequency drops below a threshold; (b) discharge the storage unit when the frequency exceeds above a threshold.

unit responds to the second-by-second frequency measurements in file `E2-27.csv`. The initial state of charge (SoC) is 50%.

(a) Identify all the charge intervals and all the discharge intervals.

(b) Suppose the storage unit is ideal. Plot the SoC curve versus time.

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