Exercises

1.1 Consider the AC circuit in Figure 1.13, which includes a voltage source:

$$v_{\text{source}} = 120\sqrt{2}\cos(2\pi \times 60 \times t). \tag{1.69}$$

(a) Obtain the current $i(t)$ as a phasor and as a sinusoidal function of time.

(b) Obtain the voltage $v(t)$ as a phasor and as a sinusoidal function of time.

1.2 Consider the circuit in Example 1.3, which consists of two AC voltage sources with the same magnitude at 120 V but different phase angles θ_1 and θ_2. Suppose $\theta_1 = 0°$ is fixed. Suppose θ_2 can vary between 0° and 90°.

(a) Express the current phasor $I\angle\phi$ in terms of parameter θ_2.

(b) Express $I\angle\phi$ in time domain in form of a sinusoidal function $i(t)$.

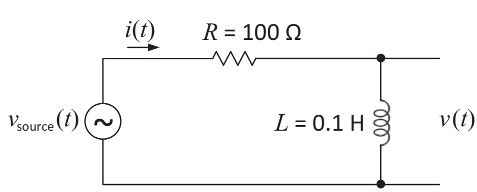


Figure 1.13 The AC circuit in Exercise 1.1.

https://doi.org/10.1017/9781108891448.002 Published online by Cambridge University Press

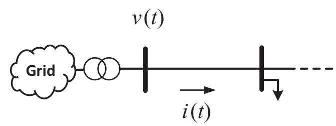


Figure 1.14 The power distribution feeder in Exercise 1.3.

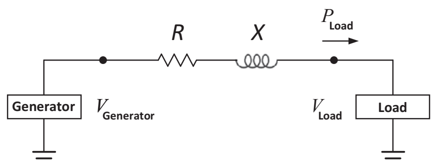


Figure 1.15 The circuit and its parameters in Exercise 1.5.

(c) Plot the instantaneous power $p(t)$ that is dissipated at the $R = 12\Omega$ line resistor. Calculate the average power P that is dissipated at this line resistor for $\theta_2 = 30^\circ$, 45°, and 60°. Explain your observations.

1.3 Consider a portion of a power distribution system as shown in Figure 1.14, where $v(t) = 7200\sqrt{2}\cos(\omega t)$ and $i(t) = 50\sqrt{2}\cos(\omega t) + 28\sqrt{2}\cos(3\omega t)$.
(a) Plot $v(t)$ and $i(t)$ over a period of 100 msec.
(b) Use (1.4) to numerically obtain the RMS value for $v(t)$ and $i(t)$.

1.4 In Exercise 1.3, obtain and plot the instantaneous power $p(t) = v(t)i(t)$ that is drawn from the substation over a period of 100 msec. Also obtain the apparent power $S = V_{\text{rms}} I_{\text{rms}}$ that is drawn from the substation.

1.5 A small generator is trying to deliver $P_{\text{Load}} = 15$ kW of real power through a 120 V (RMS) single-phase power line to a load that has a lagging power factor of 0.9. The circuit is shown in Figure 1.15. The power line has resistance $R = 0.05\ \Omega$ and inductance $X = 0.1\ \Omega$. What RMS voltage should the generator provide on its end of the power line?

1.6 Consider a balanced three-phase 1.5 MVA load with a lagging power factor of 0.75. The line-to-line voltage is 480 V. What is the size of the balanced three-phase capacitor bank that needs to be added to the load to improve the power factor to 0.9? Give your answer in Farads per phase.

1.7 Consider the definition of neutral current in three-phase systems in (1.39). Use the expressions in (1.35) to show that if the three-phase current is balanced, then the neutral current is zero.

1.8 Figure 1.16(a) shows an AC circuit with a voltage source, a resistor, and an inductor. Figure 1.16(b) shows the same AC circuit but in per unit representation. The base values for voltage and impedance are $V_{\text{base}} = 500$ kV and $Z_{\text{base}} = 250\ \Omega$, respectively.
(a) Obtain the base for current, i.e., I_{base}.
(b) Express the current I in per unit.

1.9 A load transformer rated at 1000 kVA is operating near capacity as it supplies a load that draws 900 kVA with a power factor of 0.7 [3].

https://doi.org/10.1017/9781108891448.002 Published online by Cambridge University Press

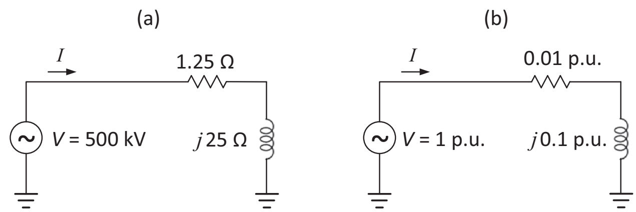


Figure 1.16 The parameters in Exercise 1.8: (a) regular representation; (b) per unit.

(a) How much active power does the load draw?

(b) How much reactive power does the load draw?

(c) How much additional active power can the load draw from this load transformer if the load's power factor remains at 0.7?

(d) How much *additional* active power can the load draw from this load transformer if the load's power factor is corrected to 0.9?

1.10 Consider the three-phase power system in Example 1.8.

(a) Plot $i_A(t)$, $i_B(t)$, and $i_C(t)$ over six cycles.

(b) Obtain and plot the neutral current over six cycles.

1.11 Consider a three-phase load transformer with turn ratio 26:1. The transformer is ideal, i.e., it has no internal power loss or leakage, and it does not cause any shift in the phase angle. Suppose the voltage phasors on the secondary side, i.e., the low-voltage side, of the transformer are as follows:

$$\begin{aligned} V_A \angle \theta_A &= 275.3\angle 14.8^\circ, \\ V_B \angle \theta_B &= 281.6\angle{-105.1}^\circ, \\ V_C \angle \theta_C &= 277.8\angle 134.9^\circ. \end{aligned} \tag{1.70}$$

(a) Obtain the line-to-neutral voltage phasors on the primary side.

(b) Obtain the line-to-line voltage phasors on the primary side.

1.12 Figure 1.17(a) shows the power generation profile of a PV unit on a cloudy day. Three points are marked on the figure. Point (1) is at early morning when solar power generation has not yet started. Point (2) is at around 10 AM when solar power generation is picking up. Point (3) is at around 1 PM when the maximum solar power generation occurs on this day. This PV unit is connected to bus 2 at a 3-bus power distribution system, as shown in Figure 1.17(b). In addition to supplying active power, the PV unit also supplies reactive power at a fixed rate of 127 kVAR. There is a fixed load of 245 kW and 184 kVAR at bus 3. The voltage at bus 1 is fixed at $V_1 = 7200\angle 0$. The impedance of each distribution line is $Z_{\text{Line}} = 0.0412 + j0.0625\ \Omega$. Obtain the voltage phasor at bus 2 and the voltage phasor at bus 3 at each of the following cases:

(a) At point (1) where solar power generation is $P_{\text{PV}} = 0$.

(b) At point (2) where solar power generation is $P_{\text{PV}} = 158$ kW.

(c) At point (3) where solar power generation is $P_{\text{PV}} = 340$ kW.

https://doi.org/10.1017/9781108891448.002 Published online by Cambridge University Press

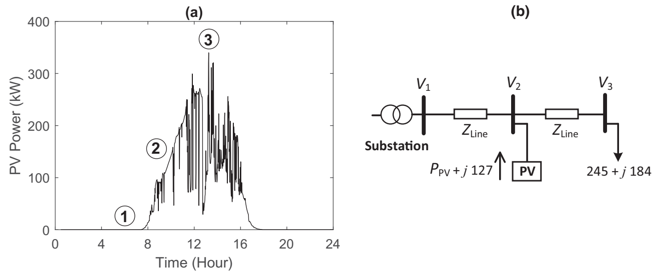


Figure 1.17 The power system in Exercise 1.12: (a) PV power generation; (b) the power distribution network with the PV unit at bus 2 and a fixed load at bus 3.

1.13 Figure 1.18 shows the square wave approximation of the sinusoidal wave in an inverter. Obtain the RMS value of this square waveform.

1.14 Consider the charge-discharge power profile of a 15 kWh energy storage system over a period of 12 hours, as shown in Figure 1.19. A positive power value indicates a charge action, and a negative power value indicates a discharge action. The energy storage system is ideal, meaning that it has 100% charge efficiency and 100% discharge efficiency. The initial state-of-charge (SoC) at the beginning of this time frame, i.e., at time zero, is 20%. Plot the SoC curve in percentage over this time period.

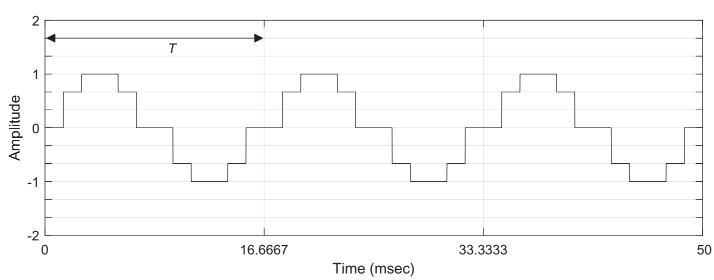


Figure 1.18 The square wave approximation of the sinusoidal wave in Exercise 1.13.

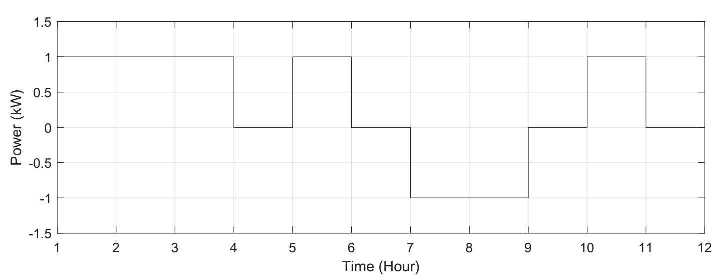
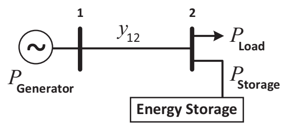


Figure 1.19 The charge-discharge power profile of the storage unit in Exercise 1.14.

https://doi.org/10.1017/9781108891448.002 Published online by Cambridge University Press



Hour	$h=1$	$h=2$	$h=3$	$h=4$
P_{Load}	1.5	1.0	4.0	1.5

Figure 1.20 The power network and the hourly load (in p.u.) in Exercise 1.15.

1.15 Consider the power grid with two buses as shown in Figure 1.20.

(a) The load factor (LF) for a load profile is defined as the ratio of the average amount of the load to the maximum load. What is the load factor for the load profile P_{Load} based on the table that is provided in this figure?

(b) We define the *net load* at bus 2 as $P_{\text{Load}} + P_{\text{Storage}}$, where P_{Storage} denotes the power exchange between the energy storage unit and bus 2. A positive P_{Storage} indicates charging, and a negative P_{Storage} indicates discharging. Suppose the storage unit is *ideal*, i.e., it experiences no loss during the charge and discharge cycles. The initial charge level of the storage unit is zero. Suppose the final charge level (i.e., at the end of hour $h = 4$) of the storage unit is also zero. What is the *minimum power rating* and the *minimum energy rating* needed to achieve a load factor of 0.8 for the *net load*? Assume that the entire charge capacity of the storage unit is usable.

(c) Obtain the required charge and discharge schedule for the storage unit to achieve 0.8 load factor for the net load in Part (b).

(d) Suppose the storage unit has 4/5 = 80% charge efficiency and 3/4 = 75% discharge efficiency. Answer the question in Part (b) in this case.

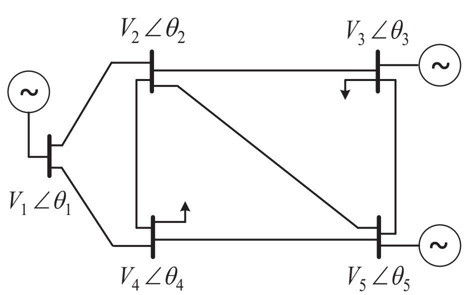
(e) Obtain the required charge and discharge schedule for the storage unit to achieve a 0.8 load factor for the net load in Part (d).

1.16 Consider the 5-bus transmission network in Figure 1.21. The admittance for each transmission line is $0.5 - j10$ p.u. The load at bus 3 is $3.0 + j1.2$ p.u. The load at bus 4 is $2.0 + j0.8$ p.u. There is no shunt capacitor at any bus. The voltage phasors at all buses are also given in the figure.

(a) How much is the active and reactive power injection by each generator?

(b) How much is the total active power loss on transmission lines?

(c) How much is the total reactive power loss on transmission lines?



Bus #	Voltage Phasor (p.u.)
1	$1.047261\angle 19.3593^\circ$
2	$1.005200\angle 16.2548^\circ$
3	$0.966485\angle 11.8524^\circ$
4	$0.991939\angle 14.5205^\circ$
5	$1.021064\angle 18.9208^\circ$

Figure 1.21 The power network and the voltage phasors in Exercise 1.16.

Table 1.1 Active and reactive power injection in Exercise 1.17.

Bus #	Active Power Injection (p.u.)	Reactive Power Injection (p.u.)
1	1.2810	1.1240
3	−1.5680	−0.6460
4	−2.1420	−0.8740
5	2.4506	0.8277

1.17 Consider the 5-bus transmission network in Exercise 1.16. Suppose we know only the voltage phasor at bus 1, which is $1.0104\angle 32.4381^\circ$. We do *not* know the voltage phasors at the rest of the buses. However, we do know the active and reactive power injection at each bus; as shown in Table 1.1.

(a) Calculate the active power flow on each transmission line.

(b) Calculate the reactive power flow on each transmission line.

(c) Calculate the total power loss on all transmission lines.

1.18 Consider the balanced three-phase power distribution system in Figure 1.22. The per-phase impedance of each distribution line is $0.0412 + j0.0625\ \Omega$. The voltage phasors at all buses are shown in Table 1.2.

(a) How much is the active and reactive power injection at each bus?

(b) How much is the active and reactive power flow on each line?

1.19 Consider the power distribution network in Exercise 1.18. We know that the voltage phasor at bus 1 is $7200\angle 0^\circ$. Bus 1 is the reference bus. We do *not* know the voltage phasors at the rest of the buses. However, we do know the active and reactive power injection at all buses, as shown in Table 1.3.

Table 1.2 Voltage phasors at all buses in Exercise 1.18.

Bus #	Voltage Phasor (V)	Bus #	Voltage Phasor (V)
1	$7200.0\angle 27.8214^\circ$	6	$7055.7\angle 27.3628^\circ$
2	$7143.9\angle 27.6195^\circ$	7	$7079.4\angle 27.4349^\circ$
3	$7097.4\angle 27.4856^\circ$	8	$7072.9\angle 27.4784^\circ$
4	$7076.8\angle 27.4269^\circ$	9	$7063.9\angle 27.4148^\circ$
5	$7066.9\angle 27.4413^\circ$		

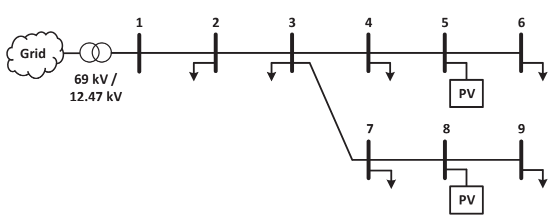


Figure 1.22 The balanced three-phase active distribution feeder in Exercise 1.18.

https://doi.org/10.1017/9781108891448.002 Published online by Cambridge University Press

Table 1.3 Active and reactive power injection in Exercise 1.19.

Bus #	Active Power Injection (kW)	Reactive Power Injection (kVAR)
2	−1104.7	−253.9
3	−592.4	−278.7
4	−1079.2	−342.6
5	993.5	−193.2
6	−1312.0	−316.0
7	−1304.3	−259.1
8	1292.2	−474.2
9	−987.8	−245.8

(a) Calculate the active power flow on each distribution line.
(b) Calculate the reactive power flow on each distribution line.
(c) Calculate the total power loss on all distribution lines.

1.20 Consider the 2-bus power transmission network with two parallel transmission lines, as shown in Figure 1.23. Admittance $y = -j10$ p.u. is fixed. The controllable admittance y_{control} can be adjusted between 0 and $-j20$ p.u. Select y_{control} such that one-fourth of the power transfer from bus 1 to bus 2 flows through the top transmission line and three-fourth of the power from bus 1 to bus 2 flows through the bottom transmission line.

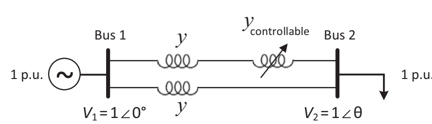


Figure 1.23 The network with controllable admittance in Exercise 1.20.

https://doi.org/10.1017/9781108891448.002 Published online by Cambridge University Press