A Unifying Market Power Measure for Deregulated Transmission-Constrained Electricity Markets

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Abstract—Market power assessment is a prime concern when designing a deregulated electricity market. In this paper, we propose a new functional market power measure, termed *transmission constrained network flow* (TCNF), that unifies three large classes of transmission constrained structural market power indices in the literature: residual supply based, network flow based, and minimal generation based. Furthermore, it is suitable for demand-response and renewable integration and hence more amenable to identifying market power in the future smart grid. The measure is defined abstractly, and allows incorporation of power flow equations in multiple ways; we investigate the current market operations using a DC approximation and further explore the possibility of including detailed AC power flow models through semidefinite relaxation, and interior-point algorithms from Matpower. Finally, we provide extensive simulations on IEEE benchmark systems and highlight the complex interaction of engineering constraints with market power assessment.

Index Terms—Market power, electricity markets.

I. INTRODUCTION

Beginning in 1990 with Chile, electricity markets in many regions have moved from being a vertically integrated regulated monopoly to a deregulated market structure that encourages innovation and competition in technology. The California energy crisis in 2000-01, however, highlights how strategic interaction can erode the benefits of competition in a deregulated market. It is estimated that about $5.55 billion was paid in excess of costs in the deregulated market in California between 1998 and 2001 alone [1]. To avoid such over-payments, monitoring and mitigating market power is essential. It is expected to become even more critical as new smart grid technologies such as intermittent renewable generation, energy storage, and demand-response programs start presenting more opportunities to exploit.

The Department of Justice defines market power as the ability of a firm to profitably alter prices away from competitive levels [2], [3]. In other words, market power is a form of market "dominance", where a player can increase its profitability by behaving independently of competitors and consumers. Market power in generic markets has been extensively studied using microeconomics, e.g., in [4]. The theory, however, does not apply directly to *electricity markets* due to various reasons, such as: (a) Unlike in most commodity markets, electricity cannot be stored cheaply; therefore generators have significant short-run capacity constraints. (b) Electricity demand is typically inelastic because of limited price-responsiveness of consumers. (c) Trade agreement between a supplier and a consumer is not enough to guarantee feasible power delivery over a transmission grid since power transfer respects physical laws as well as market outcomes. Economics or engineering alone cannot handle such issues adequately. In electricity markets, such dominance can be global, e.g., by a supplier with a large enough generation capacity, or local, e.g., by a supplier in a region which has limited ability to import less expensive electricity due to transmission constraints [5].

A. Literature on Market Power

Classically, the literature on market power is fractured. Recently, however, a principled design has begun to emerge, e.g., see [3] for a survey. The analysis of market power can be divided into three distinct categories: (a) structural analysis, (b) competition models, and (c) behavioral analysis.

Structural analysis of market power is based on an *ex ante* approach where the emphasis is on identifying firms that own "must-run" generators and hence have a strategic advantage in terms of market share, location in the network, etc. Such market power studies are also useful in the long-run to evaluate mergers, plan transmission capacity expansions, etc. Competition models analyze the electricity market either as a supply function or a Cournot competition with or without transmission constraints and establish competitive benchmarks for firm behavior using extensive simulations, e.g., see [6]–[9]. Real data is then compared *ex post* to such benchmarks to identify abuses of market power. In contrast, behavioral analysis is another *ex post* approach that detects actual supply withholding or high price-to-cost markups in the spot market as opposed to comparing it with perfectly competitive behavior. We make two observations. First, such ex post analysis indeed correlates with structural indices [10], [11]. Second, ex post analysis with real data can be highly challenging to identify intentional abuse of market power [12], [13]. Thus ex ante structural analysis helps to prevent rather than cure such abuse. In this paper, we focus on structural analysis.

Early work on structural market power analysis, emerging from microeconomics, suggested measures that focussed exclusively on market share based on generator capacities,

e.g., *Herfindahl-Hirschmann index* (HHI) [14]. The major shortcoming of such an analysis in electricity markets is in defining the *relevant market*. Due to demand variations and lack of storage, electricity across different periods of time are not substitutes. Similarly supplies that are geographically located on different ends of a congested transmission line are not substitutes. Thus, market power indices that are agnostic to demand variations and transmission constraints have limited applicability to electricity markets.

To incorporate the demand side, Bushnell et al. introduced the *pivotal supplier index* (PSI) as a binary indicator examining whether the capacity of a generator is larger than the supply surplus, i.e., the difference between the total supply and the total demand [15]. Later, Sheffrin et al. refined PSI by measuring market power on a continuous scale, and proposed the *residual supply index* (RSI) in [16]. This index is used by the California ISO to assure price competitiveness [17]. The electric reliability council of Texas (ERCOT) uses a similar measure called the *element competitiveness index* (ECI) [18], based on HHI [14].

Issues arising due to transmission constraints have also been addressed in the literature. A traditional approach uses the SS-NIP (small but significant non-transitory increase in price) test [19] to identify geographically isolated "load pockets". Many authors have studied Cournot-based or supply-function based markets with congestion, e.g., see [6]–[9], [20], [21]. Structural indices on a transmission constrained network, however, have remained fractured. We have attempted to bridge that gap in this work.

B. Contributions of this paper

In this work, we introduce a *functional* market power measure for structural analysis that unifies the theory to study the complex interactions of this measure with the engineering constraints[1]. The new measure, termed "transmission constrained network flow" unifies three broad classes of structural measures in the literature: "network flow based" [23], [24], "residual supply based" [20], [25], and "minimal generation based" [26], [27]. We introduce each of these classes in detail in Section II. Calculating the new measure in Section III requires us to solve a nonconvex optimization program resulting from the nature of the AC power flow equations. Current electricity markets use a linearized DC approximation [28], [29]. Employing this approximation, we solve a linear program (LP) to compute the market power functional and study it in detail. We further explore the possibility of including a detailed AC power flow model in this economic measure in Section IV in two ways: (a) use interior-point based methods implemented in Matpower [30], (b) use recent advances in semidefinite programming (SDP) based relaxations [31] to AC power flow equations [32]–[35]. In Section V, we provide extensive simulations on IEEE benchmark systems [30] and illustrate the impact of modeling engineering constraints in identifying market power. We extend the index to the case where firms can own generators at multiple locations in Section VI.

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[1]A preliminary version of this work has appeared in [22].

II. MARKET POWER MEASURES

Recently, many indices have been introduced to include the effect of transmission constraints in structural market power indices; we categorize them as: "residual supply based", "network flow based", and "minimal generation based". In what follows, we introduce each of them in detail.

A. Residual supply based measures

Residual supply based measures propose to quantify the maximum total load that the transmission-constrained electricity market can meet if generator of interest, s, is excluded. Following [20], [25], the *transmission-constrained residual supply index* (TCRSI) for generator s is defined as:

$$
\begin{aligned}
\text{TCRSI}_s = \underset{q,t}{\textbf{maximize}} \quad & t \\
\textbf{subject to} \quad & \mathbf{1}^\dagger q = \mathbf{1}^\dagger(\bar{d}t), \\
& -b \le H_q q - H_d(\bar{d}t) \le b, \\
& q_s = 0, \quad 0 \le q_i \le \bar{q}_i, i \ne s.
\end{aligned} \tag{1}
$$

where q is the supply vector, t is the demand scaling parameter, H_q is the generation shift factor matrix, H_d is load shift factor matrix, b is the transmission line capacity vector, $\bar{q}_i$ is the capacity of generator i, $\bar{d}_j$ is the demand of load j, $\mathbf{1}$ is a unit vector, and $\dagger$ denotes transposition. If $\text{TCRSI}_s < 1$, then generator s can potentially exercise market power. Consider the network in Figure 1. For G_1, TCRSI is $3.2/7$, the fraction of demand that can be met with available supply.
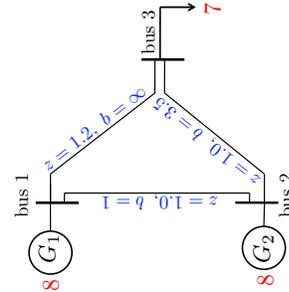

Fig. 1: A small network to illustrate market power indices. All quantities are measured in per units (p.u.). z denotes impedance and b denotes line capacity.

B. Network flow based measures

Network flow based measures are exemplified by [23], [24], which model market power in the presence of transmission constraints in terms of the *maximal network flow* (MNF) achievable without the generator of interest. Conceptually, these measures are similar to TCRSI, but they do not use power flow equations to model the underlying power systems. A key result in [23], [24] is that market power is supermodular, i.e., there is always an incentive for generators to collude. This conclusion, however, does not hold if the power flow respects impedance and follows Kirchoff's laws. See Section

VI for an example in IEEE test systems. Intuitively, one would expect that there is always an incentive to collude since any individual strategy for generators would likely be a valid strategy for a collusion of generators. Market power index measures the demand shortfall due to the absence of a generator. Consider the network in Figure 1. When G_1 withholds generation, G_2 can only supply $3.2pu$; demand shortfall is $3.8pu$. Similarly, when G_2 withholds generation, demand shortfall is $4.33pu$. When both generators withhold, shortfall is the total demand of $7pu$, which is lower than the sum of the two shortfalls computed before. Thus market power is *not* supermodular. Roughly, when power injection from two different generators lead to opposing power flows on a capacity-limited transmission line, then these two generators acting together may not be able to cause more demand shortfall than shortfalls due to each generator withholding alone. This intuition holds for the network in Figure 1.

C. Minimal generation based measures

The above two definitions of market power focus on the fraction of unmet demand when generator at bus s is not in service. An alternate approach is to calculate the minimum generation required from generator s to meet the total target demand. In particular, minimal generation based measures typically identify "must run generators", e.g., [26], [27] are exemplified by the *transmission-constrained minimal generator index* (TCMGI):

$$\text{TCMGI}_s = \underset{q}{\textbf{minimize}} \quad q_s$$
$$\textbf{subject to} \quad \mathbf{1}^\dagger q = \mathbf{1}^\dagger \bar{d}, \quad (2)$$
$$-b \le H_q q - H_d \bar{d} \le b,$$
$$0 \le q_i \le \bar{q}_i.$$

Note that in (1), we have $q_s = 0$ and the total load is scaled by a variable factor t. In (2), however, the output of generator s is a variable and the total demand is a constant. If $\text{TCMGI}_s > 0$, then generator s can exercise market power. In general, TCMGI_s does *not* equal the unmet demand in the network when generator at bus s is not operational. For example, consider the network in Figure 1. It can be checked that $\text{TCMGI}_1 = 4.2pu$ while the shortfall is actually $3.8pu$ when the same generator is not in service. TCRSI_s and TCMGI_s are indeed related; we explore this below.

III. FUNCTIONAL MEASURE OF MARKET POWER

Prior work on structural market power measures in Section II suggests that while a wide variety of measures exist, the literature lacks a unified theory that incorporates economic and engineering constraints. Here we propose a *functional* market power measure rather than a market power *index* that represents a step toward such a unifying measure.

To motivate the measure, consider the following informal definition:

$$\text{TCNF}_s(\rho) = \textbf{maximize} \quad \text{total demand met}$$
$$\textbf{subject to} \quad \text{supply from generator } s \le \rho,$$
$$\text{other network constraints.}$$

The functional TCNF_s maps every scalar $\rho \in [0, \bar{q}_i]$ into the maximum demand that can be satisfied when the (real) power output of generator s is no more than ρ. $\text{TCNF}_s(\rho)$ can also be interpreted as a measure of the minimum amount of load that has to be shed (or dispatched, through demand-side management[2]), provided that the supply of generator s is up to ρ. At different levels of ρ, it measures the relative importance of each generator to meet additional demand, abiding by the network constraints.

The definition can also be interpreted as follows. Consider the optimal power flow (OPF) problem where the objective is to only satisfy demand and the production level of generator s is upper bounded by the parameter ρ. Then, the optimal objective value of this OPF type problem is a function of that variable ρ and hence defines a "functional" measure of market power for generator s.

In the rest of this section, we provide a detailed power flow model and its linearized approximation to arrive at a unifying market power measure that is applicable to the current electricity markets and can be used for the evolving smart grid. Next, we formally define $\text{TCNF}_s(\rho)$ with the engineering constraints.

A. Definition

We begin with some notation. Let $\mathbf{i} = \sqrt{-1}$ and for any complex matrix or number z, let z^H be the complex conjugate transpose of z. Consider a network on n nodes (buses) labeled $1, 2, \ldots, n$. Let p_k^G and q_k^G be the real and reactive power generations at node k. Also let p_k^D and q_k^D be the real and reactive power demands that are met at node k. We denote $s_{kj} := p_{kj} + \mathbf{i}q_{kj}$ as the apparent power flowing from bus k to bus j, where p_{kj} and q_{kj} are the real and reactive power flows, respectively. Thus, power balance equation at each node k becomes

$$(p_k^G - p_k^D) + \mathbf{i}(q_k^G - q_k^D) = \sum_{j:j \sim k} s_{kj}, \quad (3)$$

where $j \sim k$ denotes that buses k and j are connected in the power network. The power generations are assumed to satisfy

$$0 \le p_k^G \le \bar{p}_k^G, \qquad -\beta_k \ p_k^G \le q_k^G \le \beta_k \ p_k^G, \quad (4)$$

where $\beta_k > 0$ is a known constant that depends on the technology, i.e., each generator is assumed to vary its reactive power output within a certain power factor of the real power generation. The total load to be supported at bus k has a target real demand $\bar{p}_k^D$ and a target power factor α_k. The target power factor depends on the type of load at bus k. Thus, the supported demand $p_k^D + \mathbf{i}q_k^D$ satisfies

$$0 \le p_k^D \le \bar{p}_k^D, \qquad q_k^D = \tan(\cos^{-1} \alpha_k) \ p_k^D. \quad (5)$$

Power factors typically range from 0.95 to 0.98 lagging. The apparent power flowing from bus k to bus j is s_{kj} and is

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[2]When there is a deficit in electricity supply, the system operator may call upon consumers to adjust their demand so as to match the supply − an approach which is usually referred to as demand response.

bounded by the thermal and stability limits of the transmission lines as

$$|s_{kj}| \leq f_{kj}, \tag{6}$$

where f_{kj} is the known capacity of the line between buses k and j. Let the voltage at bus k be V_k, and the admittance of the line between buses k and j be y_{kj}. The current flowing from bus k to bus j is $y_{kj}(V_k - V_j)$ and we have

$$s_{kj} = V_k \left[y_{kj}(V_k - V_j) \right]^{\mathrm{H}}. \tag{7}$$

To maintain power quality and the system stability, the voltage magnitude $|V_k|$ at bus k is required to be bounded as follows:

$$\underline{W}_k \leq |V_k|^2 \leq \overline{W}_k, \tag{8}$$

where $\underline{W}_k$ and $\overline{W}_k$ are known constants.

Using the notations introduced above, we are now ready to formally introduce a measure the market power of a generator at node s as follows:

$$
\begin{aligned}
\mathrm{TCNF}_s(\rho) = \textbf{maximize} \quad & \sum_k p_k^D \\
\textbf{subject to} \quad & p_s^G \leq \rho, \\
& (3),(4),(5),(6),(7),(8), \\
\textbf{over} \quad & p_k^G, q_k^G, p_k^D, q_k^D, k = 1, \dots, n, \\
& s_{kj}, \quad k \sim j.
\end{aligned} \tag{9}
$$

We refer to this measure as the *transmission-constrained network flow*. The constraints in (3)-(8) model the impact of the network topology, the underlying circuits, and the transmission line capacities. These constraints make our analysis different from a traditional economic approach to market power. Note that, $\mathrm{TCNF}_s(\rho)$ is a *functional* measure, i.e., it evaluates market power for every given value of parameter ρ.

In Section III-C, we describe how the measure in (9) unifies the three general classes of structural market power measures discussed in Section II. The formulation of the TCNF measure in (9) is general. Depending on how the non-convexity in the power flow equations is tackled, different variations of the TCNF measure can be derived. We focus on the most practical version, which is based on the linear DC approximation, since current electricity markets widely employ this to model the underlying power system. In the rest of this section, we describe this approximation and analyze the resulting TCNF measure in detail. To highlight the generality of the TCNF framework, we also discuss other variations of the measure in Section IV.

B. The DC approximation

Currently, most electricity markets use a linearized DC approximation of the power flow equations. Therefore, the immediate application of our proposed functional market power measure is realized when it is customized based on DC approximation, using the following common assumptions [28], [29]:

- Voltage magnitude $|V_k|$ at each node k is assumed to be at its nominal value, where $V_k = |V_k| \exp(\mathbf{i}\theta_k)$. Thus $|V_k| = 1 pu$.

- Transmission lines are assumed to be loss-less, i.e., $y_{kj} = \mathbf{i} b_{kj}$ is purely imaginary for all pairs $k \sim j$.
- For any pair of buses $k \sim j$, the voltage phase angle differences $\theta_k - \theta_j$ are assumed to be small, i.e., $\sin(\theta_k - \theta_j) \approx \theta_k - \theta_j$ and $\cos(\theta_k - \theta_j) \approx 1$.

Using this approximation, for any pair $k \sim j$, we have

$$s_{kj} = p_{kj} = b_{kj}(\theta_k - \theta_j).$$

It can be checked that there is no reactive power flow in this model; thus, ignoring the reactive power demand constraint in (5), this definition of TCNF_s coincides with the one studied in [22] and can be solved as an LP. Henceforth, we refer to this computation as the *DC case*, denoted by $\mathrm{TCNF}_s^{DC}(\rho)$.

C. Properties of TCNF_s^{DC}

Earlier, we introduced the functional measure $\mathrm{TCNF}_s(\rho)$ and its DC approximated version $\mathrm{TCNF}_s^{DC}(\rho)$ to assess market power. Now, we explore its salient features. $\mathrm{TCNF}_s^{DC}(\rho)$ generalizes network flow based and residual supply based measures. When $\rho = 0$, it indicates the maximal network flow satisfying the DC power flow constraints when generator s withholds generation.

To relate TCNF_s to the minimum generation based measure, consider the *transmission-constrained minimal generation* $\mathrm{TCMG}_s(D)$ for generator s to be defined as follows:

$$
\begin{aligned}
\mathrm{TCMG}_s(D) = \textbf{minimize} \quad & p_s^G, \\
\textbf{subject to} \quad & \sum_k p_k^D = D, \\
& (3),(4),(5),(6),(7),(8), \\
\textbf{over} \quad & p_k^G, q_k^G, p_k^D, q_k^D, k = 1, \dots, n, \\
& s_{kj}, \ k \sim j.
\end{aligned}
$$

This generalizes the minimum generation based measures in [23], [24] to a functional form. It is easy to extend the definition of $\mathrm{TCMG}_s(\cdot)$ to its DC approximated version $\mathrm{TCMG}_s^{DC}(\cdot)$. In the next result, we explore the relationship of the functions $\mathrm{TCNF}_s^{DC}(\cdot)$ and $\mathrm{TCMG}_s^{DC}(\cdot)$; proof is included in the appendix.

Theorem 1. *For each generator s, $\mathrm{TCNF}_s^{DC}(\cdot)$ is a continuous, concave, piecewise linear and non-decreasing function; $\mathrm{TCMG}_s^{DC}(\cdot)$ is a continuous, convex, piecewise linear and non-decreasing function. Moreover, $\mathrm{TCNF}_s^{DC}(\cdot)$ and $\mathrm{TCMG}_s^{DC}(\cdot)$ are inverses of each other, i.e., for any $0 \leq D \leq \mathrm{TCNF}_s^{DC}(\infty)$,*

$$\mathrm{TCNF}_s^{DC} \left[\mathrm{TCMG}_s^{DC}(D) \right] = D.$$

The inverse relationship between $\mathrm{TCNF}_s^{DC}(\cdot)$ and $\mathrm{TCMG}_s^{DC}(\cdot)$ holds for all $0 \leq D \leq \mathrm{TCNF}_s^{DC}(\infty)$. Here, $\mathrm{TCNF}_s^{DC}(\infty)$ is the total demand in the network that can be met when the power generated by generator s is not constrained (it, however, satisfies the generation capacity constraint $0 \leq p_s^G \leq \overline{p}_s^G$ in (4)). Beyond that, the network cannot satisfy the target demand and hence $\mathrm{TCMG}_s^{DC}(D)$ only exists for $0 \leq D \leq \mathrm{TCNF}_s^{DC}(\infty)$.

Next, we illustrate the result of Theorem 1 through an example. Consider the network shown in Figure 1. $\mathrm{TCNF}_s^{DC}(\cdot)$
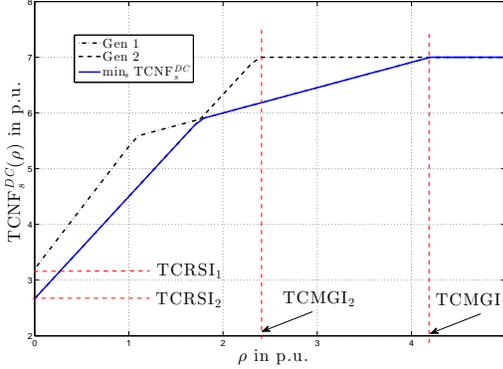
Fig. 2: $\text{TCNF}_s^{DC}(\cdot)$ of generators in the 3-bus network shown in Figure 1. Quantities are measured in per units (p.u.).

is plotted for generators at buses 1 and 2 in Figure 2. Functions $\text{TCNF}_1^{DC}(\cdot)$ and $\text{TCNF}_2^{DC}(\cdot)$ are continuous, convex, piecewise linear and non-decreasing. As noted earlier, $\text{TCNF}_s^{DC}(0)$ equals the TCRSI for generator s. Also, TCMGI for generator s is given by $\min\{\rho \geq 0 \mid \text{TCNF}_s(\rho) = \text{TCNF}_s(\infty)\}$. TCRSI and TCMGI for each generator are indicated in the figure.

Lower the value of $\text{TCNF}_s^{DC}(\cdot)$, higher the market power of generator s. Thus, we plot $\min_s \text{TCNF}_s^{DC}$ by considering the lower envelope of $\text{TCNF}_1^{DC}(\cdot)$ and $\text{TCNF}_2^{DC}(\cdot)$ to indicate the market power of the dominant generator for each $\rho \geq 0$. In this example, the generator with maximum market power changes with ρ. This suggests that market power assessment is complex and cannot be sufficiently captured through a single index.

D. Calculating TCNF_s^{DC}

Computing the DC approximated market power functional requires solving a linear program that is fast and scalable with the size of the network. From Theorem 1, we know that $\text{TCNF}_s^{DC}(\cdot)$ is continuous and piecewise linear. Then we can characterize the slopes of the linear segments of $\text{TCNF}_s^{DC}(\cdot)$ using Lagrangian duality [31]; furthermore, we can use these slopes to provide an efficient way to compute the function. Specifically, for generator s, let μ be the Lagrange multiplier for the constraint $p_s^G \leq \rho$. For any function $f(z)$ in variable z, define $(df(z)/dz)^+$ as its right-hand derivative . We can relate the slopes of the linear segments of the functions $\text{TCNF}_s^{DC}(\rho)$ as follows:

$$\left(\frac{d}{d\rho} \text{TCNF}_s^{DC}(\rho) \right)^+ = \mu_*, \qquad (10)$$

where μ_* is the Lagrange multiplier at the optimum. Recall that $\text{TCNF}_s(\rho)$ is piecewise linear and is non-differentiable at the end-points of each line segment, but the right-hand derivative in (10) is well-defined. Using (10), a recursive algorithm can be developed to compute $\text{TCNF}_s^{DC}(\rho)$ for ρ in any interval $[a, b]$; see [22] for details.

IV. AC POWER FLOW AND MARKET POWER

While DC approximation approach results in a variation of the TCNF measure that best fits the current practice in existing

electricity markets, in this section, we explore the possibility of including a detailed AC power flow model in market power assessment.

To motivate this generalization, notice that structural indices identify pivotal suppliers, i.e., generators that are crucial to meet demand subject to engineering constraints. These constraints, however, are not limited to transmission capacities of the network only and includes limits on voltage magnitudes and reactive powers. Thus, though our focus is on TCNF_s^{DC}, it is interesting to explore the impact of the underlying engineering model on the economics of electricity markets. The analysis in this section also serves to highlight that our market power functional easily generalizes with different models of power flow equations.

As noted in Section III, the calculation of TCNF_s involves the solution of a nonconvex optimization problem in (9). The role of nonconvexity of power-flow equations have played a significant role in the literature on power networks [36]. Traditionally, the engineering problems and market computations have differed in the approaches taken to deal with this nonconvexity. While market outcomes have relied on the DC approximation [15], [16], [18], [22], [23], [25], engineering problems such as real-time economic dispatch have applied heuristics or iterative techniques to reach an implementable operating point [30], [37]. The conic relaxation approach, however, is a recent development and is finding applications in both the engineering and market considerations, e.g., see [32], [33], [35] for its use in optimal power flow and see [38], [39] for its use in electricity markets. In what follows, we present these two computational approaches to assess market power with AC power flow equations: (1) using heuristic iterative nonconvex optimization techniques in Section IV-A, (2) relaxing the non-convex quadratic equality constraint to a convex semidefinite constraint and use conic program solvers in Section IV-B. Our goal in this section is to explicitly characterize the effect of engineering models of the power system to electricity market considerations.

A. Non-linear optimization technique

Many iterative techniques have been used to solve optimization problems in power systems, specifically the optimal power flow problem; see [36], [37] for surveys. Some notable examples are quadratic programming, variations of gradient methods, Newton-based techniques, sequential quadratic programming, and interior-point based methods.

In this work, we use the primal-dual interior-point solver in Matpower [30]. When this converges, we refer to it as TCNF_s^{NL} and call this computation as the *NL case*. Though it is hard to comment on the optimality of the point obtained through this heuristic, the use of Matpower solver provides insights as we explore the simulations on the IEEE benchmark systems. Interior-point methods were popularized by Karmarkar for LPs [40] and Nesterov et al. for SDPs [41]. For LPs and SDPs, it is known that interior point methods converge to a *global* optimal solution in polynomial time. For nonlinear nonconvex problems, they rather provide a heuristic approach to obtain a *local* optimal solution. Matpower has

been known to perform well for economic dispatch problems over various IEEE test systems. As we would show in Section V, the NL case often shows similarity to the DC and the AC cases and provides a yard stick to measure the performance of our proposed DC approximation and the AC relaxation approaches. However, we reiterate that computing TCNF in (9) is NP-hard and thus it is hard to comment on the optimality of the solution obtained using Matpower.

B. The SDP relaxation approach

Recently, a conic relaxation has been proposed to deal with the nonconvexity of power-flow equation in (7), e.g., see [32]–[35]. In particular, consider the $n \times n$ positive semidefinite matrix $W = VV^{\mathrm{H}}$ that has rank one (denoted as $W \succeq 0, \mathrm{rank}\ W = 1$). For each pair of buses $k \sim j$, we express s_{kj} as a linear matrix relation in W as follows. Define an $n \times n$ matrix M^{kj}, where

$$[M^{kj}]_{kk} = y_{kj}^{\mathrm{H}}, \quad [M^{kj}]_{jk} = -y_{kj}^{\mathrm{H}},$$

and rest of the entries of M^{kj} are zero. In terms of M^{kj}, the equality in (7) can be written as

$$s_{kj} = \mathrm{tr}(M^{kj}W).$$

Accordingly, the optimization problem to calculate TCNF becomes a rank-constrained SDP [31] in terms of matrix W. It still remains nonconvex due to the rank constraint. Next, we relax the rank constraint to obtain $\mathrm{TCNF}_s^{AC}(\rho)$ and refer to this computation as the *AC case*.

When the relaxation is exact, it indeed provides a global optimal solution as opposed to the heuristic NL case. The sufficient conditions for exact relaxation, however, are specific to particular network topologies and constraint patterns [33], [34]. When line-flow constraints are active, the relaxation is often inexact, as in [42] and the optimization yields a non rank-1 optimal W_*. To better understand the accuracy of the relaxation, we explore the quantity $\eta := \lambda_2(W_*)/\lambda_1(W_*)$, where $\lambda_1(W_*), \lambda_2(W_*)$ are the first and second eigenvalues of the positive semidefinite matrix W_*, respectively. A lower value for this ratio indicates a smaller optimality gap and hence more accurate results. We report the statistics of the quantity η on IEEE benchmark systems in Section V. We remark that SDP relaxation is known to scale poorly with the size of the network. However, recent results in [35], [43] suggest that the sparsity of the power network can be suitably exploited to obtain fast and scalable conic relaxations.

C. Properties of TCNF_s^{NL} and TCNF_s^{AC}

In Section III-C, we explored the property of the market power functional $\mathrm{TCNF}_s^{DC}(\rho)$. Here, we present a result similar to that of Theorem 1. For the minimum generation based measure, consider the versions of TCMG_s computed using non-linear heuristic techniques of Section IV-A as TCMG_s^{NL} and through the SDP relaxation of Section IV-B as TCMG_s^{AC}. First, we relate TCNF_s^{AC} and TCMG_s^{AC} as follows.

Theorem 2. *For each generator s, $\mathrm{TCNF}_s^{AC}(\cdot)$ is a continuous, concave, and non-decreasing function; $\mathrm{TCMG}_s^{AC}(\cdot)$ is*

a continuous, convex, and non-decreasing function. Moreover, $\mathrm{TCNF}_s^{AC}(\cdot)$ and $\mathrm{TCMG}_s^{AC}(\cdot)$ are inverses of each other, i.e., for any $0 \le D \le \mathrm{TCNF}_s^{AC}(\infty)$,

$$\mathrm{TCNF}_s^{AC}\left[\mathrm{TCMG}_s^{AC}(D)\right] = D.$$

The proof is similar to that of Theorem 1 and is omitted for brevity. From Theorems 1 and 2, it follows that the function $\mathrm{TCNF}_s^{DC}(\cdot)$ satisfies all properties of $\mathrm{TCNF}_s^{AC}(\cdot)$; in addition, it is also piecewise linear, because the optimization problem to compute TCNF_s^{DC} is a linear-parametric LP. This property does not generalize to linear-parametric SDPs; see [31] for a counterexample. The concavity and monotonicity follow from standard arguments on the feasible sets of the respective optimization programs.

Second, unlike the DC and AC approximations, $\mathrm{TCNF}_s^{NL}(\cdot)$ may not be concave since the local minimum over the nonconvex feasible set of the corresponding optimization problem may not be concave. The function $\mathrm{TCNF}_s^{NL}(\cdot)$ may not be monotonically increasing in the interval $[0, \mathrm{TCMG}_s^{NL}(\mathrm{TCNF}_s^{NL}(\infty))]$ in this case, and thus may not be invertible either.

V. CASE STUDIES

In this section, we use our proposed unifying measure to assess market power of generators in various IEEE test systems [30]. In particular, we show how the market power functional TCNF_s^{DC} can be affected by different factors such as the variation of target demand due to distributed renewable generation, changes in dispatchable load in presence of demand-response programs, etc. To investigate the significance of engineering constraints on market outcomes, we further compare the results with TCNF_s^{NL} and TCNF_s^{AC} and in particular, characterize the role of non convexity of power flow equations and uncertainty of load power factors in assessing market power.

In our simulations, we consider the IEEE 6-bus and 39-bus test systems. In each case, we look at a variety of scalings of the target demands in the test systems to understand the impact of demand fluctuation and distributed renewable generation. Specifically, target demands are scaled uniformly by a scalar $t \ge 0$, i.e., each target demand $\overline{p}_k^D$ in the database is multiplied by a factor t to obtain the new target demand for our simulations. We assume that for all generators, the minimum level of generation is zero, i.e., $p_k^G \ge 0$. Most systems have a reactive generation capability defined by $\underline{q}_k^G \le q_k^G \le \overline{q}_k^G$. We modify this box constraint on q_k^G to $-\beta_k p_k^G \le q_k^G \le \beta_k p_k^G$ as in (4), where β_k is chosen accordingly for each case study. To compute $\mathrm{TCNF}_s^{DC}(\cdot)$ and $\mathrm{TCNF}_s^{AC}(\cdot)$, we use the convex programming package CVX [44] in MATLAB with SDPT3 as the SDP solver [45]. Finally, TCNF_s^{NL} is computed using the primal-dual interior-point method in Matpower [30].

A. IEEE 6-bus Test System

The IEEE 6-bus test system has three generators at nodes 1, 2 and 3. For all generators, we assume that $\beta_k = 0.6$ and for all loads we assume that the power-factors are $\alpha_k = 0.98$ lagging. In Figure 3, we plot $\mathrm{TCNF}_s^{DC}(\rho)$, $\mathrm{TCNF}_s^{NL}(\rho)$, and $\mathrm{TCNF}_s^{AC}(\rho)$ for demand scalings of $t = 1.2$ and $t = 1.9$.
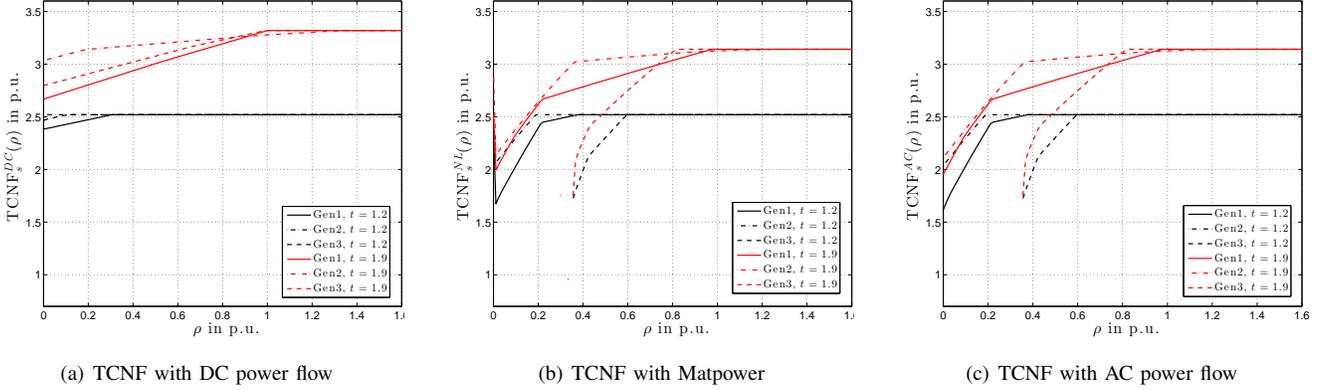
(a) TCNF with DC power flow

(b) TCNF with Matpower

(c) TCNF with AC power flow

Fig. 3: TCNF calculation based on different approaches for various generators in the IEEE 6-bus system.
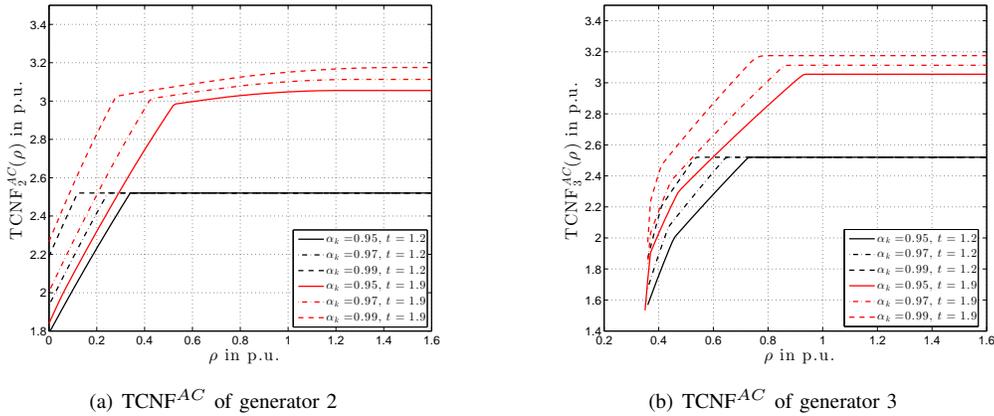

(a) TCNF^{AC} of generator 2

(b) TCNF^{AC} of generator 3

Fig. 4: TCNF$^{AC}(\rho)$ for generators 2 and 3 plotted for load power factors from 0.95 to 0.99 lagging in the IEEE 6-bus system.

For the DC case at $t = 1.9$, consider the total demand level (y-axis) of $3.2pu$, which is lower than the total target demand level. At this demand level, TCNF$_1^{DC}$ has a larger slope than TCNF$_3^{DC}$. Therefore, to satisfy an extra unit of demand at $3.2pu$, generator 1 has to supply less additional power and hence more valuable to the system operator. This means that generator 1 has more market power in an *incremental market*.

Notice, however, that there is a remarkable difference between the AC and the DC cases, while the results from the NL case are similar to that of the AC model. Therefore, in this case study, the SDP relaxation finds a feasible and close to optimal solution of the non-convex optimization problem in (9). In Theorems 1 and 2, the TCNF functions for the DC and AC cases in Figure 3 are increasing and concave for all generators. This property does not generalize for the NL case. Note that, for generator 3, the optimization problem for calculating TCNF$_s^{AC}$ remains infeasible for $\rho \leq 0.35pu$. This indicates that generator 3 is needed to supply at least $0.35pu$ in order to maintain system stability. It is interesting to note that if the SDP relaxation is infeasible, so is the non-linear optimization problem in (9) and hence the interior-point method does not converge to a feasible point for $\rho \leq 0.35pu$. We can also see that TCNF$_s^{NL}$ and TCNF$_s^{AC}$ are quite similar except for generators 1 and 2 at $\rho = 0$, where TCNF$_s^{NL}$ is greater than TCNF$_s^{AC}$. For such a non-convex optimization

problem, determining feasibility is NP-hard and hence it is hard to comment whether the problem in (9) is infeasible at $\rho = 0$. The SDP relaxation TCNF$_s^{AC}$, however, is feasible. Moreover, it is continuous at $\rho = 0$ as expected from Theorem 2.

Another key observation is about the importance of each generator at various demand levels, in presence of *dispatchable load*. In this regard, we see that at the same demand level, TCNF$_s^{DC}$ in Figure 3(a) and TCNF$_s^{AC}$ in Figure 3(c) give conflicting conclusions, while TCNF$_s^{NL}$ in Figure 3(b) agrees with TCNF$_s^{AC}$, indicating that the relaxation approach of AC power flow model is efficient in quantifying market power in the IEEE 6-bus system.

Finally, we conduct a numerical analysis to assess the sensitivity of our proposed market power measure to uncertainty in nodal reactive power values. Consider TCNF$_s^{AC}(\rho)$ in Figures 4(a) for generator 2 and in Figure 4(b) for generator 3, respectively. The plots have been generated with $t = 1.2$ and $t = 1.9$ and the load power factors have been varied from 0.95 to 0.99 lagging uniformly for all buses in each case. We can make two important observations. First, changing the power factor does have impact on market power analysis because the curves for TCNF$_2^{AC}(\rho)$ and TCNF$_3^{AC}(\rho)$ are different. Second, there is no crossing among the TCNF curves in this figure, which means, minor changes in the power factor, e.g., due to

uncertainty in measuring nodal reactive power, do *not* change our conclusions with respect to identifying the generators with highest potential to gain market power. Therefore, while using an AC power flow model may help in better assessing market power due to its inherently more accurate representation of the underlying physical power system, it is not necessarily prone to higher errors due to uncertainty in both active and reactive power measurements. Of course, whether one would prefer to use $\text{TCNF}_s^{DC}(\rho)$ or $\text{TCNF}_s^{AC}(\rho)$ may depend on several factors, such as availability of computational power and existing market practices, which are beyond the scope of this paper.

B. IEEE 39-bus Test System

We now assess our proposed approach for market power analysis in a larger IEEE test system with 39 buses. At each bus s, the value of parameter ρ in function $\text{TCNF}_s(\rho)$ can be interpreted as the amount of curtailable load that is available for dispatch at bus s, in case of losing the generator at bus s. The higher the amount of dispatchable load at a bus, the better the grid operator can handle the loss of a generator at that bus, preventing such generator from gaining market power. However, the effectiveness of the same amount of dispatchable load in mitigating market power may not be the same at different buses. In other words, dispatchable load can be more (or less) valuable at certain locations. For example, consider the simulation results in Figure 5. Here, we are considering the TCNF_s for generators at buses 31, 35 and 38. For the purpose of our analysis, we plot the lower envelope of TCNF_s only, namely $\min_{s \in \{31,35,38\}} \text{TCNF}_s(\cdot)$ for demand levels ranging from $t = 1.0$ to $t = 1.15$. For the case where $t = 1.15$, increasing the dispatchable load capacity is most beneficial when it is done at bus 38 because the generator at bus 38 has the highest potential to gain market power in this case. As another example, for the case where $t = 1.05$, if there is $1pu$ of dispatchable load capacity already in place in all generator buses, then increasing the dispatchable load capacity is most beneficial at bus 31, but if there is $3pu$ of dispatchable load capacity already in place in all generator buses, then increasing the dispatchable load capacity at bus 35 is most beneficial.

We can also make the following observations based on the results in Figure 5: (a) In the DC approximation case, depending on the value of ρ, different generators may gain the maximum market power. However, in the AC case, it is only generator 38 that always maintains the maximum market power for all values of ρ. (b) The DC and the NL cases are more similar to each other than the corresponding AC case. (c) For demand scaling of $t = 1.15$, the DC and NL cases indicate that the total demand that can be met is lower than the total target demand. In the AC case, however, the total target demand of about $71.1pu$ can be satisfied.

C. Summary of findings

We end our case studies by summarizing some of the highlights of our market power functional, as applied to IEEE benchmark systems. First, our proposed measure is suitable to incorporate the impact of demand-response in market power

Test Case	# of Scenarios	Mean η	Max η
6-bus	834	0.0015	0.0044
9-bus	900	0.0034	0.0093
39-bus	900	0.0099	0.0171

TABLE I: Statistics of η for IEEE benchmark systems.

analysis. One option is to analyze demand response by looking at the results at a certain demand level, as we explained in Section V-A. Another option is to analyze demand response in form of quantifying the value of dispatchable loads at different buses, as we explained in Section V-B. Note that, since we study structural market power, our analysis does not involve pricing. Accordingly, it does not address price-elasticity in load demand. However, our case study in Section V-B provides an example on how we can utilize dispatchable loads as an *elastic demand resource* to mitigate market power.

Second, the results in our case studies can also be used to understand the role of renewable generation. For example, similar to the analysis in Section V-B, we can assess renewable generators by examining their impact on parameter ρ. Note that, at a bus where a traditional generator is *co-located* with a renewable generator, the value of ρ is calculated as the *total* power injection by both generators combined. Therefore, we can analyze how the variations in the output of renewable generator may aggravate or mitigate market power of a co-located traditional generator.

Finally, our proposed market power functional easily generalizes to allow consideration of different power flow models. Though our focus is on the DC model, our case studies also highlight that the AC power flow equations provide valuable insights into identifying *reliability must-run* generators that are otherwise not obvious from a DC model. Though uncertainties in reactive power loads can affect the TCNF curves of the generators, the variations tend to be much smaller as compared to the DC case. This suggests that faithfully representing the engineering model significantly affects market considerations.

We end this section with a remark on the accuracy of the SDP relaxations in the computation of TCNF_s^{AC} for IEEE benchmark systems. Recall that the quantity $\eta := \lambda_2(W_*)/\lambda_1(W_*)$ is a measure of accuracy of the relaxation, where $\lambda_1(W_*), \lambda_2(W_*)$ are the first and second eigenvalues of the optimal positive semidefinite matrix W_*, respectively. A lower value corresponds to a smaller optimality gap. We tabulate η for our simulation runs on IEEE benchmark systems in Table I. We see that η is typically very small, but may not be negligible. The optimality gaps may not be accurate to find optimal operating points in economic dispatch, but as far as structural market power analysis is concerned, the results provide valuable insights to the market monitor on the complex interaction of power flow equations with market power assessment.

VI. FIRM BEHAVIOR

Our focus so far has been on identifying market power of a single generator. However, our analysis can easily be extended to the case where a single firm owns multiple generators at different locations. Let $\mathcal{S}$ denote the set of locations (buses) where the firm has a generator. The TCNF index of the

(a) TCNF with DC power flow

(b) TCNF with Matpower
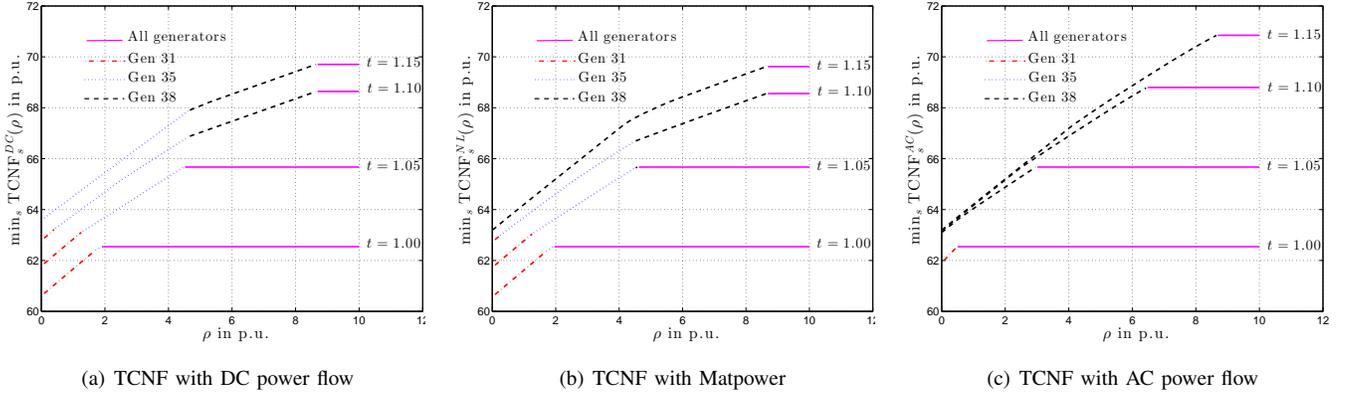
(c) TCNF with AC power flow

Fig. 5: The lower envelope of TCNF, i.e., $\min_s \text{TCNF}_s$ for selected generators in the IEEE 39-bus system.

firm can be defined using the optimization problem (9) with a modified constraint that the *total* supply of the firm's generators does not exceed ρ, i.e., $\sum_{s\in\mathcal{S}} p_s^G \leq \rho$. Similarly, the TCMG index of a firm can be defined as the minimum *total* supply needed from the generators of this firm in order to meet a certain demand level D. This index can be calculated by modifying the objective function to $\sum_{s\in\mathcal{S}} p_s^G$ in the definition of TCMG_s.

Note that, if an "adversarial" firm acts strategically to degrade the performance of the grid, then the behavior of each individual generator (of the firm) might be potentially different if it acted as a separate entity. A game theoretic analysis will be needed to measure the "worst-case" market power of an adversarial firm, which is an area left for future work.

We end this discussion with a note on supermodularity of market power. When market power is supermodular, it suggests that there is an incentive for generators to collude and form large firms. In fact, previous work in [23], [24] has suggested that there is always such an incentive. However, [23], [24] did not use power-flow equations in their study, and so we revisit this question here. Interestingly, it is indeed the case that, most of the time, market power is supermodular. This is not always the case though, e.g., for the IEEE 39-bus system, supermodularity does not hold for $\text{TCNF}_s^{DC}(0)$ for generators at nodes $s = 31$ and $s = 32$ when the line-flow limits are uniformly scaled down to 70% of their given values. Other examples can also be found. While it is often the case that firms have incentive to collude, this is not universally true.

VII. CONCLUDING REMARKS

In this paper, we propose a functional market power measure for structural analysis, called the *transmission constrained network flow*. This measure unifies three directions within market power research – residual supply based measures, network flow based measures, and minimal generation based measures. The measure is useful to study market power with demand response and renewable generation and hence suitable to the needs of the evolving smart grid. Furthermore, it generalizes to the case where firms own multiple generators across the network and can be extended to study possible mergers among generators. Motivated by current market practices, we focus

our analysis on the market power functional with a linearized DC approximation of the power flow equations. However, to highlight the generalized market power framework, we also explore the possibility of incorporating the detailed AC power flow equations in our measure. Extensive simulations on IEEE benchmark systems suggest that conclusions can vary significantly depending on the power flow model considered. This points towards the growing need of faithfully representing the physics of the underlying power system in the design and analysis of electricity markets; engineering and economics in isolation is not enough to capture the complex interaction.

APPENDIX
PROOF OF THEOREM 1

In computing TCNF_s^{DC}, the optimization is an LP lineally parameterized by ρ. Then it is well-known that the optimal objective function (in this case $\text{TCNF}_s^{DC}(\rho)$) is continuous and piecewise linear in the parameter ρ; e.g., see [46, Lemma 2]. Thus, $\text{TCNF}_s^{DC}(\rho)$ is a continuous and piecewise linear function of $\rho \geq 0$. For $0 \leq \rho_1 < \rho_2$, the feasible set for the optimization problem to compute $\text{TCNF}_s^{DC}(\rho_1)$ is a subset of that of $\text{TCNF}_s^{DC}(\rho_2)$ and thus $\text{TCNF}_s^{DC}(\rho)$ is non-decreasing in $\rho \geq 0$. Let the optimal points for problems $\text{TCNF}_s^{DC}(\rho_1)$ and $\text{TCNF}_s^{DC}(\rho_2)$ be x_1 and x_2, respectively. For any $0 \leq \gamma \leq 1$, the point $\gamma x_1 + (1-\gamma)x_2$ is a feasible point for the problem $\text{TCNF}_s^{DC}(\gamma\rho_1 + (1-\gamma)\rho_2)$. Then it follows that $\text{TCNF}_s^{DC}(\rho)$ is concave.

Next, we show that $\text{TCNF}_s^{DC}(\cdot)$ and $\text{TCMG}_s^{DC}(\cdot)$ are inverses of each other. For any $\rho \geq 0$, consider the optimal point for the optimization problem to compute $\text{TCNF}_s^{DC}(\rho)$. This optimum is feasible for the optimization problem $\text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\rho)]$ and we have

$$\text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\rho)] \leq \rho. \tag{11}$$

Similarly, it can be checked that for any $0 \leq D \leq \text{TCNF}_s^{DC}(\infty)$,

$$\text{TCNF}_s^{DC}[\text{TCMG}_s^{DC}(D)] \geq D. \tag{12}$$

For $\rho \in [0, \text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\infty)]$, replacing $D = \text{TCNF}_s^{DC}(\rho)$ in (12), we obtain

$$\text{TCNF}_s^{DC}\left[\text{TCMG}_s^{DC}(\text{TCNF}_s^{DC}(\rho))\right] \geq \text{TCNF}_s^{DC}(\rho). \tag{13}$$

Now, for $\rho \in [0, \text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\infty)]$, we have $\text{TCNF}_s^{DC}(\rho)$ is concave and non-decreasing. Then it is easy to check that $\text{TCNF}_s^{DC}(\rho)$ is monotonically increasing in this interval and hence from (13), it follows that

$$\text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\rho)] \geq \rho.$$

Combining the above relation with (11), we have

$$\text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\rho)] = \rho. \tag{14}$$

The rest follows from the fact that for $0 \leq \rho \leq \text{TCMG}_s^{DC}[\text{TCNF}_s^{DC}(\infty)]$, the map $\text{TCNF}_s^{DC}(\rho)$ is monotonically increasing and hence one-one in this interval.

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