

Tackling Co-existence and Fairness Challenges in Autonomous Demand Side Management

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Abstract—Consider a smart grid system in which every user may or may not choose to participate in Demand Side Management (DSM). This will lead to a general co-existence problem between participant and non-participant users. To gain insights, first, we show that some existing electricity billing mechanisms suffer from severe fairness and co-existence defects. Next, we propose an alternative billing mechanism that can tackle the co-existence and fairness problems by taking into account not only the users' total load, but also the exact shape of their load profiles. Our analytical results provide mild sufficient conditions on the choice of system parameters to assure fairness. Furthermore, our simulation results confirm that the proposed billing mechanism significantly improves the fairness index of the DSM system.

Keywords: Autonomous Demand Side Management, Energy Consumption Scheduling, Co-existence, Fairness, Game Theory.

I. INTRODUCTION

Demand side management (DSM) commonly refers to programs implemented by utility companies to control the energy consumption at the consumer side of the meter [1]. One approach in DSM is direct load control (DLC). In residential DLC, based on an agreement between the utility company and customers, the utility or an aggregator, can remotely control the operations and energy consumption of certain appliances in a household [2]. An alternative for DLC is smart pricing, where users are encouraged to *individually* and *voluntarily* manage their loads, e.g., by reducing their consumption at peak hours [3]. This can be done using automated Energy Consumption Scheduling (ECS) units that are embedded in users' smart meters, as suggested in [4]. For each user, the ECS unit finds the best load schedule to minimize the user's electricity bill while fulfilling the user's energy needs.

The literature on DSM with smart pricing is extensive. One thread of research, e.g., in [5]–[8], focuses on an individual user's load to minimize his daily or monthly energy expenditure. In this viewpoint, the potential impact of each user's load profile on the price of electricity and consequently on other users' electricity expenses is *not* considered. The common analytical tool in such studies is optimization [9].

Another thread of research that has emerged only recently, e.g., in [4], [10]–[13], rather focuses on energy consumption management in a *group* of users that share an energy source or are connected to a shared electric bus of the power grid. Such studies do consider the impact of users' load profiles on other users' energy expenditure. The common analytical

tool in these studies is Game Theory [14]. Both competitive and cooperative game theoretic frameworks have been recently considered and various solution concepts, in particular Nash Equilibrium (NE), have been investigated.

A common assumption in the existing literature on game theoretic analysis of DSM is that *all users* are rational and they are willing to participate in DSM programs. The usual argument to support this assumption is that since participation has financial benefits, users have no reason not to participate. However, it is clear that full penetration of ECS devices will not happen overnight and it is likely to see users who will hesitate to switch to new technologies. Therefore, it is of practical importance to investigate scenarios where only a subset of users in each neighborhood participate in DSM. This will lead to a general *co-existence* problem between *participant* and *non-participant* users that can particularly raise some concerns with respect to DSM fairness.

In this paper, we focus on an autonomous DSM system, similar to the one studied in [4] using game theory. We show that the design in [4] has major drawbacks when it comes to co-existence and fairness. Furthermore, we show that those drawbacks are essentially due to the type of the billing model adopted that charges each user only based on his *total* energy consumption. While the billing model in [4] is optimal and minimizes the total energy cost in the system, it falls short taking into account the shape of the users' exact load profiles. This makes the system unfair in the sense that, two users with the same total load pay the same bill even if they receive energy in different-priced hours. In addition, when DSM participant users co-exist with non-participant users, non-participant users may unfairly benefit from the participant users' efforts to reduce the cost of electricity in the system without having any contribution in minimizing the cost. Such users can eventually become *free riders*. As an alternative for the electricity billing model in [4], here in this paper, we propose a strategy that takes into account not only the user's total load, but also the exact shape of his load profile. The proposed scheme is in fact an improved version of the billing scheme in [4] to obviate the mentioned drawbacks with respect to co-existence and fairness. The rest of this paper is organized as follows. The system model is explained in Section II. The proposed billing scheme is presented in Section III and compared by that of [4] in a simple illustrative example. An analytical case study is provided in Section IV to gain insights

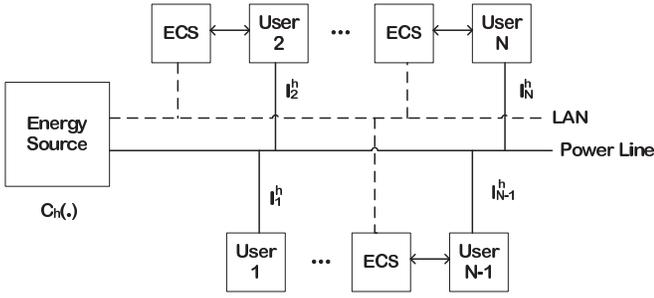


Fig. 1. A power system of N users and one energy source. User 1 does not participate in DSM. Users $2, \dots, N$ use ECS units and participate in DSM.

about the performance of the proposed billing scheme. The simulation results are presented in Section V. Conclusions and future work are discussed in Section VI.

II. SYSTEM MODEL

Consider a smart power grid with N users and one energy source, as shown in Fig. 1 [4]. In this system, each user can choose to be equipped with an ECS unit which can communicate with other users' ECS units as well as the energy source, via a communication infrastructure. In this figure, user 1 does not have an ECS unit and does not participate in demand side management. At every hour h , the cost of electricity $C_h(\cdot)$ is a function of the total load at hour h . As an example, for thermal generators, we have [15]

$$C_h(L_h) = a_h L_h^2 + b_h L_h + c_h,$$

where L_h is the total load at hour h and cost parameters $a_h, b_h > 0$ and $c_h \geq 0$ are fixed for every hour h .

The ECS units manage the users' controllable load for the next H hours, e.g., $H = 24$ for a daily scheduling, in order to reduce the users' electricity bills. For example, washer, dryer, dishwasher, and plug-in electric vehicles are controllable and their operations can be shifted over time. For the ease of presentation, here we assume that each user has exactly one load, e.g., one appliance, which is controllable. The operation of such load can be shifted over time. Let E_n denotes the total energy needed to finish the operation of the load for user $n \in \{1, \dots, N\}$. The load of user n should be scheduled within the time interval between α_n and β_n to be indicated by the user. For example, the user may set $\alpha_n = 1:00$ PM and $\beta_n = 5:00$ PM for the operation of a dishwasher after the lunch table and before diner. Let $(\ell_n^1, \ell_n^2, \dots, \ell_n^H)$ denote user n 's load profile, where ℓ_n^h is user n 's scheduled energy consumption at hour h . To assure on time operation of appliances, for each user n , it is required that user n 's ECS fulfill the following constraints

$$\sum_{h=\alpha_n}^{\beta_n} \ell_n^h = E_n \quad (1)$$

and

$$\ell_n^h = 0 \text{ for all } h \notin \{\alpha_n, \alpha_n + 1, \dots, \beta_n\}. \quad (2)$$

User n 's ECS unit should seek to find the optimal load profile that satisfies (1) and (2) and minimizes user n 's individual

electricity bill. Since users choose their load profiles in a distributed manner and given the fact that each user's bill amount is affected not only by his load but also by other users' loads due to their impact on the energy cost in the system, we can identify the following DSM game among users.

- Players: Users $n = 1, 2, \dots, N$.
- Sets of Actions: For every user n , the set of actions is the set of load profiles $(\ell_n^1, \dots, \ell_n^H)$ satisfying (1) and (2).
- Payoff Functions: For every user n , the payoff function is $-B_n$ where B_n is the electricity bill of user n .

Clearly, different billing schemes may yield to different N.E. in the above game. For example, in [4], users' electricity bills are proportional to their total energy consumption. That is, for every two users n and m , we have

$$\frac{\tilde{B}_n}{\tilde{B}_m} = \frac{E_n}{E_m}, \quad (3)$$

where \tilde{B}_n is the bill of user n according to the billing mechanism in [4]. From this, together with the budget balance requirement that users' total bills should match the total energy cost in the system, each user n 's bill was formulated in [4] as

$$\tilde{B}_n = \frac{E_n}{\sum_{m=1}^N E_m} \times \sum_{h=1}^H C_h \left(\sum_{n=1}^N \ell_n^h \right). \quad (4)$$

From (4), any two users with equal *total load* will pay equally on their bills. This holds even if one user is a participant user and the other user is non-participant. This may encourage users *not* to participate or do *not* show major flexibility in their load profile as the exact load profile of a user does not have any impact on how he will be billed. Next, we propose an alternative billing scheme that fixes these problems at N.E..

III. FAIR BILLING FOR DEMAND SIDE MANAGEMENT

A. Illustrative Example

Assume that $N = 3$ users share an energy source. We have $E_1 = E_2 = 10$ kw and $E_3 = 12.5$ kw. The users want to schedule their load for the next $H = 4$ hours. User 1 is *not* flexible and insists to operate his load within the first hour $h = 1$. That is, $\alpha_1 = \beta_1 = 1$. User 2 is *partially flexible* and allows load distribution within the first two hours $h = 1, 2$. That is, $\alpha_2 = 1$ and $\beta_2 = 2$. User 3 is *completely flexible* and allows load distribution at any time. That is, $\alpha_3 = 1$ and $\beta_3 = 4$. According to the billing scheme in [4], we have

$$\tilde{B}_1 = \tilde{B}_2 = \frac{10}{32.5} \sum_{h=1}^4 C_h \left(\sum_{n=1}^3 \ell_n^h \right)$$

and

$$\tilde{B}_3 = \frac{12.5}{32.5} \sum_{h=1}^4 C_h \left(\sum_{n=1}^3 \ell_n^h \right).$$

Clearly, the bills of every user is minimized if and only if the total cost of energy in the system is minimized. This will encourage users to make efforts to minimize the total cost of the system. In fact, this is the main advantage of the billing system in [4]. Next, assume that the hourly cost functions are $C_1(x) = C_2(x) = 0.01x^2 + 2x$ and $C_3(x) = C_4(x) =$

TABLE I

THE N.E. FOR THE ILLUSTRATIVE EXAMPLE WHEN BILLING IS AS IN [4].

user n	load schedule of user n				bill \tilde{B}_n
	ℓ_n^1	ℓ_n^2	ℓ_n^3	ℓ_n^4	
1	10	0	0	0	17.49
2	0	10	0	0	17.49
3	0	0	6.25	6.25	21.86

$0.03x^2 + x$. The N.E. strategies and the users' bills can be obtained as shown in Table I. The total cost of energy in the system at N.E. becomes \$56.84. We can see that although user 2 is more flexible than user 1, users 1 and 2 end up paying equally on their bills. Next, we will explain how this unfair aspect can be solved using an alternative billing mechanism.

B. Alternative Billing Mechanism

To solve the problem with respect to fairness in Section III-A, next we propose an alternative billing scheme that incorporates the exact shape of each user's load profile. According to this new billing mechanism, for each user n , we have

$$B_n = \sum_{h=1}^H B_n^h,$$

where B_n^h is user n 's bill at hour h . The hourly bills are set such that for every users n and m , we have

$$\frac{B_n^h}{B_m^h} = \frac{\ell_n^h}{\ell_m^h}. \quad (5)$$

From (5) and given the budget balance requirement that total hourly bills should match the total hourly cost of electricity, user n 's hourly bill at hour h is obtained as

$$\begin{aligned} B_n^h &= \frac{\ell_n^h}{\sum_{m=1}^N \ell_m^h} \sum_{m=1}^N B_m^h \\ &= \frac{\ell_n^h}{\sum_{m=1}^N \ell_m^h} C_h \left(\sum_{m=1}^N \ell_m^h \right). \end{aligned}$$

Consequently, user n 's daily electricity bill is calculated as

$$B_n = \sum_{h=1}^H \frac{\ell_n^h}{\sum_{m=1}^N \ell_m^h} C_h \left(\sum_{m=1}^N \ell_m^h \right). \quad (6)$$

Comparing (4) and (6), and also (3) and (5), we can see that the alternative billing scheme proposed in this section incorporates the exact *hour-by-hour* load profile of each user.

Next, consider the example in Section III-A. Employing the billing scheme in (6), the N.E. strategies and the bill of every user are changed as shown in Table II. We can see that unlike the billing mechanism (4) that charged users 1 and 2 equally, simply because they have equal total energy consumption, the proposed alternative billing scheme in (6) charges user 2 about 1.8% less than user 1 due to user 2's more flexibility in his energy consumption. Furthermore, while user 3 has 25% higher total daily energy consumption compared to users 1

TABLE II

THE N.E. FOR THE ILLUSTRATIVE EXAMPLE WHEN BILLING IS AS IN (6).

user n	load schedule of user n				bill B_n
	ℓ_n^1	ℓ_n^2	ℓ_n^3	ℓ_n^4	
1	10	0	0	0	21.25
2	2.50	7.50	0	0	20.87
3	0	0	6.25	6.25	14.84

and 2, it is charged 40% less due to its complete flexibility in energy consumption scheduling. These results can motivate users to be more flexible and to remain a participant user. The examples in this section suggest that the proposed alternative billing scheme can potentially fix the fairness and co-existence problems of the billing mechanism in [4].

IV. ANALYTICAL RESULTS

Consider the DSM system in Section II and assume that $H = 2$ hours and the cost function is in the form of

$$C_h(x) = a_h x^2 + b_h x, \quad (7)$$

for each hour $h = 1, 2$. User 1 is not flexible in his load. Therefore, it does not participate in DSM. In fact, we have $\alpha_1 = \beta_1 = 1$. For any other user $n = 2, \dots, N$, they are participant users and we have $\alpha_n = 1$ and $\beta_n = 2$. In this setup, the load profile of user 1 is $(E_1, 0)$ and the load profile of any other user n is in the form of $(\ell_n^1, E_n - \ell_n^1)$, where $\ell_n^1 \in [0, E_n]$. Every participant user n , more specifically his ECS unit, wants to determine ℓ_n^1 such that his bill B_n is minimized. Without loss of generality, we sort users indexes such that

$$E_2 \leq E_3 \leq \dots \leq E_N. \quad (8)$$

Next, we characterize the N.E. of the DSM game.

Theorem 1: Consider the above system model and assume that the electricity billing scheme is as in (6).

a) If $E_1 \geq \frac{b_2 - b_1}{a_1}$, then $(l_2^1, l_3^1, \dots, l_N^1)$ is a N.E. for the DSM game, where for each user $n = 2, \dots, q - 1$, we have

$$\ell_n^{1*} = 0, \quad (9)$$

and for each user $n = q, \dots, N$, we have

$$\begin{aligned} \ell_n^{1*} &= \frac{1}{N - q + 2} \left[-E_1 + \frac{a_2 \sum_{m=1}^{q-1} E_m - b_1 + b_2}{a_1 + a_2} \right] \\ &\quad + \frac{a_2}{a_1 + a_2} E_n. \end{aligned} \quad (10)$$

Here q denotes the smallest $n = 2, \dots, N$ such that

$$\frac{a_2}{a_1 + a_2} E_n \geq \frac{1}{N - n + 2} \left[E_1 - \frac{a_2 \sum_{m=1}^{n-1} E_m - b_1 + b_2}{a_1 + a_2} \right]. \quad (11)$$

b) If $E_1 < \frac{b_2 - b_1}{a_1}$, then $(l_2^1, l_3^1, \dots, l_N^1)$ is a N.E. for the DSM game, where for each user $n = 2, \dots, p - 1$, we have

$$\ell_n^{1*} = E_n, \quad (12)$$

and for each user $n = p, \dots, N$, we have

$$\ell_n^{1*} = \frac{1}{N-p+2} \left[\frac{-a_1 \sum_{m=1}^{p-1} E_m - b_1 + b_2}{a_1 + a_2} + \frac{a_2}{a_1 + a_2} E_n \right] \quad (13)$$

Here p denotes the smallest $n = 2, \dots, N$ such that

$$\frac{a_1}{a_1 + a_2} E_n \geq \frac{1}{N-n+2} \left[\frac{-a_1 \sum_{m=1}^{n-1} E_m - b_1 + b_2}{a_1 + a_2} \right]. \quad (14)$$

The complete proof of Theorem 1 is omitted due to page limit. Instead, the sketch of the proof is provided in Appendix A.

Now, assume that there exists a participant/flexible user $n \in \{2, \dots, N\}$ such that $E_n = E_1$. Given the fact that users 1 and n have the same total load but user n is more flexible in his energy consumption schedule, the proposed alternative billing mechanism is *fair* only if at N.E. we have $B_n \leq B_1$. That is, the participant/flexible user n should not be charged more than the non-participant/non-flexible user 1. The equality $B_n = B_1$ should occur only if users 1 and n have exactly the same load profile, i.e., when we have $l_1^{1*} = l_n^{1*} = E_n = E_1$.

Theorem 2: The DSM system explained in this section is fair if we have $b_2 \leq b_1$, where b_1 and b_2 are defined in (7).

The proof of Theorem 2 is given in Appendix B. From Theorem 2, *the proposed billing scheme guarantees fairness if proper cost functions are employed by the utility.*

Remark 1: Interestingly, if $b_2 \leq b_1$, then achieving fairness does not depend on the other cost parameters a_1 and a_2 .

Remark 2: Constraint $b_2 \leq b_1$ is a sufficient, not a necessary, condition for fairness. In fact, if $b_2 > b_1$, then depending on the values of energy consumptions E_1, E_2, \dots, E_N and other parameters, the system may or may not be fair.

V. SIMULATION RESULTS

While extending the analytical results in Section IV to more general DSM scenarios is a challenging future task and beyond the scope of this conference paper, using computer simulations we can still investigate the performance of the proposed electricity billing mechanism and compare it with the one in [4], under various system parameters. First, we define a fairness index to facilitate quantitative fairness evaluation.

A. Fairness Index

In a fair DSM system, we expect rewarding users with more flexible load. First, assume that we come up with a measure I_n to evaluate flexibility/inflexibility of user n 's load. In that case, a fairness index associated with the billing mechanism of interest B_1, B_2, \dots, B_N can be defined as

$$F = \sum_{n=1}^N \left| \frac{B_n}{\sum_{m=1}^N B_m} - \frac{I_n}{\sum_{m=1}^N I_m} \right|. \quad (15)$$

From (15), a lower F indicates a more fair billing. Here, the fairness index is defined as the variational distance between normalized billing vector and normalized inflexibility vector. However, other distance measures may also be considered.

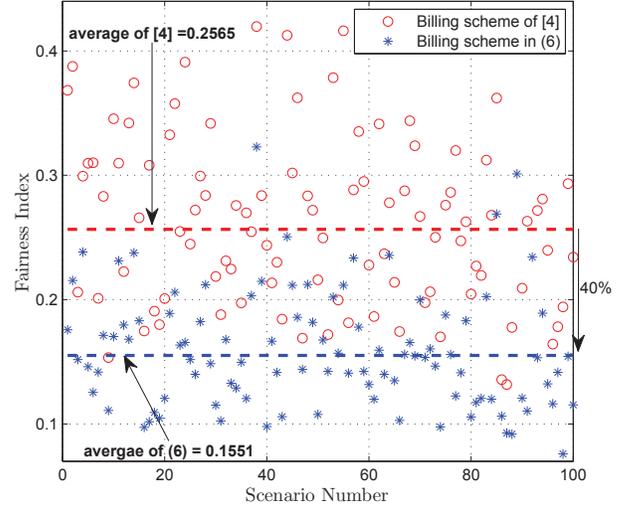


Fig. 2. Comparing the proposed billing and the one in [4] in terms of fairness.

Next, we define the individual flexibility/inflexibility index I_n for each user n . In particular, we introduce a model to assess user n 's load inflexibility as

$$I_n = \frac{E_n}{\beta_n - \alpha_n + 1} \sum_{h=\alpha_n}^{\beta_n} K(h), \quad (16)$$

where for each $1 \leq h \leq H$, we have

$$K(h) = \sum_{m: \alpha_m \leq h \leq \beta_m} \frac{E_m}{\beta_m - \alpha_m + 1}.$$

We note that $K(h)$ indicates the total load in the system at hour h if each user m evenly distributes his total energy consumption E_m across the feasible interval (α_m, β_m) . Therefore, hours h with higher $K(h)$ can be interpreted as peak hours. Therefore, index I_n can be seen as a way to evaluate whether or not user n has chosen α_n and β_n such that it would concentrate its load at peak hours. Furthermore, from (16), the inflexibility measure I_n increases as user n selects inflexible energy consumption scheduling intervals (α_n, β_n) , i.e., when the number $\beta_n - \alpha_n + 1$ is small. Combining (15) and (16) the fairness index F can be calculated.

B. Performance Comparison

In our benchmark smart grid system, there are $N = 20$ users scheduling their loads for the next $H = 24$ hours. The total load E_n for each user n is randomly selected between 0 and 40. Also, in our simulation model, the values of α_n and β_n are randomly generated such that the overall load profile of the system looks similar to the practical load profile, e.g., we have two peak hours around 11 AM and 19 PM. The energy cost functions are considered to be $C_h(x) = 0.01x^2 + 2x$ for $h < 12$ and $C_h(x) = 0.03x^2 + x$ for $h \geq 12$.

The simulation results on the average fairness index of the proposed billing scheme as well as the electricity billing scheme in [4] are shown in Fig. 2. We can see that, by using

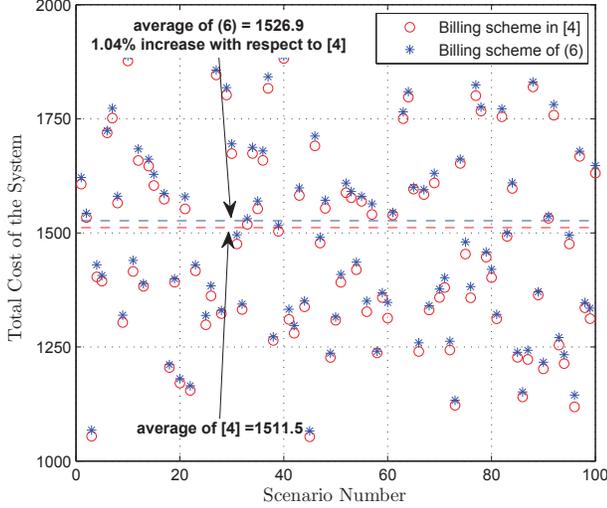


Fig. 3. Comparing the proposed billing mechanism and the one in [4] in terms of minimizing the total energy cost in the system. We can see, while our design achieves fairness, its energy cost optimality loss is very minor.

the billing mechanism that we proposed in (6), the average fairness index of the DSM system is improved by 40%. As mentioned before, the billing mechanism in [4] aims to enforce minimizing the total energy cost of the DSM system. Thus, we expect that in our alternative billing approach, we face a trade-off between achieving fairness and minimizing the total energy cost in the DSM system. Nevertheless, the simulation results show that the increase of total energy cost in comparison with the least-cost billing strategy in [4] is minor and around only 1% on average across different scenarios, as shown in Fig. 3.

Next, we assess fairness in DSM co-existence scenario. The results are shown in Fig. 4. In this figure, for different percentage of users' participation, we have plotted the average fairness index for both billing mechanisms. The results show that for different participation percentages, our billing can significantly improve fairness in comparison with the billing mechanism in [4]. In particular, when the percentage of users' participation is low, fairness improvement is more significant.

VI. CONCLUSION AND FUTURE WORK

This paper represents a first step towards tackling the co-existence and fairness problems in autonomous demand side management. To gain insights, we considered a smart grid system with an energy source that is shared by a group of users who may choose to participate in a DSM program. Each participant user tries to minimize his bill by shifting his load from peak hours to off-peak hours. Using a game theoretic analysis, we tackled the fairness and co-existence problems among participant and non-participant users at Nash Equilibrium of the formulated DSM game. In this regard, we proposed a novel billing mechanism that takes into account not only the user's total load, but also the shape of his load profile. Our analytical results provided mild sufficient conditions on the choice of system parameters to assure fairness. Furthermore, our simulation results confirmed that

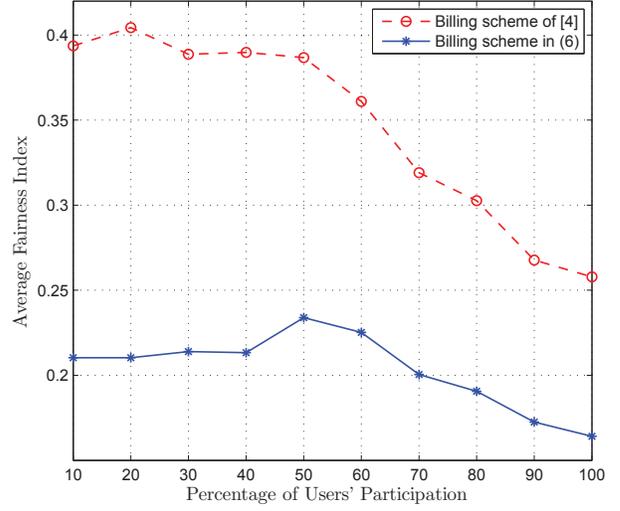


Fig. 4. The average fairness index versus the percentage of users' participation in DSM program for the proposed billing scheme and the one in [4].

the proposed billing improves the average fairness index of the system in return for only a very minor increase in total energy cost in the system, suggesting a reasonable trade-off between efficiency and fairness in DSM systems.

The ideas and results in this paper can be extended in various directions. First, one can extend the analytical results to cover more general system models. Second, the users' payoff functions can be adjusted to include both electricity bills and also users' comfort. Finally, while the proposed billing mechanism in this paper is promising, the problems of co-existence and fairness can be further investigated for potentially other and more advanced billing methods.

APPENDIX

A. Sketch of the Proof of Theorem 1

In case **a**, we need to show that for every user n , the best response is $\ell_n^{1*} \in [0, E_n]$ if all other users $m \neq n$ choose action ℓ_m^{1*} , where ℓ_m^{1*} and ℓ_n^{1*} are defined by (9) and (10). Because of the differentiability of B_n , it is sufficient to show that

- 1) $0 \leq \ell_n^{1*} \leq E_n$ for all $2 \leq n \leq N$.
- 2) $\frac{dB_n}{d\ell_n^1} \geq 0$ for $n \leq q-1$ and $\frac{dB_n}{d\ell_n^1} = 0$ for $n \geq q$.

Using (6) and (7), we can show that

$$\frac{dB_n}{d\ell_n^1} = (a_1 + a_2)(\ell_n^1 + L_T) - a_2(E_T + E_n) + b_1 - b_2, \quad (17)$$

where L_T and E_T are the total load and the total need for energy that is, $L_T = \sum_{m=1}^N \ell_m^1$ and $E_T = \sum_{m=1}^N E_m$. From (9) and (10), the value of L_T in our proposed billing strategy is obtained as

$$L_T^* = \frac{E_1}{N-q+2} + \frac{a_2}{a_1+a_2} \sum_{m=q}^N E_m + \frac{N-q+1}{N-q+2} \times \frac{a_2 \sum_{m=1}^{q-1} E_m - b_1 + b_2}{a_1+a_2}. \quad (18)$$

Substituting (10) and (18) in (17), for each $n \geq q$, we have

$$\frac{dB_n}{d\ell_n^1} = (a_1 + a_2)(\ell_n^{1*} + L_T^*) - a_2(E_T + E_n) + b_1 - b_2 = 0. \quad (19)$$

For each $n \leq q - 1$, from (9) and (17), we can write

$$\begin{aligned} \frac{dB_n}{d\ell_n^1} &= (a_1 + a_2)(L_T^*) - a_2(E_T + E_n) + b_1 - b_2 \\ &= \frac{a_1 + a_2}{N - q + 2} \left[E_1 + \frac{-a_2 \sum_{m=1}^{q-1} E_m + b_1 - b_2}{a_1 + a_2} \right] - a_2 E_n. \end{aligned} \quad (20)$$

On the other hand, from the definition of q in (11), we have

$$\frac{a_2 E_{q-1}}{a_1 + a_2} < \frac{1}{N - q + 3} \left[E_1 - \frac{a_2 \sum_{m=1}^{q-2} E_m - b_1 + b_2}{a_1 + a_2} \right]. \quad (21)$$

Subtracting $\frac{1}{N - q + 3} \times \frac{a_2}{a_1 + a_2} E_{q-1}$ from both sides in (21), yields

$$\frac{a_2 E_{q-1}}{a_1 + a_2} < \frac{1}{N - q + 2} \left[E_1 + \frac{-a_2 \sum_{m=1}^{q-1} E_m + b_1 - b_2}{a_1 + a_2} \right]. \quad (22)$$

From (20) and (22), for each $n \leq q - 1$, we have

$$\frac{dB_n}{d\ell_n^1} > a_2 E_{q-1} - a_2 E_n \geq 0, \quad (23)$$

where the last inequality comes from (8). From (19) and (23), it is concluded that ℓ_n^{1*} is the best response of user n .

To complete the proof, we must show that $0 \leq \ell_n^{1*} \leq E_n$. Clearly, it is true for $n \leq q - 1$. Using (22), for each $n \geq q$

$$\begin{aligned} \ell_n^{1*} &= \frac{1}{N - q + 2} \left[-E_1 + \frac{a_2 \sum_{m=1}^{q-1} E_m - b_1 + b_2}{a_1 + a_2} \right] + \frac{a_2}{a_1 + a_2} E_n \\ &< -\frac{a_2}{a_1 + a_2} E_{q-1} + \frac{a_2}{a_1 + a_2} E_n < E_n. \end{aligned}$$

On the other hand, from (8) and (11), for each $n \geq q$, we have

$$\frac{a_2}{a_1 + a_2} E_n \geq \frac{1}{N - q + 2} \left[E_1 + \frac{-a_2 \sum_{m=1}^{q-1} E_m + b_1 - b_2}{a_1 + a_2} \right],$$

which means $\ell_n^{1*} \geq 0$ and the proof is complete.

The proof for case **b** is similar and is omitted due to space limitation.

B. Proof of Theorem 2

Assume $E_n = E_1$ for some participant/flexible user $n \geq 2$. From (6) and (7), the bills of users 1 and n are equal to

$$B_1 = a_1 E_1 L_T + b_1 E_1$$

and

$$B_n = a_1 \ell_n^1 L_T + b_1 \ell_n^1 + a_2 (E_n - \ell_n^1) (E_T - L_T) + b_2 (E_n - \ell_n^1),$$

respectively, where L_T and E_T are the total load and energy consumption of all users. From Theorem 1 and since $E_n = E_1$, at N.E. we have

$$B_1^* - B_n^* = (E_1 - \ell_n^{1*}) [(a_1 + a_2) L_T^* - a_2 E_T + b_1 - b_2]$$

where L_T^* is the value of L_T at N.E.. By definition, $E_1 - \ell_n^{1*} \geq 0$. Therefore, fairness in the billing system is achieved if we have

$$(a_1 + a_2) L_T^* - a_2 E_T + b_1 - b_2 > 0. \quad (24)$$

If $b_2 \leq b_1$, then we always have $E_1 \geq \frac{b_2 - b_1}{a_1}$. In this case L_T^* is as (18). Therefore, we can rewrite (24) as

$$E_1 > \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \sum_{m=2}^{q-1} E_m. \quad (25)$$

On the other hand, using the definition of q in (11), we have

$$\frac{a_2}{a_1 + a_2} E_{q-1} < \frac{-1}{N - q + 3} \left[-E_1 + \frac{a_2 \sum_{m=1}^{q-2} E_m - b_1 + b_2}{a_1 + a_2} \right]$$

which results in (25) as shown bellow:

$$\begin{aligned} E_1 &> \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \left[(N - q + 3) E_{q-1} + \sum_{m=2}^{q-2} E_m \right] \\ &= \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \left[(N - q + 2) E_{q-1} + \sum_{m=2}^{q-1} E_m \right] \\ &> \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \sum_{m=2}^{q-1} E_m. \end{aligned}$$

Hence, (24) is satisfied and the system is fair. \blacksquare

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