Achieving Optimality and Fairness in Autonomous Demand Response: Benchmarks and Billing Mechanisms

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***Abstract*—Autonomous demand response (DR) programs are scalable and result in a minimal control overhead on utilities. The idea is to equip each user with an energy consumption scheduling (ECS) device to automatically control the user's flexible load to minimize his energy expenditure, based on the updated electricity pricing information. While most prior works on autonomous DR have focused on coordinating the operation of ECS devices in order to achieve various system-wide goals, such as minimizing the total cost of generation or minimizing the peak-to-average ratio in the load demand, they fall short addressing the important issue of *fairness*. That is, while they usually guarantee optimality, they do not assure that the participating users are rewarded according to their *contributions* in achieving the overall system's design objectives. Similarly, they do not address the important problem of *co-existence* when only a sub-set of users participate in a deployed autonomous DR program. In this paper, we seek to tackle these shortcomings and design new autonomous DR systems that can achieve both optimality and fairness. In this regard, we first develop a centralized DR system to serve as a benchmark. Then, we develop a smart electricity billing mechanism that can enforce both optimality and fairness in autonomous DR systems in a decentralized fashion.**

***Keywords*: Autonomous demand response, optimality, fairness, co-existence, billing mechanism, load scheduling, game theory.**

I. Introduction

Demand response (DR) programs are implemented by utilities to control the energy consumption at the consumer side of the meter in response to changes in grid operating conditions. One approach in DR is direct load control (DLC), where the utility remotely controls energy consumption for certain high-load household appliances such as air-conditioners and water heaters [1]. An alternative for DLC is smart pricing, where users are encouraged to *individually* and *voluntarily* manage their loads, e.g., by reducing their consumption at peak hours [2]. This can be done using automated Energy Consumption Scheduling (ECS) units that are embedded in users' smart meters, as suggested in [3]. For each user, the ECS unit finds the best load schedule to minimize the user's electricity bill while fulfilling the user's energy needs. This can lead to *autonomous* DR programs that are self-organizing and burden a minimal control overhead on utilities.

The literature on autonomous DR using smart pricing is extensive, e.g., see [3]–[11]. The common analytical tool that is used to study autonomous DR systems is Game Theory [12]. Both competitive and cooperative game theoretic frameworks have been considered. Scheduling energy consumption for a wide range of appliances, including electric vehicles [5], have been addressed. Moreover, autonomous DR systems have also been used to help integrating renewable energy sources [5].

Autonomous DR systems are typically designed to minimize either the total cost of power generation [3]–[6], [10], [11] or the peak-to-average ratio of the aggregate load demand [13]. While the prior autonomous DR designs have been *optimal*, i.e., successful in achieving these system-wide objectives; most of them fall short in addressing *fairness*. In fact, up to our knowledge, only [9]–[11] have discussed fairness, although with viewpoints different than ours. In [9], social fairness is addressed such that users pay for electricity based on their income levels. In [10], a billing mechanism is proposed that reflects users' flexibility in *purchasing delay*. However, it is centralized and requires users to send their demand information to the utility. In [11], the users' loads are scheduled using water-filling method and fairness is defined as assigning the same long-term (e.g., monthly) average delay to each user.

In this paper, we seek to answer the following three questions. First, while maintaining optimality, how can we assure fairness by rewarding users based on their contribution in achieving the system-wide design objectives? Second, how can we assess each user's contribution in reaching such objectives? Since full penetration of ECS devices does not happen overnight, it is of practical importance to investigate scenarios where only a subset of users participate in DR. This will lead to a general *co-existence* problem between *participant* and *non-participant* users [14]. Thus, the third question is: how should we treat non-participant users who become free-riders, without over-punishing them? We answer these questions within two design frameworks. First, a centralized design to serve as a *benchmark*. Second, a decentralized design which requires developing a *smart billing mechanism* to enforce both optimality and fairness in autonomous DR systems.

The rest of this paper is organized as follows. The system model is explained in Section II. A benchmark centralized design to achieve optimality and fairness is introduced in Section III. A fair and close-to-optimal billing mechanism is proposed in Section IV. An analytical case study is presented in Section V. More general simulation results are presented in Section VI. The paper is concluded in Section VII.

II. System Model

Consider a smart power grid with a set of $\mathcal{N} = \{1, \ldots, N\}$ users that share an energy source, as shown in Fig. 1. Assume that time is divided into equal-length time slots $\mathcal{H} = \{1, \ldots, H\}$. For example, in a daily operation of the

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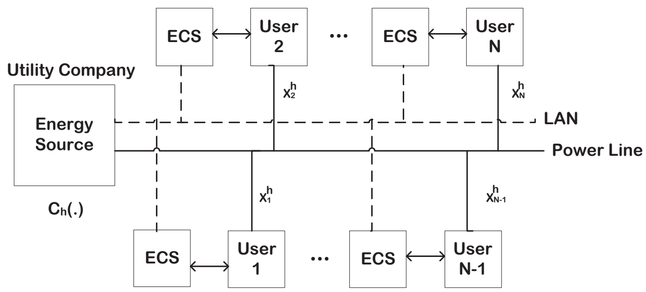

Fig. 1. A power system of N users and one shared energy source. Each user can be equipped with an ECS unit to participate in autonomous DR.

grid, each time slot may take one hour and we have $H = 24$. At each hour $h \in \mathcal{H}$, the cost of power generation is calculated using a generation cost function $C_h(L_h)$, where $L_h \geq 0$ denotes the total load in the system at hour h. As an example, for a thermal power generator, we may have [15]:

$$C_h(L_h) = a_h\, L_h^2 + b_h\, L_h + c_h, \tag{1}$$

where $a_h > 0$ and $b_h, c_h \geq 0$ at each hour $h \in \mathcal{H}$.

Under the autonomous DR paradigm [3], user n's ECS seeks to schedule energy consumption for user n such that his bill is minimized. Without loss of generality, we assume that each user $n \in \mathcal{N}$ has one shiftable load. Let x_n^h denote user n's load at hour h. We define user n's load scheduling vector as

$$\mathbf{x}_n = [x_n^1, x_n^2, \ldots, x_n^H]. \tag{2}$$

Let E_n denote the total energy needed to finish the operation of user n's time shiftable appliance. The operation of such appliance needs to be scheduled within a time frame $[\alpha_n, \beta_n]$, where $1 \leq \alpha_n < \beta_n \leq H$. These parameters are set by user n based on his energy consumption needs. For example, user n may set $\alpha_n =$ 1:00 PM and $\beta_n =$ 5:00 PM for the operation of a dishwasher after lunch and before diner. Therefore, user n's ECS device must fulfill the following constraint

$$\sum_{h=\alpha_n}^{\beta_n} x_n^h = E_n. \tag{3}$$

It is required to also set $x_n^h = 0$ for all $h \in \mathcal{H} \backslash \mathcal{H}_n$, where

$$\mathcal{H}_n = \{\alpha_n, \ldots, \beta_n\} \quad \forall\, n \in \mathcal{N}. \tag{4}$$

In this regard, we can define a *feasible* energy consumption scheduling set corresponding to user n as follows:

$$\mathcal{X}_n = \left\{ \mathbf{x}_n \,\middle|\, \textstyle\sum_{h=\alpha_n}^{\beta_n} x_n^h = E_n;\ x_n^h = 0,\ \forall\, h \in \mathcal{H} \backslash \mathcal{H}_n \right\}. \tag{5}$$

An energy consumption schedule calculated by the ECS unit in user n's smart meter is valid only if we have $\mathbf{x}_n \in \mathcal{X}_n$.

Note that, the operation of the ECS devices has direct impact on the cost of power generation in the system. In fact, we have

$$L_h = \sum_{n=1}^{N} x_n^h, \qquad \forall h \in \mathcal{H}. \tag{6}$$

In order to achieve optimality, the operation of ECS devices must be coordinated such that the cost of power generation, i.e., $\sum_{h=1}^{H} C_h(L_h)$, is minimized. To also assure fairness, users must be rewarded in their bills based on their contribution in minimizing the cost of power generation in the system.

III. Benchmark for Optimality and Fairness

In this section, we provide a centralized benchmark design to achieve optimality and fairness in the system in Section II.

A. Achieving Optimality

First, we define a notation that we will use in this as well as the next section. For each set $\mathcal{M} \subseteq \mathcal{N}$, we define

$$C_{\mathcal{M}}^* = \underset{\mathbf{x}_n \in \mathcal{X}_n}{\textbf{minimum}} \sum_{h=1}^{H} C_h \left(\sum_{n \in \mathcal{M}} x_n^h \right). \tag{7}$$

Clearly, the optimal system performance is achieved if the total cost of power generation in the system becomes $C_{\mathcal{N}}^*$. Assuming that the generation cost function is as in (1), problem (7) is a convex optimization problem and can be solved easily using convex programming techniques [16]. Therefore, if a *centralized design* is feasible, optimality is achieved when each ECS device schedules the load for its corresponding user according to the solution of problem (7) when $\mathcal{M} = \mathcal{N}$.

B. Achieving Fairness

Unlike the case for optimality, it is *not* immediately clear how we may evaluate each individual user's contribution in achieving the system-wide design objectives such as minimizing the total power generation cost in the system. Thus, we first address tackling this challenge and then we use the results to indicate how users must be charged to assure fairness.

Consider two users $n, m \in \mathcal{N}$. Recall that user n selects energy consumption scheduling parameters E_n, α_n, β_n and user m selects energy consumption scheduling parameters E_m, α_m, β_m. From (5) and (7), the choice of these parameters has direct impact in forming the optimal total cost of power generation in the system, i.e., $C_{\mathcal{N}}^*$. Intuitively, we can say that there are two factors that have to be considered.

- *Load Flexibility*: If users n and m have equal total load but user n is more flexible in his load, i.e., we have
$$\alpha_n < \alpha_m \leq \beta_m < \beta_n \quad \text{and} \quad E_m = E_n, \tag{8}$$
then user n must not be charged any higher than user m.
- *Total Load*: If users n and m have equal load flexibility but user n has less total load than user m, i.e., we have
$$\alpha_n = \alpha_m \leq \beta_m = \beta_n \quad \text{and} \quad E_n < E_m, \tag{9}$$
then user n must not be charged any higher than user m.

While these statements are helpful to assess the relative role of each user to minimize power generation cost, they are applicable only to some special cases. In particular, they cannot compare users n and m if one has more load flexibility but the other one has less total load. To solve this problem, we can employ the concept of *Shapley value* in cooperative games. In this regards, every user n is charged based on his impact on the total cost of the system, which is calculated by comparing the optimal power generation cost in the system in the following two scenarios. First, the case when user n is part of the grid. Second, the case when user n leaves the grid. Clearly, the power generation cost is higher in the former, compared to the latter. That is, $C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash n} \geq 0$, for all $n \in \mathcal{N}$. Now, consider users n and m and assume that we have

$$C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash n} < C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash m}. \tag{10}$$

Note that, the inequality in (10) can happen because user n is more flexible or he has less load, or both. But other reasons may also be applicable. For example, even if $E_n = E_m$ and $\beta_n - \alpha_n = \beta_m - \alpha_m$, it is possible that the load scheduling range $[\alpha_n, \beta_n]$ for user n is located in an off-peak hour, e.g., α_n = 1:00 AM and β_n = 5:00 AM, while the load scheduling range $[\alpha_m, \beta_m]$ for user m is located in a peak hour, e.g., α_m = 6:00 PM and β_m = 10:00 PM. Given these observations, we propose to use comparisons similar to (10) to assess how users should be charged in order to assure fairness.

Assume that optimization problem (7) is solved for $\mathcal{M} = \mathcal{N}$ in a centralized fashion and optimal power generation cost $C^*_{\mathcal{N}}$ is achieved. Let B^*_n denote the total daily charge to user n for his electricity bill. To assure fairness, we should have

$$\frac{B^*_n}{B^*_m} = \frac{C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash n}}{C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash m}}. \tag{11}$$

Furthermore, to have a budget balance system, i.e., to assure that the total charges to users matches the total power generation cost in the system, it is required that

$$\sum_{n=1}^{N} B^*_n = C^*_{\mathcal{N}}. \tag{12}$$

From (11) and (12), a fair billing mechanism is obtained as

$$B^*_n = \frac{C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash n}}{\sum_{m=1}^{N} \left(C^*_{\mathcal{N}} - C^*_{\mathcal{N}\backslash m} \right)} \times C^*_{\mathcal{N}}. \tag{13}$$

In this regard, if energy consumption scheduling is centralized, then the grid operator will first solve optimization problem (7) and inform the ECS devices about the optimal solutions such that they schedule energy consumption for their corresponding users accordingly. Then, the operator will use the billing mechanism in (13) and charge each user n according to B^*_n.

Theorem 1: Consider the billing mechanism in (13). (a) If (8) holds, then $B^*_n \leq B^*_m$. (b) If (9) holds, then $B^*_n \leq B^*_m$.

The proof of Theorem 1 is given in Appendix A. It shows that the billing mechanism in (13) satisfies the two fairness conditions that we discussed earlier in this section. Although (13) can be implemented only in a centralized energy consumption scheduling scenario, i.e., when the operator knows E_n, α_n, β_n for each user n to calculate $C^*_{\mathcal{N}}$ and $C^*_{\mathcal{N}\backslash n}$, it can still provide a benchmark to assess fairness in more practical billing models that we will discuss in Section IV.

C. Fairness Index

The fair billing mechanism that we introduced in Section III-B can be used as a benchmark to assess fairness in other billing mechanisms. Let B_n denote the billing amount for user n in an arbitrary billing mechanism. We define the fairness index corresponding to this billing mechanism as

$$F = \sum_{n=1}^{N} \left| \frac{B_n}{\sum_{m=1}^{N} B_m} - \frac{B^*_n}{C^*_{\mathcal{N}}} \right| \tag{14}$$

Here, the fairness index is defined as the variational distance between normalized billing vector for billing mechanism B and normalized billing vector for billing mechanism B^*. From (14), a lower index F indicates a more fair billing.

IV. Billing Mechanism for Optimal and Fair Autonomous Demand Response

In this section, we propose to use a fair and close to optimal billing method which, unlike the benchmark design in Section III, can be implemented within the autonomous DR framework. In the framework, given the billing function B_n, each user n seeks to minimize his energy expenses by solving the following local optimization problem in his ECS device:

$$\underset{\mathbf{x}_n \in \mathcal{X}_n}{\textbf{minimize}} \sum_{h=1}^{H} B^h_n \left(\mathbf{x}_n; \mathbf{x}_{-n} \right), \tag{15}$$

where $\mathbf{x}_{-n} = (\mathbf{x}_1, \ldots, \mathbf{x}_{n-1}, \mathbf{x}_{n+1}, \ldots, \mathbf{x}_N)$ denotes the load scheduling vector for all users other than user n. Clearly, the amount of bill for each user n depends on not only his own load profile but also other users' load profiles as they all affect the total cost of the system. Therefore, we can identify the following autonomous DR game among users [3]–[6], [17]:

- **Players**: Users $n = 1, 2, \ldots, N$.
- **Actions**: For every user n, the vector $\mathbf{x}_n \in \mathcal{X}_n$.
- **Payoffs**: For every user n, minus his bill, i.e., $-B_n$.

Different billing methods yield to different Nash equilibrium in the above game. Next, we examine fairness at the Nash equilibrium based on a common billing choice in the current autonomous DR literature. We show that the resulted fairness index is high, indicating an unfair billing. Then, we propose to use an hour-by-hour billing method that can improve fairness while maintaining a close to optimal system performance.

A. Billing Mechanism Example in Literature

Consider the following billing mechanism that has been commonly used in the autonomous DR literature [3]–[5], [17]:

$$\tilde{B}_n = \frac{E_n}{\sum_{m=1}^{N} E_m} \times \sum_{h=1}^{H} C_h \left(\sum_{n=1}^{N} x^h_n \right), \tag{16}$$

where $\tilde{B}_n$ is the bill of user n. As shown in [3, Theorem 2], the billing mechanism in (16) can lead to reaching an optimal performance, i.e., minimum total power generation cost $C^*_{\mathcal{N}}$. However, this billing mechanism does not seem to charge users based on their contributions in achieving minimum power generation cost in the system. In fact, from (16), any two users with equal *total load* will pay equally on their bills regardless of the shape of their load profiles. This holds even if one user participates in DR and another user does not participate in DR, or even if both users participate in DR but they have different load flexibilities. Next, we assess the fairness and optimality of the billing mechanism in (16) via a numerical example.

Assume that $N = 3$ users share an energy source. We have $E_1 = E_2 = 10$ kWh and $E_3 = 12.5$ kWh. The users want to schedule their load for the next $H = 4$ hours. User 1 is *not* flexible and insists to operate his load within the first hour

TABLE I
THE NASH EQUILIBRIUM WHEN BILLING IS AS IN (16).

user n	load schedule of user n				bill $\tilde{B}_n$
	x_n^1	x_n^2	x_n^3	x_n^4	
1	10	0	0	0	17.49
2	0	10	0	0	17.49
3	0	0	6.25	6.25	21.86

TABLE II
THE NASH EQUILIBRIUM WHEN BILLING IS AS IN (23).

user n	load schedule of user n				bill B_n	bill B_n^*
	x_n^1	x_n^2	x_n^3	x_n^4		
1	10	0	0	0	21.25	21.31
2	2.50	7.50	0	0	20.87	20.82
3	0	0	6.25	6.25	14.84	14.71

$h = 1$. That is, $\alpha_1 = \beta_1 = 1$. User 2 is *partially flexible* and allows load distribution within the first two hours $h = 1, 2$. That is, $\alpha_2 = 1$ and $\beta_2 = 2$. User 3 is *completely flexible* and allows load distribution at any time. That is, $\alpha_3 = 1$ and $\beta_3 = 4$. According to the billing scheme in [3], we have

$$\tilde{B}_1 = \tilde{B}_2 = \frac{10}{32.5} \sum_{h=1}^{4} C_h \left(\sum_{n=1}^{3} x_n^h \right), \tag{17}$$

$$\tilde{B}_3 = \frac{12.5}{32.5} \sum_{h=1}^{4} C_h \left(\sum_{n=1}^{3} x_n^h \right). \tag{18}$$

Clearly, the bill of every user is minimized if and only if the total cost energy in the system is minimized. In fact, this is the main advantage of the billing system in [3] as it encourages users to contribute in minimizing the energy cost. Next, assume that the hourly cost functions are $C_1(L) = C_2(L) = 0.01L^2 + 2L$ and $C_3(L) = C_4(L) = 0.03L^2 + L$. At Nash equilibrium, the users' strategies and their bills are as shown in Table I. The total generation cost in the system at Nash equilibrium becomes \$56.84. We can see that although user 2 is more flexible than user 1, users 1 and 2 end up paying equally on their bills. The fairness index is obtained as $F = 0.2515$ which is high. Next, we will explain how we can improve fairness using an alternative billing mechanism.

B. Alternative Billing Mechanism

To solve the problem with respect to fairness in Section IV-A, we propose to use an alternative billing scheme that incorporates the exact shape of each user's load profile. The bill of every user n is calculated hour-by-hour. That is,

$$B_n = \sum_{h=1}^{H} B_n^h, \tag{19}$$

where B_n^h is user n's bill at hour h. The hourly bills are set such that for every users n and m, we have

$$\frac{B_n^h}{B_m^h} = \frac{x_n^h}{x_m^h}. \tag{20}$$

From (20) and given the budget balance requirement that total hourly bills should match the total hourly cost of electricity, user n's hourly bill at hour h is obtained as

$$B_n^h = \frac{x_n^h}{\sum_{m=1}^{N} x_m^h} \sum_{m=1}^{N} B_m^h \tag{21}$$

$$= \frac{x_n^h}{\sum_{m=1}^{N} x_m^h} C_h \left(\sum_{m=1}^{N} x_m^h \right). \tag{22}$$

Consequently, user n's daily electricity bill is calculated as

$$B_n = \sum_{h=1}^{H} \frac{x_n^h}{\sum_{m=1}^{N} x_m^h} C_h \left(\sum_{m=1}^{N} x_m^h \right). \tag{23}$$

Comparing with (16), we can see that the billing scheme in (23) incorporates the exact *hour-by-hour* load profile of each user. It charges users at a higher rate if they schedule their load at peak-hours and at a lower rate if they move their load to off-peak hours. In other words, the hour-by-hour alternative billing mechanism takes into account both total load and load flexibility. Therefore, we expect that it can improve fairness.

Next, consider the example in Section IV-A. If we use the billing method in (23), at Nash equilibrium, the strategies and the bill amount of users become as in Table II. Unlike the billing mechanism in (16) that charges users 1 and 2 equally, simply because they have equal total load, the alternative billing scheme in (23) charges user 2 about 1.8% less than user 1 due to user 2's more flexibility in his energy consumption. Furthermore, while user 3 has 25% higher total load compared to users 1 and 2, it is charged 40% less due to its complete load scheduling flexibility. We can easily calculate the fairness index in this case and see that it reduces to $F = 0.0038$, which is 65 times less (i.e., better) than the fairness index for the billing mechanism in (16). These results can motivate users to be more flexible and stay as a DR participant user.

V. ANALYTICAL CASE STUDY

In this section, we investigate some of the properties of the hour-by-hour billing mechanism in (23), through an analytical case study. In particular, we assess fairness. To do so, we will need to first find the users' strategies at steady state, i.e., at Nash equilibrium of the defined game. More general results will be presented through simulations in Section VI.

For the case study in this section, consider the quadratic generation cost function in (1) and assume that $c_h = 0$, for all $h \in \mathcal{H}$. Furthermore, assume that $H = 2$. That is, energy consumption scheduling is focused only over two time slots, e.g., two hours. There are a total of $N \geq 2$ users in the system. User 1 is not flexible in his load. Therefore, it does not participate in DR. In fact, we have $\alpha_1 = \beta_1 = 1$, meaning that user 1 insists to schedule its appliance in the first time slot. For any other user $n \in \{2, \ldots, N\}$, they do participate in DR and we have $\alpha_n = 1$ and $\beta_n = 2$. In this setup, the load profile of user 1 is $(E_1, 0)$ and the load profile of any other user n is in the form of $(x_n^1, E_n - x_n^1)$, where $x_n^1 \in [0, E_n]$. Every participant user n, more specifically his ECS unit, wants to determine x_n^1 such that his bill B_n is minimized. For the purpose of having a more clear presentation, and without loss of generality, we sort users' indexes such that

$$E_2 \leq E_3 \leq \ldots \leq E_N. \tag{24}$$

Next, we characterize the Nash equilibrium of the autonomous DR game. Note that, since user 1 does not participate in DR, the game is played only by the rest of the $N-1$ users.

Theorem 2: Consider the above system model and assume that the electricity billing mechanism is as in (23). (a) If

$$E_1 \geq \frac{b_2 - b_1}{a_1}, \tag{25}$$

then $(x_2^{1*}, x_3^{1*}, \ldots, x_N^{1*})$ is a Nash equilibrium of the DR game, where for each user $n = 2, \ldots, q-1$, we have

$$x_n^{1*} = 0, \tag{26}$$

and for each user $n = q, \ldots, N$, we have

$$x_n^{1*} = \frac{1}{N-q+2}\left[-E_1 + \frac{a_2 \sum_{m=1}^{q-1} E_m - b_1 + b_2}{a_1 + a_2}\right] + \frac{a_2}{a_1 + a_2} E_n. \tag{27}$$

Here, q denotes the smallest $n = 2, \ldots, N$ such that

$$\frac{a_2}{a_1+a_2} E_n \geq \frac{-1}{N-n+2}\left[-E_1 + \frac{a_2 \sum_{m=1}^{n-1} E_m - b_1 + b_2}{a_1 + a_2}\right]. \tag{28}$$

(b) If (25) does not hold, then at Nash equilibrium of the DR game, for each user $n = 2, \ldots, p-1$, we have

$$x_n^{1*} = E_n, \tag{29}$$

and for each user $n = p, \ldots, N$, we have

$$x_n^{1*} = \frac{1}{N-p+2}\left[\frac{-a_1 \sum_{m=1}^{p-1} E_m - b_1 + b_2}{a_1 + a_2}\right] + \frac{a_2}{a_1 + a_2} E_n. \tag{30}$$

Here, p denotes the smallest $n = 2, \ldots, N$ such that

$$\frac{a_1}{a_1+a_2} E_n \geq \frac{1}{N-n+2}\left[\frac{-a_1 \sum_{m=1}^{n-1} E_m - b_1 + b_2}{a_1 + a_2}\right]. \tag{31}$$

The proof of Theorem 2 is given in Appendix B. In this Theorem, the users' strategies are formulated based on the relationships among the system parameters as in (25). If (25) holds then the N.E. strategies are as in (26) and (27). If (25) does not hold, then the N.E. strategies are as in (29) and (30).

Next, we want to investigate the fairness of the system at N.E.. Assume that there exists a participant/flexible user $n \in \{2, \ldots, N\}$ such that $E_n = E_1$. Given the fact that users 1 and n have the same total load but user n is more flexible in his energy consumption schedule, the billing mechanism (23) is *fair* only if at N.E. we have $B_n \leq B_1$. That is, the participant/flexible user n should not be charged more than the non-participant/non-flexible user 1. The equality $B_n = B_1$ should occur only if users 1 and n have exactly the same load profile, i.e., when we have $x_1^{1*} = x_n^{1*} = E_n = E_1$.

Theorem 3: Consider the autonomous DR system explained in this section. Assume that $b_2 \leq b_1$, where b_1 and b_2 are defined in (1). If (8) or (9) hold for $m = 1$, then $B_n \leq B_m$.

The proof of Theorem 3 is given in Appendix C. Comparing the results in Theorems 3 and 1, the alternative billing mechanism in Section IV-B and the benchmark billing mechanism in Section III-B have some common fairness properties.

Remark 1: Interestingly, if $b_2 \leq b_1$, then achieving fairness does not depend on the other cost parameters a_1 and a_2.

Remark 2: If $b_2 > b_1$, then depending on the values of system parameters, the system may or may not be fair.

VI. Simulation Results

The analytical case study in the last section suggests that the hour-by-hour billing mechanism in Section IV-B may perform closely to the benchmark design in Section III-B. We will further investigate this through simulations in this section. Unless we state otherwise, the simulation setting is as follows. There are $N = 20$ users in the system and energy consumption scheduling is done for the next $H = 24$ hours. The total load E_n for each user n is randomly selected between 0 and 40 kWh. The values of α_n and β_n are randomly generated such that the overall load profile of the system looks similar to a typical aggregate load profile, e.g., with one peak hour around 11 AM and another peak hour around 19 PM. The power generation cost functions are $C_h(L_h) = 0.01L_h^2 + 2L_h$ for each $h < 12$ and $C_h(L_h) = 0.03L_h^2 + L_h$ for each $h \geq 12$.

The simulation results on average fairness index of the billing mechanism in [3] as well as the alternative one in (23), in different scenarios, are shown in Fig. 2(a). We can see that, by using the billing mechanism in (23), the average fairness index reduces from 0.171 to 0.046. That is, it reduces by 73%. Recall from Section III-C that a lower fairness index indicates a more fair system. Next, we note that the billing mechanism in [3] aims to enforce optimality with minimum total generation cost. However, optimality is not guaranteed for our alternative billing mechanism. Nevertheless, the simulation results in Fig. 2(b) show that the optimality gap is less than 1%. In other words, while the hour-by-hour billing mechanism significantly improves fairness, it only losses less than 1% in optimality. That is, there is a trade-off between fairness and optimality. In Fig. 2(b), we can see that for both billing mechanism in (23) and [3], more users' flexibility (in their time intervals) yields to more decrease in the average total cost of the system. Note that, the energy consumptions of the users are assumed fixed in all scenarios. Thus, the decrease in the total cost of the system is due to only increase in the users' flexibility.

Next, we assess fairness in an autonomous DR scenario, where only a subset of users participate in demand response. That is, the case where a group of participant users *co-exist* with a group of non-participant users. For non-participant users, either they do not have an ECS device, or they program their ECS devices to start energy consumption as soon as they turn an appliance on. That is, for each non-participant user n, the operation of appliance is scheduled at hour α_n. On the other hand, for each participant user n, the operation of appliance is scheduled some time between hour α_n and hour β_n. The results are shown in Fig. 3, where we plotted the average fairness index versus different percentage of users' participation, for the hour-by-hour billing mechanism in (23)

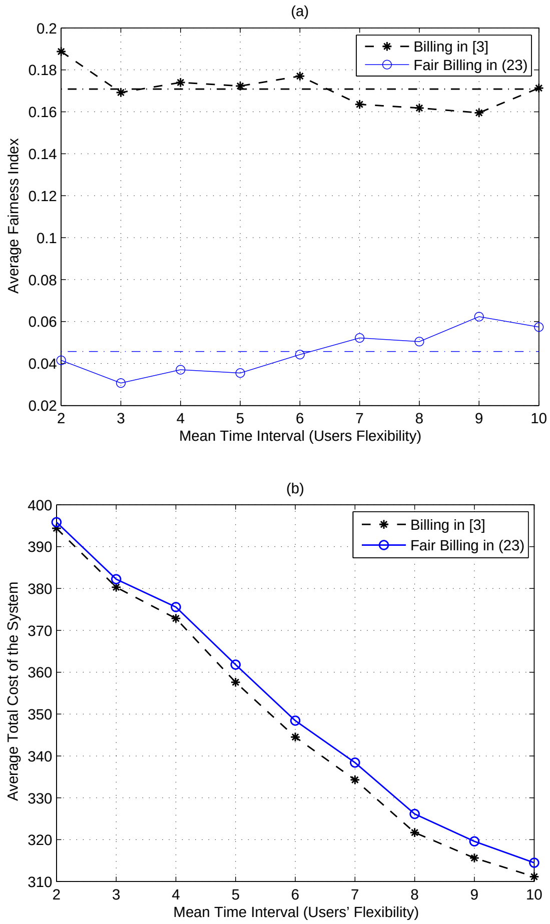

Fig. 2. Comparing the hour-by-hour billing and the one in [3] in terms of (a) fairness, and (b) minimizing the total energy cost in the system.

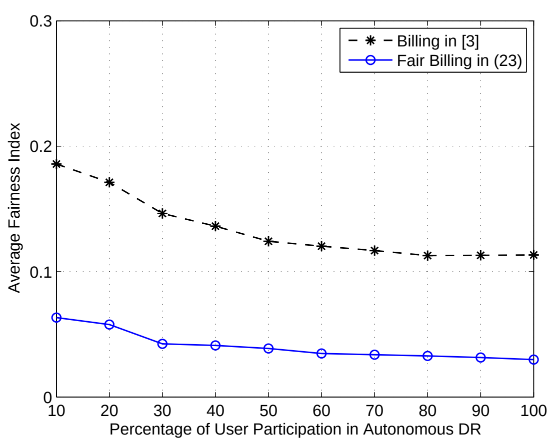

Fig. 3. The average fairness index versus the percentage of users participating in autonomous DR for the hour-by-hour billing mechanism and the one in [3].

and the one in [3]. We can see that, for all participation percentages, the billing in (23) can significantly improve fairness in comparison with the billing mechanism in [3].

To investigate the benefit of load flexibility on each user's individual bill, assume that $N = 10$ and we have $E_n = E_m$ for each two $n, m \in \mathcal{N}$. We also assume that users 1 to 5 are flexible in their load, i.e., they act as participant users, while users 6 to 10 do not show flexibility, i.e., they act as non-participant users. We can see in Fig. 4, that the participant users' bills are much less than that of non-participant users. Since all users have equal total load, the reduced bill amounts are directly due to the participant users' contribution in minimizing the total power generation cost in the system. Note that, although users 1 to 5 are all flexible users, their allocated cost is different as they have different levels of flexibility. For example, user 1 and 2 in Fig. 4 are both flexible with equal energy $E_2 = E_1$, but user 2 is more flexible than user 1 because $\alpha_2 < \alpha_1 < \beta_1 < \beta_2$. Thus, user 2 is charged less, i.e., $B_2 < B_1$. On the other hand, for the billing mechanism in [3], all users pay equally as billing is solely based on total load. Finally, we can see that the difference between the users' bills under the hour-by-hour billing mechanism in (23) and the benchmark billing mechanism in (13) is very minor.

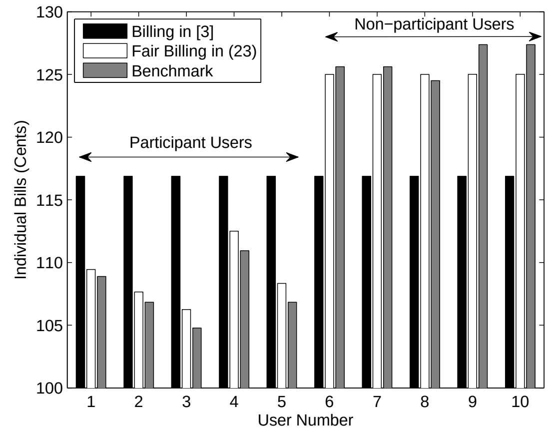

Fig. 4. Users' bill when only a subset of users participate in autonomous DR.

VII. Conclusions

While most prior work on autonomous demand response have focused on achieving optimality, e.g., in terms of minimizing the cost of power generation, here in this paper, we examined the possibility of achieving both optimality and fairness. First, we developed a benchmark by introducing a novel fairness index. Then, we showed that some common billing mechanisms in the autonomous DR literature are not fair. Thus, we proposed using an alternative billing model to improve fairness while maintaining a close to optimal overall system performance. We confirmed the advantages of our proposed design by analytical case studies and simulations.

Appendix

A. *Proof of Theorem 1*

a) From (13), we need to show that $C^*_{\mathcal{N}\backslash n} \geq C^*_{\mathcal{N}\backslash m}$. That is,

$$\begin{aligned} &\underset{\substack{\mathbf{x}_i \in \mathcal{X}_i \; i \neq n,m \\ \mathbf{x}_m \in \mathcal{X}_m}}{\textbf{minimum}} \; \sum_{h=1}^{H} C_h \left(\sum_{i=1, i \neq n,m}^{N} x_i^h + x_m^h \right) \\ &\geq \underset{\substack{\mathbf{x}_i \in \mathcal{X}_i \; i \neq n,m \\ \mathbf{x}_n \in \mathcal{X}_n}}{\textbf{minimum}} \; \sum_{h=1}^{H} C_h \left(\sum_{i=1, i \neq n,m}^{N} x_i^h + x_n^h \right). \end{aligned} \tag{32}$$

The two sides of the above inequality differ only due to the difference between $\mathcal{X}_n$ and $\mathcal{X}_m$. In fact, the inequality in (32) holds if $\mathcal{X}_m \subseteq \mathcal{X}_n$. However, this relationship is evident from the definition of $\mathcal{X}_n$ and $\mathcal{X}_m$ in (5) and the conditions in (8).
b) Let $\bar{\mathbf{x}}_i \in \mathcal{X}_i$, for all $i \neq n, m$ and $\bar{\mathbf{x}}_m \in \mathcal{X}_m$ denote the solution of the minimization on the left hand side in (32). Consider $\hat{\mathbf{x}}_n$ such that $\sum_{\alpha_n}^{\beta_n} \hat{x}_n^h = E_n$ and $\hat{x}_n^h \leq \bar{x}_m^h$ for all $\alpha_n = \alpha_m \leq h \leq \beta_n = \beta_m$. From (5) and (9), $\hat{\mathbf{x}}_n \in \mathcal{X}_n$. Since the generation cost function in (1) is non-decreasing, we have

$$\sum_{h=1}^{H} C_h\left(\sum_{i=1, i\neq n,m}^{N} \bar{x}_i^h + \bar{x}_m^h\right) \geq \sum_{h=1}^{H} C_h\left(\sum_{i=1, i\neq n,m}^{N} \bar{x}_i^h + \hat{x}_n^h\right) \quad (33)$$

The left hand side in (33) is *equal* to the left hand side in (32) and the right hand side in (32) is *upper bounded* by the right hand side in (33). Therefore, (33) directly results in (32). ■

B. Proof of Theorem 2

a) If (25) holds, then we need to show that for each user n, the best response is $x_n^{1*} \in [0, E_n]$ if all other users $m \neq n$ choose action x_m^{1*}. From (23), we can show that

$$\frac{dB_n}{dx_n^1} = (a_1 + a_2)(x_n^1 + L_T) - a_2(E_T + E_n) + b_1 - b_2, \quad (34)$$

where L_T and E_T are the total load and the total need for energy that is, $L_T = \sum_{m=1}^{N} x_m^1$ and $E_T = \sum_{m=1}^{N} E_m$.

From (26) and (27), we have

$$\begin{aligned} L_T^* = E_1 + \sum_{m=q}^{N} x_m^{1*} &= \frac{E_1}{N-q+2} + \frac{a_2}{a_1+a_2} \sum_{m=q}^{N} E_m \\ &+ \frac{N-q+1}{N-q+2} \times \frac{a_2 \sum_{m=1}^{q-1} E_m - b_1 + b_2}{a_1 + a_2}. \end{aligned} \quad (35)$$

Substituting (27) and (35) in (34), for each $n \geq q$, we have

$$\frac{dB_n}{dx_n^1} = (a_1 + a_2)(x_n^{1*} + L_T^*) - a_2(E_T + E_n) + b_1 - b_2 = 0. \quad (36)$$

For each $n \leq q-1$, from (26) and (34), we can write

$$\begin{aligned} \frac{dB_n}{dx_n^1} &= (a_1 + a_2)(L_T^*) - a_2(E_T + E_n) + b_1 - b_2 \\ &= \frac{a_1 + a_2}{N-q+2}\left[E_1 + \frac{-a_2 \sum_{m=1}^{q-1} E_m + b_1 - b_2}{a_1 + a_2}\right] - a_2 E_n. \end{aligned} \quad (37)$$

On the other hand, from the definition of q in (28), we have

$$\frac{a_2}{a_1+a_2} E_{q-1} < \frac{-1}{N-q+3}\left[-E_1 + \frac{a_2 \sum_{m=1}^{q-2} E_m - b_1 + b_2}{a_1 + a_2}\right].$$

Subtracting $a_2 E_{q-1}/(N-q+3)(a_1+a_2)$ from both sides yields

$$\frac{a_2 E_{q-1}}{a_1 + a_2} < \frac{1}{N-q+2}\left[E_1 + \frac{-a_2 \sum_{m=1}^{q-1} E_m + b_1 - b_2}{a_1 + a_2}\right]. \quad (38)$$

From (37) and (38), for each $n \leq q-1$, we have

$$\frac{dB_n}{dx_n^1} > a_2 E_{q-1} - a_2 E_n \geq 0, \quad (39)$$

where the last inequality comes from (24). From (36) and (39), x_n^{1*} is the best response for user n. However, to complete the proof, we also need to show that $0 \leq x_n^{1*} \leq E_n$. Clearly, this is true for any $n \leq q-1$. From (38), for each $n \geq q$, we have

$$\begin{aligned} x_n^{1*} &= \frac{1}{N-q+2}\left[-E_1 + \frac{a_2 \sum_{m=1}^{q-1} E_m - b_1 + b_2}{a_1 + a_2}\right] \\ &+ \frac{a_2}{a_1 + a_2} E_n < -\frac{a_2}{a_1 + a_2} E_{q-1} + \frac{a_2}{a_1 + a_2} E_n < E_n. \end{aligned}$$

Furthermore, from (24) and (28), for each $n \geq q$, we have

$$\frac{a_2}{a_1 + a_2} E_n \geq \frac{1}{N-q+2}\left[E_1 + \frac{-a_2 \sum_{m=1}^{q-1} E_m + b_1 - b_2}{a_1 + a_2}\right],$$

which means $x_n^{1*} \geq 0$. Thus, for each $n = 2, \ldots, N$, we have $0 \leq x_n^{1*} \leq E_n$. This completes the proof for part (a).
b) If (25) does not hold, from (29) and (30), the value of L_T in the hour-by-hour billing strategy is obtained as

$$\begin{aligned} L_T^* = \sum_{m=1}^{p-1} E_m + \sum_{m=p}^{N} x_m^{1*} &= \frac{\sum_{m=1}^{p-1} E_m}{N-p+2} + \frac{N-p+1}{N-p+2} \\ &\times \frac{a_2 \sum_{m=1}^{p-1} E_m - b_1 + b_2}{a_1 + a_2} + \frac{a_2}{a_1 + a_2} \sum_{m=p}^{N} E_m. \end{aligned} \quad (40)$$

Substituting (30) and (40) in (34), for each $n \geq p$, we have

$$\frac{dB_n}{dx_n^1} = (a_1 + a_2)(x_n^{1*} + L_T^*) - a_2(E_T + E_n) + b_1 - b_2 = 0. \quad (41)$$

For each $n \leq p-1$, using (29) and (34), we can write

$$\frac{dB_n}{dx_n^1} = a_1 E_n + \frac{1}{N-p+2}\left[a_1 \sum_{m=1}^{p-1} E_m + b_1 - b_2\right]. \quad (42)$$

On the other hand, from the definition of p in (31), we have

$$\frac{a_1}{a_1 + a_2} E_{p-1} < \frac{1}{N-p+3}\left[\frac{-a_1 \sum_{m=1}^{p-2} E_m - b_1 + b_2}{a_1 + a_2}\right].$$

Subtracting $a_1 E_{p-1}/(N-p+3)(a_1+a_2)$ from both sides yields

$$\frac{a_1}{a_1 + a_2} E_{p-1} < \frac{-1}{N-p+2}\left[\frac{a_1 \sum_{m=1}^{p-1} E_m + b_1 - b_2}{a_1 + a_2}\right]. \quad (43)$$

From (42) and (43), for each $n \leq p-1$, we have

$$\frac{dB_n}{dx_n^1} < a_1 E_n - a_1 E_{p-1} \leq 0, \quad (44)$$

where the last inequality is due to (24). From (41) and (44), x_n^{1*} is the best response of user n. To complete the proof, we also need to show that $0 \leq x_n^{1*} \leq E_n$. Clearly, this is true for any $n \leq p-1$. From (43), for each $n \geq p$, we have

$$\begin{aligned} x_n^{1*} &= \frac{-1}{N-p+2}\left[\frac{a_1 \sum_{m=1}^{p-1} E_m + b_1 - b_2}{a_1 + a_2}\right] + \frac{a_2}{a_1 + a_2} E_n \\ &> \frac{a_1}{a_1 + a_2} E_{p-1} + \frac{a_2}{a_1 + a_2} E_n > 0. \end{aligned}$$

Furthermore, from (24) and (31), for each $n \geq p$, we have

$$\frac{a_1}{a_1 + a_2} E_n \geq \frac{1}{N-p+2}\left[\frac{-a_1 \sum_{m=1}^{p-1} E_m - b_1 + b_2}{a_1 + a_2}\right],$$

which means

$$x_n^{1*} \leq \frac{a_1}{a_1 + a_2} E_n + \frac{a_2}{a_1 + a_2} E_n = E_n. \quad (45)$$

Thus, for each $n = 2, \ldots, N$, we have $0 \leq x_n^{1*} \leq E_n$. ■

C. Proof of Theorem 3

Assume that $E_n = E_1$ for some participant/flexible user $n \geq 2$. From (1) and (23), for users 1 and n we have

$$B_1 = a_1 E_1 L_T + b_1 E_1,$$
$$B_n = a_1 x_n^1 L_T + b_1 x_n^1 + a_2(E_n - x_n^1)(E_T - L_T) + b_2(E_n - x_n^1),$$

where L_T and E_T are the total load and energy consumption of all users. From Theorem 2 and since $E_n = E_1$, at the Nash equilibrium of the DR game we have

$$B_1^* - B_n^* = (E_1 - x_n^{1*}) \left[(a_1 + a_2) L_T^* - a_2 E_T + b_1 - b_2 \right].$$

By definition, $E_1 - x_n^{1*} \geq 0$. Therefore, fairness in the hour-by-hour billing system is achieved if we have

$$(a_1 + a_2) L_T^* - a_2 E_T + b_1 - b_2 > 0. \tag{46}$$

If $b_2 \leq b_1$, then we always have $E_1 \geq (b_2 - b_1)/a_1$. In this case, L_T^* is as (35). Therefore, we can rewrite (46) as

$$E_1 > \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \sum_{m=2}^{q-1} E_m. \tag{47}$$

On the other hand, using the definition of q in (28), we have

$$\frac{a_2}{a_1 + a_2} E_{q-1} < \frac{-1}{N - q + 3} \left[-E_1 + \frac{a_2 \sum_{m=1}^{q-2} E_m - b_1 + b_2}{a_1 + a_2} \right]$$

which results in (47) as shown bellow:

$$\begin{aligned} E_1 &> \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \left[(N - q + 3) E_{q-1} + \sum_{m=2}^{q-2} E_m \right] \\ &= \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \left[(N - q + 2) E_{q-1} + \sum_{m=2}^{q-1} E_m \right] \\ &> \frac{b_2 - b_1}{a_1} + \frac{a_2}{a_1} \sum_{m=2}^{q-1} E_m. \end{aligned}$$

Hence, (46) is satisfied and the DR system is fair. ■

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