Joint Sparse Estimation of Sensitivity Distribution Factors and Power Flows in Low-Observable Power Distribution Systems

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***Abstract*—Sensitivity distribution factors (SDFs) have diverse applications in power system operation. In particular, when SDFs are obtained in a data-driven fashion based on measurements, they can eliminate the need to repeatedly solve the computationally complex non-linear power flow equations. However, in a typical *low-observable* power distribution system, where the measurements are not sufficient to achieve full-observability, it is a major challenge to estimate SDFs using the available measurements. This challenge is addressed in this paper. Specifically, a new method is proposed for *joint* estimation of SDF and power flows in power distribution systems that lack full-observability. The proposed method requires measuring the nodal injection power and line power flow at *only a few* locations on the power distribution feeder. The proposed method is physics-aware. It is built upon extracting *physics-based sparsity features* of power distribution feeders. In this regard, the aforementioned joint estimation problem is formulated as a *sparse matrix completion* problem. The advantages of the proposed method in comparison with the existing methods are investigated via numerical results.**

***Keywords* – Sensitivity distribution factor, injection shift factor, power flow estimation, sparse recovery, low-observability, power distribution systems, computation.**

I. Introduction

Sensitivity distribution factors (SDFs), such as *injection shift factors* (ISF) and *power transfer distribution factors* (PTDF) are powerful computational tools to help grid operators to obtain the solution of the power flow problem *without* the need to repeatedly solve the non-linear power flow equations. SDFs have various applications, such as in contingency analysis [1], energy dispatch [2], and voltage regulation [3].

In general, there are two different types of methods to obtain the SDF matrices: *model-based* and *measurement-based*. In model-based methods, SDF matrices are obtained from the Jacobian matrix of the power flow equations. Based on the application, the power flow equations might be in form of DC approximations [4] or non-linear AC equations [5], [6]. Because of the higher ratio of R/X in power distribution lines, DC approximation typically does *not* work well. As for the methods that use non-linear power flow equations, the accuracy of the analysis depends on the operating points of the system, which either may not be necessarily known or might be changing frequently. Needless to say that, non-linear power flow models also suffer from higher computation cost.

In measurement-based (i.e., data-driven) methods, the SDF matrices are obtained by applying linear regression to the available measurements [7], [8]. For instance, to estimate the ISF matrix, which is the main focus in this work, the typical required measurements include the changes in the nodal power injection at *every* node, and the changes in the line power flow in *every* line segment. One advantage of the measurement-based methods over the model-based methods is that the measurement-based methods do *not* need any prior knowledge about the power system topology, parameters of the lines, or the time-varying operating points of the system.

However, the accuracy of the measurement-based methods highly depends on the sufficiency of the available measurements. One issue is whether or not the measurements are available at *every* bus or *every* line segment, such that a regression analysis can be established based on the available measurements. At a power transmission network, such comprehensive measurements might be available due to measurement redundancy. However, this is *not* the case in a typical power distribution network. Distribution feeders often suffer from low-observability [9]–[12]. Therefore, unlike the existing measurement-based methods that are developed for use at transmission level, one should take into account the issue of low-observability when estimating the SDF matrices using the measurement-based methods in power distribution systems.

To the best of our knowledge no previous work in the literature has looked into this aspect of the problem.

Another aspect about the sufficiency of the available measurements is the sufficiency of the *measurement samples*. If there are *not* enough measurement samples, then the measurement matrices are *not* full-rank. In that case, special methods are needed to deal with rank deficiency. In [13], a method based on singular value decomposition (SVD) is proposed to obtain the sensitivity matrix from the low-rank measurement matrices. In [2], a recursive partial least square method is used to address the same issue. In [14], the rows of the SDF matrix are sparsified based on the electrical distance of the buses and the lines of the power transmission network, to obtain the SDF matrix from insufficient measurements via sparse recovery.

In this paper, our focus is on power distribution feeders with their typical radial topology. We show that such radial topology can create *physics-based sparsity features*, whose utilization in this low-observability problem makes up for the low-rank properties of the measurement matrices. Accordingly, we address both issues about the sufficiency of measurements in a low-observable power distribution network. We propose a novel joint sparse estimation method for the SDF matrices and power flows in a low-observable power distribution system. We seek to estimate the ISF, which is an important class of SDFs. Unlike the existing methods in the literature, which are either model-based or measurement-based, the proposed method is

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hybrid, since a model-based approach is used with respect to the physics-aware component of our design; while all other aspects of our approach is measurement-based.

Our method works based on extracting the *sparsity features* of the SDF matrix and power flows based on the typical radial topology of the power distribution networks and by augmenting the estimation problem formulation based on the extracted features to make up for the lack of observability.

We also develop a novel iterative solution to deal with the non-linearity of the optimization problem that is caused because of the missing measurements in the regression model.

II. Problem Statement

A. System Model

Let us represent a radial power distribution feeder by graph $\mathcal{G} := (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ denotes the set of nodes and $\mathcal{L}$ denotes the set of line segments. Let Δx_i^t denote the change in the nodal power injection at node i at time t. Accordingly, let $\Delta \mathbf{x}^t \in \mathbb{R}^{1\times n}$ denote the vector that captures Δx_i^t for all buses in $\mathcal{N}$ at time t, where n is the number of nodes in the system. Also, let Δy_j^t denote the change in the power flow of line segment j at time t. Similarly, let $\Delta \mathbf{y}^t \in \mathbb{R}^{1\times l}$ denote the vector which contains Δy_j^t for all line segments in $\mathcal{L}$ at time t, where l is the number of line segments in the system. Our goal is to obtain the linear sensitivity ISF matrix such that:

$$\Delta \mathbf{y}^t = \Delta \mathbf{x}^t\, \mathbf{H}, \tag{1}$$

where $\mathbf{H} \in \mathbb{R}^{n\times l}$ is the ISF matrix. Based on equation (1), the change in the power flow of line segment j, for any $j \in \mathcal{L}$, can be obtained from the product of the sensitivity factors by their corresponding change in the nodal power injection as:

$$\Delta y_j^t = \Delta x_1^t\, h_{1j} + \cdots + \Delta x_n^t\, h_{nj}. \tag{2}$$

Here, $h_{1j}, \ldots, h_{nj}$ are the entries of column j in matrix $\mathbf{H}$.

B. Problem Statement

The current measurement-based models in the literature, such as in [7], [8], [13], [14], assume that the changes in the nodal power injection, i.e., $\Delta \mathbf{x}^t$, and the changes in the line power flow, i.e., $\Delta \mathbf{y}^t$, are *known* quantities measured by a variety of sensors that are installed across the power system. Hence, they form a linear regression model to obtain the unknown entries of the ISF matrix. In this measurement-based approach, the total number of unknowns is $n \times l$. But, the number of equations in (1) is only l. Therefore, at least $n \times (l-1)$ more independent measurements are needed, such that the unique solution of (1) can be obtained.

Next, let $\Delta \mathbf{Y} \in \mathbb{R}^{T\times l}$ be the matrix that includes all the measurement samples of the changes in the line power flows, i.e., the matrix whose rows are vectors of $\Delta \mathbf{y}^t$ from time $t = 1$ to time $t = T$. Similarly, let $\Delta \mathbf{X} \in \mathbb{R}^{T\times n}$ be the matrix which includes all the measurement samples of the change in the nodal power injection. We can rewrite equation (1) as:

$$\Delta \mathbf{Y} = \Delta \mathbf{X}\, \mathbf{H}. \tag{3}$$

If the measurement matrix $\Delta \mathbf{X}$ is a full-row-rank matrix, then the system of equations in (3) can be solved by applying a

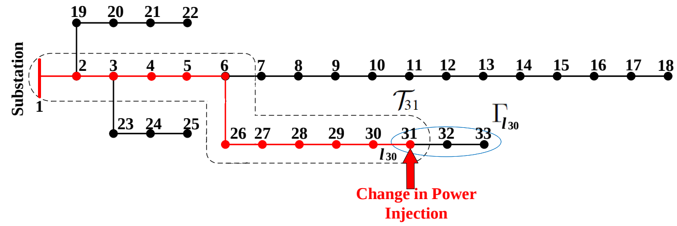

Fig. 1. An illustrative example for substation connector path in a radial power distribution network. The red path corresponds to the injection at bus 31.

conventional least square method, such as in [7], [15], to obtain $\mathbf{H}$. However, having measurements at *every* location is *not* a common setting in power distribution systems. Therefore, we need to develop a new model which can directly address the issue of low-observability in the estimation of the ISF matrix.

C. Sparsity Pattern of Power Flows and Distribution Factors

Consider the radial power distribution system in Fig. 1. Suppose a change in the nodal power injection occurs at bus 31. Let us call the path that connects bus 31 to the substation as the *substation connector path* associated with bus 31. This path is marked in red on the figure. Based on the Compensation Theorem in Circuit Theory [16], when a change happens in the power injection of a node, then we can replace the element that causes such change by an *equivalent current source* which injects a current to the circuit that causes the same changes on the operating points of the system. Therefore, approximately, the equal amount of current flows from bus 31 all the way up to the substation through the substation connector path, see [17]. The reason is that the impedance of the Thevenin equivalent circuit of the power grid as seen by the distribution feeder at the substation is much less than the impedance of the loads [18]. As a result, in a radial power distribution feeder, once a change in the nodal power injection happens, it only changes the power flow of those distribution line segments that are on the substation connector path, while the power flow for the rest of the line segments remains approximately unchanged.

For each node $i \in \mathcal{N}$, let us define tree $\mathcal{T}_i := (\mathcal{V}_i, \mathcal{E}_i)$ as the sub-graph which includes the nodes and lines that are on the substation connector path. Based on the above discussion, once a deviation in the power injection of node i happens, it only changes the power flow in those line segments that belong to set $\mathcal{E}_i$, while the power flow in the rest of line segments remains unchanged. To take advantage of such physics-based approximation, let us denote the set which includes all the nodes that line segment j belong to their associated set $\mathcal{E}_i$ as:

$$\Gamma_j = \{\forall i \in \mathcal{N} \mid j \in \mathcal{E}_i\}. \tag{4}$$

A major observation here is that, if the change in the nodal power injection for every node in Γ_j is zero, then the change in the line power flow of line j would be zero. This observation can be used in the estimation of power flows. It can also play an important role in the estimation of sensitivity distribution factors. The above statement means that the line segments which do not belong to set $\mathcal{E}_i$ are *insensitive* to a change in the nodal power injection of node i. Therefore, we have:

$$h_{ij} = 0, \quad \forall j \notin \mathcal{E}_i. \tag{5}$$

III. Physics-Aware Measurement-Based Model

In this section, we consider three scenarios for the availability of the measurements. We develop our proposed method to estimate the ISF matrix in each scenario accordingly.

A. Scenario 1: Nodal Power Injections and Line Power Flows are Fully Measured

Suppose we can measure the power flow at every node and every line of the power distribution feeder. As discussed in section II-A, this does not necessarily make the system of equations in (3) overdetermined; depending on the number of measurement samples. Based on the discussion in Section II-C, we know that the ISF matrix is a *sparse* matrix, and we have already extracted the sparsity pattern of this matrix in (5). Therefore, we can formulate the problem in Scenario 1 as:

$$\begin{aligned} \underset{\mathbf{H}}{\text{minimize}} \;\; & ||\Delta\mathbf{Y} - \Delta\mathbf{X}\mathbf{H}||_F^2 + \lambda \sum_{j=1}^{l} ||\mathrm{c}(\mathbf{H}, j)||_1 \\ \text{subject to} \;\; & \mathbf{A}\mathbf{H} = \mathbf{0}, \end{aligned} \tag{6}$$

where $||.||_F$ is the Frobenius norm, $\mathrm{c}(\mathbf{H}, j)$ is the operator which returns the j-th column of matrix $\mathbf{H}$, λ is the sparsity regularization parameter, and matrix $\mathbf{A}$ is constructed based on the sparsity patterns in equation (5). The minimization problem in (6) is convex and can be solved by any convex optimization solver, such as the CVX toolbox [19].

B. Scenario 2: Nodal Power Injections are Fully Measured but Line Power Flows are not Fully Measured

In this scenario, we consider a more realistic and more challenging case, where the nodal power injection measurements are available for every node, but the line power flow measurements come from only a few lines. Due to the setting of the problem in this scenario, some entries in the measurement matrix $\Delta\mathbf{Y}$ are unknown, because they are not directly measured. Instead, they have to be estimated simultaneously.

To address Scenario 2, let us split the measurement matrix for line power flows into two sub-matrices as:

$$\Delta\mathbf{Y} = \Delta\mathbf{Y}_u + \Delta\mathbf{Y}_k, \tag{7}$$

where $\Delta\mathbf{Y}_u$ denotes the *unknown* part and $\Delta\mathbf{Y}_k$ denotes the *known* part. Based on the discussion in Section II-C, we know that matrix $\Delta\mathbf{Y}$ is a sparse matrix, and its sparsity pattern is a function of the sparsity pattern of matrix $\Delta\mathbf{X}$. In particular, since we know which entries of matrix $\Delta\mathbf{X}$ are non-zero, we also know which entries of the unknown matrix $\Delta\mathbf{Y}_u$ are zero. Therefore, we can modify the minimization problem in (6) as:

$$\begin{aligned} \underset{\mathbf{H}, \Delta\mathbf{Y}_u}{\text{minimize}} \;\; & ||\Delta\mathbf{Y}_u + \Delta\mathbf{Y}_k - \Delta\mathbf{X}\mathbf{H}||_F^2 + \lambda \sum_{j=1}^{l} ||\mathrm{c}(\mathbf{H}, j)||_1 \\ & + \gamma \sum_{i=1}^{T} ||\mathrm{c}(\Delta\mathbf{Y}_u^\top, i)||_1 \\ \text{subject to} \;\; & \mathbf{A}\mathbf{H} = \mathbf{0} \\ & \mathbf{B}\Delta\mathbf{Y}_u = \mathbf{0} \end{aligned} \tag{8}$$

where γ is the sparsity regularization parameter, and $\mathbf{B}$ is the matrix which captures the sparsity pattern of the line power flows based on the values in measurement matrix $\Delta\mathbf{X}$.

The optimization problem in (8) is a convex minimization problem which can be solve by a commercial solver.

C. Scenario 3: Neither the Line Power Flows nor the Nodal Power Injections are Fully Measured

In this scenario, we consider the most challenging case, where only a subset of line power flows and only a subset of nodal power injections are measured. Therefore, similar to equation (7), we split the matrix of nodal power injection measurements into two sub-matrices as follows:

$$\Delta\mathbf{X} = \Delta\mathbf{X}_u + \Delta\mathbf{X}_k, \tag{9}$$

where $\Delta\mathbf{X}_u$ denotes the *unknown* part and $\Delta\mathbf{X}_k$ denotes the *known* part. Unlike the minimization problem in (8), we cannot define a matrix similar to $\mathbf{B}$, because some of the values of the changes in the nodal power injection are unknown to us. Therefore, we need to develop a way such that the sparsity pattern of the power flows can still be utilized.

To do so, let us define a binary variable b_i^t. It is 1, if the node i experiences a change in the nodal power injection at time t, otherwise it is zero. At each time t, let us stack up all the binary variables into vector $\mathbf{b}^t \in \mathbb{R}^{1\times n}$. We have:

$$-M.\mathbf{b}^t \leq \Delta\mathbf{x}^t \leq M.\mathbf{b}^t, \tag{10}$$

where M is a large number. Based on matrix $\Delta\mathbf{X}_k$, we already know the value for some of the binary variables. However, for the rest of the nodes that we do not measure their power injection directly, the associated binary variable is unknown.

For each line segment j, from the definition of Γ_j in (4) in Section II-C, we know that if all of the binary variables that are associated with the nodes in set Γ_j are zero, then the change in the line power flow of line segment j is zero. We can mathematically express this observation as:

$$-M.\max\{\mathbf{b}^t_{\Gamma_j}\} \leq \Delta y_j^t \leq M.\max\{\mathbf{b}^t_{\Gamma_j}\}, \quad \forall j \in \mathcal{L}. \tag{11}$$

Now, we can rewrite the minimization problem in (8) as:

$$\begin{aligned} \underset{\mathbf{H}, \mathbf{b}, \Delta\mathbf{Y}_u, \Delta\mathbf{X}_u}{\text{minimize}} \;\; & ||\Delta\mathbf{Y}_u + \Delta\mathbf{Y}_k - (\Delta\mathbf{X}_u + \Delta\mathbf{X}_k)\mathbf{H}||_F^2 \\ & + \lambda \sum_{j=1}^{l} ||\mathrm{c}(\mathbf{H}, j)||_1 + \gamma \sum_{i=1}^{T} ||\mathrm{c}(\Delta\mathbf{Y}_u^\top, i)||_1 \\ & + \omega \sum_{i=1}^{T} ||\mathrm{c}(\Delta\mathbf{X}_u^\top, i)||_1 \\ \text{subject to} \;\; & \mathbf{A}\mathbf{H} = \mathbf{0} \\ & \text{Eqs. (10)} - \text{(11)} \end{aligned} \tag{12}$$

where ω is the sparsity regularization parameter.

The minimization problem in (12), is non-convex, because of the product (i.e., bilinear) term in the objective function. Therefore, to be able to work with this formulation, we propose to break down the minimization problem in (12) into the following two optimization problems:

$$\begin{aligned}\underset{\mathbf{b},\Delta\mathbf{Y}_u,\Delta\mathbf{X}_u}{\text{minimize}}\ & ||\Delta\mathbf{Y}_u+\Delta\mathbf{Y}_k-(\Delta\mathbf{X}_u+\Delta\mathbf{X}_k)\mathbf{H}^0||_F^2\\ & +\gamma\sum_{i=1}^{T}||\mathrm{c}(\Delta\mathbf{Y}_u^\top,i)||_1+\omega\sum_{i=1}^{T}||\mathrm{c}(\Delta\mathbf{X}_u^\top,i)||_1\\ \text{subject to}\ & \text{Eqs. }(10)-(11)\end{aligned} \tag{13}$$

and

$$\begin{aligned}\underset{\mathbf{H}}{\text{minimize}}\ & ||\Delta\mathbf{Y}^*-(\Delta\mathbf{X}^*)\mathbf{H}||_F^2++\lambda\sum_{j=1}^{l}||\mathrm{c}(\mathbf{H},j)||_1\\ \text{subject to}\ & \mathbf{AH}=\mathbf{0}.\end{aligned} \tag{14}$$

In the optimization problem in (13), we obtain the solution of $\mathbf{b}$, $\Delta\mathbf{Y}_u$, and $\Delta\mathbf{X}_u$ for a given initial value for matrix $\mathbf{H}$, which is denoted by $\mathbf{H}^0$. Once the estimations of the power flows are obtained by solving (13), next, we solve the optimization problem in (14) in order to update matrix $\mathbf{H}$. After that, we will continue to iterate between (13) and (14) until we convergence to a solution. It is worth to mention that, the choice of the initial value for the entries of matrix $\mathbf{H}$ has impact on both convergence and the accuracy of the obtained solution. One good choice to obtain the initial matrix $\mathbf{H}^0$ is to hypothetically assume that the power distribution system is lossless, and set all the non-zero entries equal to one. Another option, which might be more accurate, is to use the estimated values for the previous time slots as the initial choice for $\mathbf{H}^0$. More discussion on this issue will be given in the next section.

IV. Case Studies

In this section, we test the performance of our proposed method in jointly estimating both the uknown power flows and the ISF matrix. The case studies are completed by simulating the IEEE 33-bus distribution test network which has a radial topology. We use the default values the of loads per the IEEE 33-bus test system as the operating points of the system. We assume that the measurement noise is Gaussian with zero mean and standard deviation of 0.01.

A. Performance Evaluation

For all the three scenarios in Section III, we consider a range of 20 to 200 measurement samples to be available for each measurement. For Scenarios 2 and Scenario 3, we assume that between 10% to 50% of the power flow measurements are unknown. We compare our proposed method with the conventional least square estimation only for Scenario 1, as it cannot address the cases in Scenario 2 and Scenario 3 with unknown entries in measurement samples. We use Mean Absolute Percentage Error (MAPE) as the index for this assessment. The result for this comparison is shown in Fig 2.

As we can see, the proposed method outperforms the conventional least square method in Scenario 1. Also, from the curve of the Scenario 2 and Scenario 3 in which 30% of the measurements are unknown, we can see that missing measurements can cause a degradation on the performance of the proposed method; however, it still outperforms the conventional least square method which is not built to deal with rank deficient matrices. Moreover, as we can see, increasing the number of measurement samples for all methods leads to improvement in the performance of estimation.

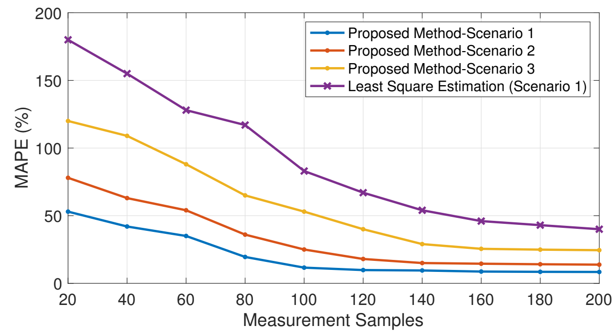

Fig. 2. The comparison for estimation accuracy of the ISF matrix for different methods. The LS method is applicable to only Scenario 1.

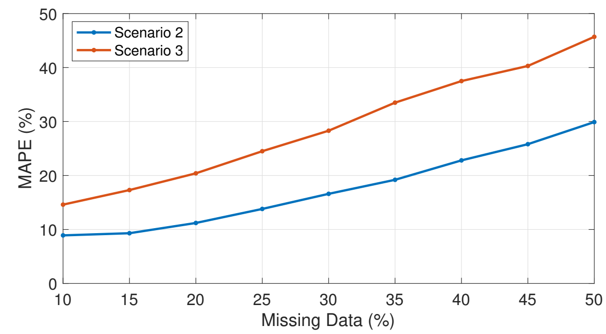

Fig. 3. The performance of the proposed method under different levels of low-observability severance, i.e., different percentage of missing measurement data. Scenario 1 is not included; because it assumes no missing data.

We also investigate the impact of missing data percentage on the accuracy of the proposed method in Scenario 2 and Scenario 3. To the best of our knowledge, we are the first who studies this problem under the settings in these two scenarios. Therefore, we only compare the accuracy of estimation for the missing values with respect to their true value for different level of missing data ranging from 10% to 50%. The results are shown in Fig. 3, where the number of measurement samples is 200. As we can see, higher percentage of missing data causes the accuracy of the proposed method to degrade.

B. Importance of Initial Guess for the ISF Matrix

As mentioned earlier, solving the non-linear optimization in (12) through the proposed iterative approach in (13) and (14) depends on the choice for the initial guess of the decision variables, in particular the choice of $\mathbf{H}^0$. An inappropriate initial guess can cause convergence issue. Also, it drastically impact the accuracy of the obtained solution. In this section, we study this impact by comparing the convergence rate and the value of the objective function for multiple initial guesses.

As explained in Section III-C, a good choice for the initial guess is to start from the lossless system, and assume every non-zero entry in the ISF matrix to be one. The second choice is to use the estimated ISF matrix from the previous time

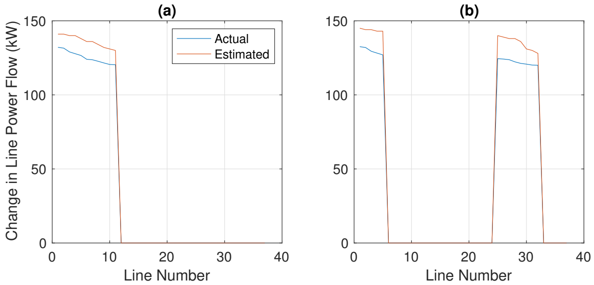

Fig. 4. The application of estimating the ISF matrix in a low-observable network on estimating the impact of DER failure at buses a) 12, b) 33.

instances $t = 1$ to $t = T$, as the initial estimation at time $t = T+1$. Besides the above two options, we have also randomly selected two other initial guesses as the third and the fourth selections. The performance of the above four initial guesses on the convergence and accuracy of the results are shown on Table 1, where 25% of the measurements are unknown and the total number of available measurement samples is 200. We can see that, when we get the initial guess based on estimated values of the previous time slots, proposed method converges to the solution in lower number of iterations, and additionally, its accuracy is higher. Also, random selections of initial guess can cause either to convergence and divergence.

C. Application in Contingency Analysis: DER Failure

An example for the application of estimating ISF is in contingency analysis. In this section, we consider the failure of a major DER and we seek to examine how the estimated ISF can help estimate the impact of such DER failure on the amount of power flow across the power distribution feeder, *without the need to solve the power flow equations*.

In our base case scenario, we assume that there are four DERs installed at buses 6, 12, 26, and 33, and each one has the capacity of 120 kW. Our goal is to find out how the line power flows would change if one of the DERs suddenly fails. The results for this analysis are shown in Fig. 4; focusing on the failures of DERs at buses 12 and 33. As we can see the estimated ISF matrix has reasonably estimated the change in line power flow in each case. In Fig. 4(a), the average error is 2.45% and the maximum error is 9.32%. In Fig. Fig. 4(b), the average error is 4.65% and the maximum error is 12.08%.

V. CONCLUSIONS

A new physics-aware measurement-based approach was proposed for joint estimation of the ISF matrix and the power flows in power distribution systems with low-observability. Due to low-observability, the measurement matrices, whether for measuring nodal power injection or line power flow, have several unknown entries. To address low-observability, sparsity patterns of the ISF matrix and the changes in power flows were approximated based on the radial topology of the power distribution systems. Three scenarios were defined based on the availability of measurements: 1) nodal power injections and line power flows are fully measured; 2) nodal power injections are fully measured but line power flows are not fully measured; 3) neither the line power flows nor the nodal power injections are fully measured. The optimization problem was formulated and solved in each case. The performance of the proposed method was verified via case studies, with respect to the impact of the number of measurement samples, the extent of low-observability, and application in contingency analysis.

TABLE I
IMPACT OF INITIAL GUESS ON THE CONVERGENCE AND ACCURACY OF ESTIMATION

Initial Guess	Converged	No. Iterations	MAPE
Point 1	Yes	382	31%
Point 2	Yes	566	26%
Point 3	No	-	-
Point 4	Yes	573	45%

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