Event Signatures in H-PMU Measurements: An Information-Theoretic Analysis of Real-World Data

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***Abstract*—A harmonic phasor measurement unit (H-PMU) is a technological evolution of the conventional PMU. Unlike a conventional PMU that solely captures fundamental phasors, an H-PMU encompasses the measurements of both fundamental and harmonic phasors. So far, the application of H-PMU measurements has been on the analysis of steady-state characteristics of harmonic phasors, such as for harmonic source identification or harmonic state estimation. However, in this paper, we take a rather unique and multi-disciplinary approach to harness the additional information provided by *harmonic phasor signatures* to better analyze power system *events*. The proposed approach is *data-driven* and from the view point of *information theory*, and based on real-world H-PMU measurements. Our analysis reveals the presence of significant independent information content in the extracted features from the event signatures in harmonic phasor measurements. This study also explores the applications of utilizing such additional information content, such as to optimally select the orders of the harmonic phasors for the analysis of power system events, as well as to enhance the performance in the task of event clustering in power systems situational awareness.**

***Keywords*: Harmonic phasor measurements, event signatures, power system situational awareness, H-PMU, information theory.**

I. Introduction

A. Background and Motivations

Data from Phasor Measurement Units (PMUs) have been widely used in recent years to detect, characterize, identify, and classify *events* in power systems. An event in this field is defined broadly and may refer to load switching, capacitor bank switching, connection or disconnection of distributed energy resources, inverter malfunction, momentary oscillations, a minor fault, a signature for an incipient fault, etc. Analysis of events has major applications in power system situational awareness [1]–[3], equipment condition monitoring [4], cybersecurity [5], and modeling power system dynamics [6].

Traditionally, PMUs provide phasor measurements based on the *fundamental* component of voltage or current. However, the fundamental phasor measurements may *not* fully capture the rich information content that is embedded in the changes that occur in voltage and current during an event.

This area has recently received a boost with the development of *Harmonic Phasor Measurement Units* (H-PMUs), which are a new class of smart grid sensors. H-PMUs can provide not only the phasor measurements for the fundamental component (same as in the traditional PMUs), but also the phasor measurements for the harmonic components. We refer to [7] for more details about the recent developments in the field of H-PMUs.

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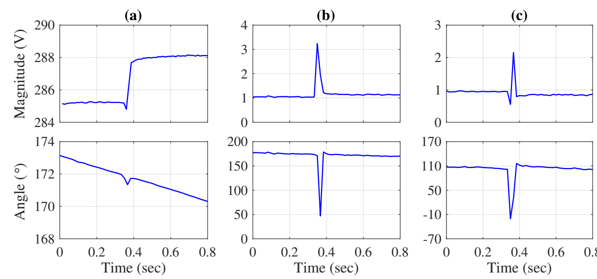

Fig. 1. Signatures of an event in: (a) fundamental phasor measurements, (b) third harmonic phasor measurements, (c) fifth harmonic phasor measurements.

Accordingly, in this paper, we seek to examine the *event signatures* as captured not only in the fundamental phasor measurements but also in the harmonic phasor measurements.

Throughout this paper, we use real data from a test site in California. An example is shown in Fig. 1. The measurements are three-phase, but only one phase is shown here. Fig. 1(a) shows the signature of an event in the fundamental phasor measurements. Figs. 1(b) and (c) show the signatures of the *same* event in the third and the fifth harmonic phasor measurements, respectively. These signatures demonstrate important features, both in transient changes and in steady-state changes, as well as both in magnitude and in phase angle.

The type of harmonic phasor signatures that are shown in Figs. 1(b) and (c) are currently unexplored as they have *not* been used in the literature to study power system events. However, when available, the further information that is provided about an event by these additional phasor measurements can significantly enhance our ability to make inferences.

B. Related Work

PMU measurements have been widely used for the analysis of power system events. Various methods have been developed for examining the event signatures in the fundamental phasor measurements, such to do event detection [2], event location identification [8], and event type classification [9], [10].

As for the literature on harmonic phasor measurements, the focus so far has *not* been on the analysis of event signatures. It has been rather on the following three general categories. First, there are studies that focus on the design of H-PMU devices and the signal processing methods to accurately obtain the harmonic phasors, such as by using matrix pencil method [11]. Second, there are studies that seek to identify the sources of harmonics in power systems, such as based on Harmonic State Estimation (HSE) [12], [13]. This line of work also includes methods to assess the daily harmonic variations in power systems [14]. Importantly, the work in this category is only concerned with the *steady state* analysis of harmonics.

It is *not* concerned with the analysis of power system events. Third, there are studies that focus on other (less traditional) applications of harmonic phasor measurements, such as in topology identification [15], fault location identification [16], and detection of wideband oscillations in power systems [17].

There are also studies that investigate which certain harmonic orders are created as the result of which certain physical phenomena; albeit with focus on steady-state harmonics. For example, the third harmonic is common when there are issues in three-phase systems without a neutral, while the fifth harmonic can be due saturation in the transformers' cores [18].

Different from the above-mentioned literature, in this paper, we take a rather unique approach to harness the additional information provided by harmonic phasor signatures to better analyze power system events. The only other study that has touched on a similar idea is the recent work in [19], in which Graph Learning is used to investigate events. In [19], the main focus is still on fundamental phasors; yet an example is presented for the case where some harmonic phasor measurements are included in the analysis. Notably, the example in [19] was based on *computer simulations*, and *not* real-world data. Here, we take a new and more fundamental approach by using information theory and by analyzing real-world data.

II. Problem Statement

The purpose of this study is to investigate the hypothesis that the event signatures in harmonic phasor measurements can uncover some significant insights about power system events, that are *not* captured by the event signatures in the conventional fundamental phasor measurements. Suppose a conventional PMU provides the following vectors of *fundamental* voltage and current phasor measurements during an event:

$$\mathbf{V}_1\angle\boldsymbol{\theta}_1, \mathbf{I}_1\angle\boldsymbol{\phi}_1, \tag{1}$$

where

$$\begin{aligned} \mathbf{V}_1 &= \left[\, V_1[1] \;\ldots\; V_1[n] \,\right]^T, \quad \mathbf{I}_1 = \left[\, I_1[1] \;\ldots\; I_1[n] \,\right]^T \\ \boldsymbol{\theta}_1 &= \left[\, \theta_1[1] \;\ldots\; \theta_1[n] \,\right]^T, \quad \boldsymbol{\phi}_1 = \left[\, \phi_1[1] \;\ldots\; \phi_1[n] \,\right]^T \end{aligned} \tag{2}$$

are the time-series of the magnitude of the fundamental voltage phasor, the magnitude of the fundamental current phasor, the phase angle of the fundamental voltage phasor, and the phase angle of the fundamental current phasor, respectively. Parameter n is the number of phasor measurements that are recorded in the window of time series that captures each event.

Next, suppose we replace the conventional PMU with an H-PMU. In addition to providing the fundamental voltage and current phasor measurements in (2), the H-PMU can also provide the following complex-valued vectors of *harmonic* voltage and current phasor measurements during the event:

$$\begin{aligned} &\mathbf{V}_2\angle\boldsymbol{\theta}_2, \mathbf{V}_3\angle\boldsymbol{\theta}_3, \ldots, \mathbf{V}_m\angle\boldsymbol{\theta}_m \\ &\mathbf{I}_2\angle\boldsymbol{\phi}_2, \mathbf{I}_3\angle\boldsymbol{\phi}_3, \ldots, \mathbf{I}_m\angle\boldsymbol{\phi}_m \end{aligned} \tag{3}$$

Here, the harmonic phasors are reported by the H-PMU up to harmonic order m. In practice, the H-PMU may not report all the harmonic phasors. For example, it may only report the third and the fifth harmonics. Or it may only report the two most dominant harmonics; see [20, Section 4.5]. Furthermore, the H-PMU may or may not report both the harmonic voltage phasors and the harmonic current phasors. For example, for the phasor measurements in Fig. 1, the H-PMU only reported $\mathbf{V}_3\angle\boldsymbol{\theta}_3$ and $\mathbf{V}_5\angle\boldsymbol{\theta}_5$, in addition to reporting $\mathbf{V}_1\angle\boldsymbol{\theta}_1$.

It is clear that an H-PMU provides *more data* than a conventional PMU. However, our question is on whether (and to what extent) the event signatures in the harmonic phasor measurements in (3) provide *more information* than the event signatures in the fundamental phasor measurements in (2), *as far as the analysis of the power system events is concerned*. The presence and the extent of such additional information can depend on the type of the event that is captured and the order of the harmonic phasor that is measured. We seek to address this open problem by using concepts from information theory.

The nature of this study is inherently *data-driven*. Therefore, we leverage a real-world dataset from a substation in California. The measurements are made at the secondary side of a 69 kV to 12.47 kV transformer that supplies a power distribution feeder. The dataset covers one whole year of power system events, from March 1, 2022 to February 28, 2023. A total of 2400 events were recorded during this period. All events are three-phase and often unlabeled. For each event, the voltage and current phasors are recorded, both at the fundamental and harmonic frequencies.

III. Methodology: An Information Theoretic Approach

A. Entropy and Information Content

We commence our proposed approach by introducing the concept of *entropy*, which is the foundation of information theory [21]. For a random variable, entropy measures the inherent uncertainty or randomness of its outcomes. For a discrete random variable A, entropy $H(A)$ is defined as:

$$H(A) = -\sum_{a\in\mathcal{A}} P_A(a)\log P_A(a), \tag{4}$$

where P_A is the probability mass function of discrete random variable A over its support set $\mathcal{A}$, which is the set of all possible values that A can take with a non-zero probability.

Given another discrete random variable B, the notion of conditional entropy can be similarly defined as [22]:

$$H(A|B) = \sum_{a\in\mathcal{A}}\sum_{b\in\mathcal{B}} P_{A,B}(a,b)\log\left(\frac{P_{A,B}(a,b)}{P_A(a)}\right), \tag{5}$$

where $P_{A,B}$ is the joint probability mass function of A and B. The conditional entropy measures the average residual uncertainty about variable A once variable B has been observed.

Together, the above two concepts lay the groundwork for the definition of Mutual Information (MI), a measure of the reduction in uncertainty about random variable A when random variable B is observed [21], or in other words, the *overlap in information content* between two variables:

$$\text{MI}(A;B) = H(A) - H(A|B). \tag{6}$$

We note that MI is symmetric in A and B, i.e., $\text{MI}(A;B) = \text{MI}(B;A)$. It is zero when A and B are statistically independent, indicating that observing A provides no additional information about B and vice versa [21]. A Normalized Mutual Information (NMI) has been introduced in [23] as:

$$\mathrm{NMI}(A;B) = \frac{\mathrm{MI}(A;B)}{H(A)+H(B)}, \tag{7}$$

which takes a value between 0 and 1. If there is no shared information content between A and B, then $\mathrm{NMI}(A;B) = \mathrm{MI}(A;B) = 0$. The more the overlap in information content of A and B, the closer the NMI approaches a value of 1.

The formulations in (6) and (7) can be extended to also measure the overlap in the information content among sets of variables. For instance, the NMI between random variable A and the pair of random variables B and C is obtained as:

$$\mathrm{NMI}(A;B,C) = \frac{\mathrm{MI}(A;B,C)}{H(A)+H(B,C)}, \tag{8}$$

where

$$\mathrm{MI}(A;B,C) = H(A) - H(A|B,C). \tag{9}$$

Note that $H(A|B,C)$ is the conditional entropy of A, given both B and C. Again, the value of $\mathrm{NMI}(A;B,C)$ is always between 0 and 1, where 0 implies no shared information content between A and (B,C). The closer $\mathrm{NMI}(A;B,C)$ is to 1, the more information content is shared by A and (B,C).

B. Information Content of Features in Phasor Measurements

In order to apply the above concepts to the context of our study, we take two steps. First, we represent the event signatures, whether in the fundamental phasor measurements or the harmonic phasor measurements, based on *features*, which represent a summary statistic of time series measurements. Second, we discretize the extracted features from the first step.

With regards to feature extraction, we start from the existing literature in the analysis of event signatures in phasor measurements. Specifically, we focus on extracting the features from the following time-series on each of the three phases [10]:

$$V_i, I_i, \cos(\theta_i - \phi_i), \quad i = 1, \ldots, m. \tag{10}$$

Note that, for $i=1$, i.e., for the fundamental phasors, the term $\cos(\theta_i - \phi_i)$ in (10) is the same as power factor. However, for any $i > 1$, i.e., for harmonic phasor measurements, the term $\cos(\theta_i - \phi_i)$ in (10) can too be viewed as a notion of power factor, but based on harmonic phasors. Importantly, it is common *not* to directly use the phase angles of voltage and current in the analysis of events. Instead, the cosine of their difference is used to eliminate the impact of the fluctuations in the frequency of the power system, see [20, p. 114].

Let $X(t)$ denote a time-series from the list in (10)Suppose $\bar{X}_{\text{pre}}$ and $\bar{X}_{\text{post}}$ denote the *average* of $X(t)$ *before* and *after* the event, respectively. Furthermore, let $X_{\min}$ and $X_{\max}$ denote the *minimum* value and the *maximum* value of $X(t)$ during the event. We define two *features* with respect to each $X(t)$:

$$S = \bar{X}_{\text{post}} - \bar{X}_{\text{pre}} \tag{11}$$

and

$$T = \begin{cases} X_{\max} - \bar{X}_{\text{pre}} & \text{if } |X_{\max} - \bar{X}_{\text{pre}}| \geq |X_{\min} - \bar{X}_{\text{pre}}| \\ X_{\min} - \bar{X}_{\text{pre}} & \text{otherwise,} \end{cases} \tag{12}$$

TABLE I
NORMALIZED MI AMONG CERTAIN PAIRS OF FEATURES

	V	I	$\cos(\theta-\phi)$
NMI(T1;T3)	0.1034	0.0445	0.0208
NMI(T1;T5)	0.0653	0.0389	0.0217
NMI(S1;S3)	0.0524	0.0417	0.0191
NMI(S1;S5)	0.1386	0.0544	0.0203

where S is the change in the *steady-state* conditions of the time-series, before and after the event; and T is the maximum change in the *transient* conditions of the time-series during the event, either as overshoot or as undershoot during the event.

Importantly, for each event, the above two features are extracted from not only the event signatures in the *fundamental* phasor data but also the event signatures in the *harmonic* phasor data. Accordingly, a total of $6m$ features are extracted from the phasor data in (10) on each phase for each event.

The discretization of the extracted features is done by dividing the range of each continuous-valued feature into a number of bins (equal to the square root of the number of data points). This choice balances the objectives that discretization effects remain negligible for NMI measurements and that the numerical NMI estimates are sufficiently accurate.

We can incorporate each pair of the extracted discretized features as random variables A and B to obtain NMI as in (7). For example, A can be the transient change in the event signature in the fundamental phasor measurements and B can be the transient change in the event signature in the harmonic phasor measurements of the third harmonic. Accordingly, we can investigate the information content of the extracted features in the real-world power system events in the dataset.

In this paper, we estimate the joint probability functions as well as the marginal probability distribution functions by discretizing the extracted features into bins, and using bivariate or multivariate histogram bin counts depending on the number of features. The results are then normalized based on the number of events. Marginal distributions are obtained by summing the joint probabilities for the two features.

IV. CASE STUDIES USING FIELD DATA

A. Analysis of Pairwise Information Content

Suppose the harmonic phasor measurements are limited to the third harmonic phasors, and the fifth harmonic phasors. The results are shown in Table I. Each row provides the NMI between a feature from the fundamental phasor measurements and a comparable feature from the harmonic phasor measurements. These results are based on taking the average of the NMI across all the power system events in the dataset. The features are extracted by (11) and (12). For example, T1 is the transition change in the event signature based on the fundamental phasor data, and S3 is the steady-state change in the event signature based on the third harmonic phasor data.

All the NMI values in Table I are close to 0, highlighting that every feature for every harmonic phasor that is listed in this table carries distinct information. The varying levels of NMI values in this table suggest that the information overlap between these features are different for different features and

TABLE II
NORMALIZED MI FOR TWO DIFFERENT SCENARIOS FOR CHOOSING A FIXED NUMBER OF HARMONIC PHASORS

Selection Scenario 1	NMI(T1; T3, T5, T7, T9)	0.2463
	NMI(S1; S3, S5, S7, S9)	0.2620
Selection Scenario 2	NMI(T1; T2, T3, T4, T5)	0.1334
	NMI(S1; S2, S3, S4, S5)	0.1383

for different harmonics. A lower value for NMI means that the second feature provides more additional information to the first feature. For example, consider the lowest value of NMI in the column under V, which is NMI(S1,S3) = 0.0524. This means that by using the steady-state changes in the voltage magnitude of the third harmonic phasors, we can most significantly increase the information content of the features, compared to the case where we only use the steady-state changes in the voltage magnitude of the fundamental phasors.

B. Application in Optimal Selection of Harmonic Phasors to Maximize Information Content in Event Signatures

Recall from Section II that an H-PMU may provide harmonic phasor measurements only for a small and specific number of harmonics. One may ask: *if we can only measure a few harmonic phasors, which ones should we pick for the analysis of events?* Next, we seek to answer this question.

Specifically, we compare two scenarios, which contain an equal number of harmonic phasors within their feature sets. Without loss of generality, we assume that only the magnitudes of the voltage phasors are used in this case study. Scenario 1 exclusively employs odd harmonic phasors:

$$\mathbf{V}_1\angle\boldsymbol{\theta}_1,\ \mathbf{V}_3\angle\boldsymbol{\theta}_3,\ \mathbf{V}_5\angle\boldsymbol{\theta}_5,\ \mathbf{V}_7\angle\boldsymbol{\theta}_7,\ \mathbf{V}_9\angle\boldsymbol{\theta}_9. \tag{13}$$

Scenario 2 employs both odd and even harmonic phasors:

$$\mathbf{V}_1\angle\boldsymbol{\theta}_1,\ \mathbf{V}_2\angle\boldsymbol{\theta}_2,\ \mathbf{V}_3\angle\boldsymbol{\theta}_3,\ \mathbf{V}_4\angle\boldsymbol{\theta}_4,\ \mathbf{V}_5\angle\boldsymbol{\theta}_5. \tag{14}$$

The above two scenarios use the same number of phasors, i.e., five. The question is: *which scenario carries more information about the event?* We can answer this question by conducting a *multivariate* mutual information analysis corresponding to the formulation in (8), but based on five variables to account for the number of phasors in (13) and (14). The results are shown in Table II. We observe that, whether with respect to the transient features or with respect to the steady-state features, Scenario 2 has a lower NMI than Scenario 1 on average:

$$\begin{aligned}&\text{NMI(T1; T2, T3, T4, T5)} < \text{NMI(T1; T3, T5, T7, T9)}\\&\text{NMI(S1; S2, S3, S4, S5)} < \text{NMI(S1; S3, S5, S7, S9)}.\end{aligned} \tag{15}$$

That means that the features in Scenario 2 have less information overlap with the features of the fundamental phasors than those in Scenario 1. Thus, the event signatures in Scenario 2 are expected to be more informative with respect to the characteristics of the events than the event signatures in Scenario 1. This approach can help with systematic and optimal selection of the harmonic phasors to maximize their information content.

We note that the above results are in contrast to the traditional analysis of harmonics in the field of power quality, where even harmonics are almost never considered due to the often symmetric nature of voltage waveforms in steady-state conditions. However, when it comes to the analysis of power system events, the above results suggest that the use of even harmonics, as in Scenario 2, can be beneficial.

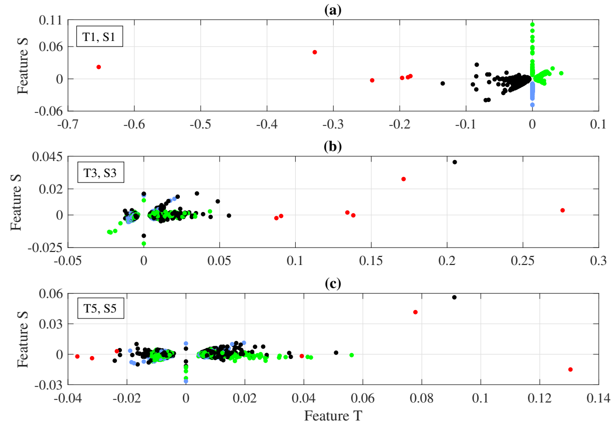

Fig. 2. (a) Fundamental phasor signature feature space, (b) Third harmonic phasor signature feature space, (c) Fifth harmonic phasor signature feature space. In all subfigures, the color coding is based on clustering of the events with respect to their S and T features in the fundamental phasors.

C. Application in Event Clustering

The additional information content of H-PMU measurements can improve the performance of event-based tasks in power systems. An example is in the field of event clustering and event classification, where we seek to identify the type or cause of an event based on its signatures in the measurements.

Fig. 2 depicts the use of harmonic phasor measurements in event clustering. The clusters are obtained by using K-means clustering for different choices of the feature space. Different subfigures demonstrate different harmonic phasor signature feature spaces, while sharing a *common color-coding*. The color coding is based on clustering of the events with respect to their S and T features as captured in Fig. 2(a) based on the fundamental phasor measurements. This representation provides a baseline scenario to assess how the events that may seemingly belong to the same cluster appear very differently based on their features in the higher harmonic spaces.

To see this, consider the feature spaces in Figs. 2(b) and (c), which are based on the features that are extracted from the third and the fifth harmonic phasor measurements, respectively. The crucial aspect to note here is that the colors represent the *same clusters* that were identified in Fig. 2(a), based on the features from the *fundamental* phasor measurements.

The comparison of Figs. 2(a), (b), and (c) confirms that the clusters based on the features in the fundamental phasor measurements are *not* valid for the higher harmonic feature spaces. This discrepancy implies that clustering based purely on fundamental phasors may not fully capture the characteristics that are hidden in the higher harmonic phasor features.

Since the event data in real-world power systems is predominantly *unlabeled*, we propose to use the silhouette value to assess the performance in event clustering, leveraging the various features derived in our analysis. Silhouette value indicates how well each object lies within its cluster. Specifically, it is a measure of how similar an object is to its own cluster (cohesion) compared to other clusters (separation) [24].

The silhouette value ranges between -1 and 1. A higher value indicates a better *clustering quality* of the power system events. Without loss of generality, we assume that the number of clusters is fixed at four. That is, the target in each clustering task is to create four clusters based on certain features.

If we use only the features from the fundamental phasors, i.e., T1 and S1, then the silhouette values are obtained as:

$$\text{Silhouette Value: } \begin{array}{ccc} \text{I} & \text{V} & \cos(\theta-\phi) \\ \hline 0.174 & 0.2151 & 0.2248 \end{array} \tag{16}$$

If we use the features from the fundamental phasors and the third harmonic phasors, i.e., T1, S1, T3, and S3, then we have:

$$\text{Silhouette Value: } \begin{array}{ccc} \text{I} & \text{V} & \cos(\theta-\phi) \\ \hline 0.2884 & 0.4526 & 0.3432 \end{array} \tag{17}$$

Finally, if we use the features from the fundamental phasors, the third harmonic phasors, and the fifth harmonic phasors, i.e., T1, S1, T3, S3, T5, S5, then the silhouette value becomes:

$$\text{Silhouette Value: } \begin{array}{ccc} \text{I} & \text{V} & \cos(\theta-\phi) \\ \hline 0.3547 & 0.677 & 0.5011 \end{array} \tag{18}$$

By comparing the results in (16), (17), and (18), we can conclude that the silhouette values are highest in all variables when all features are included in the event clustering task. This outcome supports the premise that the inclusion of the features from the event signatures in the harmonic phasor measurements, particularly both the third and fifth harmonics, can significantly enhance the event clustering performance.

V. Conclusions

The event signatures in harmonic phasor measurements that are recorded by H-PMUs can uncover new information about power system events that are *not* captured by the event signatures in fundamental phasor measurements from conventional PMUs. Based on the results in this paper, while the use of fundamental phasors is necessary, having access also to harmonic phasors as additional phasor measurements can be highly beneficial. This can be shown systematically by applying techniques from information theory to real-world phasor measurements. One key issue here is to decide which exact orders of the harmonic phasors we may need to analyze when it comes to event signatures. While the traditional analysis of harmonics often focuses on odd harmonics due to the often symmetric shape of the waveforms in steady-state conditions, the information content of event signatures can be valuable from both odd and even harmonic phasor measurements. Another application of the proposed methods is in event clustering. Future research can further explore the applications of this new direction in power system monitoring.

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