Physics-Aware Sparse Harmonic State Estimation in Power Distribution Systems

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***Abstract*—Achieving reliable harmonic state estimation (HSE) is becoming increasingly important in smart grid development due to the growing penetration of power electronics devices, non-linear loads, and distributed energy resources. A major challenge in this field is the limited availability of power quality measurements in power distribution feeders. This inevitably results in *low-observability* conditions in the HSE problem, where harmonic phasors are measured at only a very few locations on the distribution feeder. We address this challenge by proposing a novel *physics-aware* sparse HSE method. It is built upon extracting some unique sparsity patterns in power distribution systems based on their radial topology and physical characteristics. Based on the various extracted individual and group sparsity features, we formulate and solve the physics-aware sparse HSE problem *with* and also *without* knowing the location of the harmonic source. The effectiveness of the proposed method is confirmed through multiple case studies and sensitivity analysis.**

***Keywords*—Harmonic state estimation, physics-aware method, data-driven method, low-observability, sparse recovery, radial topology, harmonic phasor measurements.**

I. Introduction

With the growing penetration of power electronics devices, non-linear loads, and distributed energy resources, it is becoming increasingly crucial to achieve situational awareness about harmonic distortions in power distribution systems.

Harmonic state estimation (HSE) is a key step in achieving situational awareness about harmonic distortions. However, a major challenge in this field is the limited availability of power quality measurements in power distribution feeders. Although the cost of power quality sensors is dropping, it is still impractical to maintain full-observability about harmonic distortions by placing power quality sensors at every bus.

We seek to address this open problem in this manuscript.

Although there is a rich literature on the study of HSE at power transmission networks [1], [2], developing HSE methods at power distribution networks requires addressing its own unique issues. For a radial or weakly-meshed power distribution system, numerous harmonic measurement devices are needed to make the system fully-observable. Therefore, due to the lack of extensive monitoring at the distribution level, a necessary requirement in any HSE method at distribution level is to address the issue of low-observability [3]–[6].

A common approach to make up for the lack of measurements in the HSE problem is to use pseudo-measurements such as historical data. But in this case, the error and uncertainty in the historical data can severely affect the accuracy of HSE.

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Another approach is not to use pseudo-measurements, but rather to directly deal with the low-observability conditions by using mathematical techniques to solve the HSE problem as an undetermined system of equations. In [4], the HSE problem is solved by using singular value decomposition, where the HSE problem is formulated as a least square optimization and solved by obtaining the pseudo-inverse of the low-rank measurement matrix. In [3], a method is proposed to solve the HSE problem based on sparse Bayesian learning which involves regression analysis for power flow calculation and recurrent neural network models for demand prediction in power distribution systems. In [7], the HSE problem is formulated as a constrained sparsity maximization problem which is solved by using linear programming.

However, to the best of our knowledge, the prior studies in the literature have not taken the unique physics-based features of the power distribution system into account, such as its radial topology, as the main tool in achieving sparse recovery. As a result, they still require a considerably large number of sensors to be deployed. Thus, there is still a need to explore some of the most important sparsity patterns in the state variables of the power distribution system, while the physics-based relations and constraints among the state variables are being considered.

This paper proposes a novel physics-aware sparse HSE method in power distribution systems with very few power quality sensors. The proposed method is built upon extracting some unique sparsity patterns in power distribution systems based on their radial topology and other physical characteristics. The main contributions of this paper are as follows:

1) The physics-based individual or group sparsity patterns are identified for harmonic nodal injection current phasors, harmonic line current phasors, and harmonic nodal voltage phasors. Importantly, it is assumed that the power distribution feeder has only very few harmonic measurement sensors and there is only one harmonic source of each harmonic order in the system.
2) Two scenarios are considered. First, we formulate and solve the sparse HSE problem when the location of the harmonic source for each harmonic order is known. We formulate a constrained weighted-Lasso optimization problem that incorporates the mathematical implications of the various identified physics-based sparsity patterns.
3) We also formulate and solve the sparse HSE problem under a more challenging scenario where the location of harmonic source is *not* known. Accordingly, we combine the proposed sparse recovery formulation with a proper exhaustive search in order to enforce the various identified sparsity features despite the lack of prior knowledge about the location of the harmonic source.

II. Problem Formulation

Let $\mathcal{G} := (\mathcal{N}, \mathcal{L})$ denote the graph representation of a power distribution feeder, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ is the set of distribution lines. For a harmonic order h, let $\mathbf{I}_N(h)$ denote the vector which contains the nodal harmonic injection current for all the nodes in set $\mathcal{N}$. Also, let $\mathbf{I}_L(h)$ denote the vector of harmonic line currents for all the line segments in set $\mathcal{L}$ for harmonic order h. Let $\mathbf{V}(h)$ denote the vector of nodal harmonic voltage phasors for all the nodes in set $\mathcal{N}$ for harmonic order h. In our problem formulation, the vector of state variables for each harmonic order h is denoted by

$$\mathbf{X}(h) = [\, \mathbf{I}_N(h) \;\; \mathbf{I}_L(h) \;\; \mathbf{V}(h) \,]^\top. \tag{1}$$

Suppose there are only a few harmonic phasor measurement units (H-PMUs) available across the distribution feeder to measure the harmonic nodal voltage phasors $\mathbf{V}^m(h)$ and the harmonic line current phasors $\mathbf{I}_L^m(h)$ at the locations of H-PMU installations. Let $\mathbf{Z}(h)$ denote the measurement vector:

$$\mathbf{Z}(h) = [\, \mathbf{V}^m(h) \;\; \mathbf{I}_L^m(h) \,]^\top. \tag{2}$$

The goal in HSE is to estimate the state variables $\mathbf{X}(h)$ based on the available measurements $\mathbf{Z}(h)$.

A. Basic Equations

Since all the harmonic measurements are in phasor domain, the following relationship holds between the harmonic phasor measurements and the harmonic state variables:

$$\mathbf{Z}(h) = \mathbf{H}(h)\, \mathbf{X}(h), \tag{3}$$

where $\mathbf{H}(h)$ is the measurement matrix at harmonic order h. Next, we explain the construction of matrix $\mathbf{H}(h)$.

The first type of equations in matrix $\mathbf{H}(h)$ are associated with harmonic voltage phasor measurements. The following relationship holds between the nodal harmonic injection current phasors and the harmonic voltage phasor measurements:

$$\mathbf{V}^m(h) = \mathbf{Y}^{-1}(h)\, \mathbf{I}_N(h), \tag{4}$$

where $\mathbf{Y}(h)$ is the admittance matrix for harmonic order h. In addition to (4), the harmonic voltage phasor measurements can be mapped also to their associated entries in the vector of harmonic voltage phasors through an identity mapping:

$$\mathbf{V}^m(h) = \mathbf{U}\, \mathbf{V}(h), \tag{5}$$

where $\mathbf{U}$ is a diagonal matrix, where a diagonal entry is 1 if its corresponding state variable is a harmonic voltage phasor that is directly measured; otherwise it is 0.

The second type of equations in matrix $\mathbf{H}(h)$ are associated with the harmonic line current measurements. The harmonic line current measurements are mapped to the vector of nodal harmonic voltage phasors as follows:

$$\mathbf{I}_L^m(h) = \mathbf{Y}_{\text{prim}}(h)\; \mathbf{V}(h), \tag{6}$$

where $\mathbf{Y}_{\text{prim}}(h)$ is the primitive admittance matrix [8], which includes the line admittances only for the line segments whose harmonic current phasors are measured. Harmonic line current phasor measurements can also be related to the vector of the harmonic line current phasors through an identity mapping:

$$\mathbf{I}_L^m(h) = \mathbf{U}\, \mathbf{I}_L(h), \tag{7}$$

where $\mathbf{U}$ is a diagonal matrix, where a diagonal entry is 1 if the corresponding state variable is a harmonic current phasor that is directly measured; otherwise it is 0.

B. Additional Equations

The equations in (4)-(7) capture all the basic relationships between the entries of vector $\mathbf{Z}(h)$ and those of vector $\mathbf{X}(h)$. We can use the equations in (4)-(7) to construct matrix $\mathbf{H}$. However, due to the limited measurements, which is due to the limited number of H-PMU installations in practice, matrix $\mathbf{H}(h)$ is a *low-rank* matrix. Thus, the system of equations in (3) has an infinite number of solutions; which is not desirable.

The main remedy to the above issue is the use of sparsity analysis as we will see in Section III. However, it is useful to also add more equations to the basic set of equations in (4)-(7). In particular, we need to create more couplings among the state variables, even though such additional couplings does *not* make matrix $\mathbf{H}(h)$ full-rank. To do so, for the line segments that do not come with a direct harmonic phasor measurement, we propose to write an equation similar to (6), as follows:

$$\mathbf{0} = \mathbf{Y}_{\text{prim}}(h)\, \mathbf{V}(h) - \mathbf{I}_L(h). \tag{8}$$

Unlike in (6), the equations in (8) are *not* based on measurements; but they do serve the purpose of further coupling the state variables. Here they act as auxiliary equations.

We can treat the zeros on the left hand side in (8) as virtual measurements to revise the vector of measurements in (2) as:

$$\mathbf{Z}(h) = [\, \mathbf{V}^m(h) \;\; \mathbf{I}_L^m(h) \;\; \mathbf{0} \,]^\top. \tag{9}$$

C. Original HSE Formulation

If the power distribution feeder is fully observable at harmonic order h, then we can formulate the HSE problem as:

$$\min_{\mathbf{X}(h)} \;\; \|\mathbf{Z}(h) - \mathbf{H}(h)\, \mathbf{X}(h)\|_2^2. \tag{10}$$

However, if the network is *not* fully observable, i.e., if we only have a few H-PMUs, then solving the above problem does not lead to a meaningful solution. Therefore, for the rest of this paper, we seek to address this open problem by making use of concepts from sparse recovery in signal processing.

III. Physics-Based Sparsity Features

From the theory of sparse recovery in signal processing, when it comes to an undetermined system of linear equations, such as the one in (3), if the unknown vector is *sparse*, then we might be able to obtain the unique solution of the undetermined system of linear equations despite the low observability conditions [9]. To do this, we need to first identify and extract the inherent sparsity patterns in the physical system.

Consider the IEEE 33-bus distribution test network in Fig. 1. Suppose there is a harmonic current source at bus 13. As it is previously shown in [10]–[12], the harmonic current in this power distribution feeder almost entirely flows through the

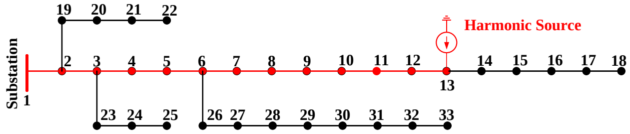

Fig. 1. An example distribution feeder with one harmonic source. In practice, the harmonic current almost entirely flows through the substation.

substation and *not* through the loads. The reason is that the impedance in the Thevenin equivalent of the substation that is seen by the distribution feeder is much less than the impedance of the loads across the distribution feeder. Therefore, almost the entire harmonic current that is injected by the harmonic source at bus 13 passes through the line segments that are marked with red color, all the way up to the substation.

The above physical observation can be used as the foundation to introduce sparsity to harmonic state estimation. Based on the notations that we defined in Section II, the sparsity is primarily in the vector of nodal harmonic current injection, i.e. $\mathbf{I}_N(h)$. Since only one harmonic source of each harmonic order is assumed to be in the network, only one entry in $\mathbf{I}_N(h)$ is non-zero, which is associated with the node of the harmonic source. The rest of the entries in $\mathbf{I}_N(h)$ are zero.

There is also a major sparsity in the vector of harmonic line current phasors, i.e., $\mathbf{I}_L(h)$. Recall from Fig. 1 that only the line segments on the red path between the harmonic source and the substation carry harmonic current. Therefore, only the entries in $\mathbf{I}_L(h)$ that are associated with the line segments on the red path are non-zero. All the other entries in $\mathbf{I}_L(h)$ that are associated with the line segments that are outside the red path are almost zero. Moreover, all the line segments that are on the red path have almost *equal* harmonic current; because the harmonic current almost entirely flows to the substation.

The above analysis also has implications for the harmonic voltage phasors. Since there is harmonic line current on the red path in Fig. 1, the nodal voltage for all the buses on this red path include *some* level of harmonic distortion associated with the same harmonic source. However, the story is a bit different for the nodes that are *outside* the red path.

To see this, let us first *group* the nodes that are *outside* the red path such that all the nodes that are laid on the same lateral are put in the same group; see [13] for a similar grouping idea. For example, buses 26, 27, 28, 29, 30, 31, 32, and 33 form one group in Fig. 1. Bus 6 is the *boundary node* for this group. For any such group, if there is no (almost no) harmonic component in the voltage at the boundary node, then there is zero (almost zero) harmonic component in the voltage of *all* the nodes in the group; otherwise, there is non-zero harmonic component in the voltage of *all* the nodes in the group. If all of the harmonic voltage phasors are non-zero in a group, they are *equal* to the harmonic voltage phasor at the boundary node.

For example, again consider buses 26 to 33 in Fig. 1 which are outside the red path and on the same lateral. They form one group. For all the nodes in this group, all would have *equal* nodal harmonic voltage phasors. If there is zero (or almost zero) harmonic component in the voltage at the boundary node, i.e., bus 6, then there would be zero (or almost zero) harmonic component in the voltages at buses 26 to 33. If there *is* a considerable harmonic component in the voltage at bus 6, then there would be an equal harmonic component in the voltages at buses 26 to 33. This is because there is *no* harmonic current flowing on any of the lines between the nodes in the above group of nodes. Thus, in addition to the sparsity in harmonic current phasors $\mathbf{I}_N(h)$ and $\mathbf{I}_L(h)$, there is also a group sparsity in the nodal harmonic voltage phasors $\mathbf{V}(h)$. We can enforce all these various sparsity patterns by constructing the following additional equality constraint in our problem formulation:

$$\mathbf{A}\,\mathbf{X}(h) = \mathbf{0}. \tag{11}$$

The rows are corresponding to two types of equality constraints: 1) the harmonic line current phasors are equal for all the line segments on the path between the substation and the node where the harmonic source is located; and 2) the harmonic voltage phasors are equal at the nodes within each group, including the boundary node of the group.

For instance, in the example that we mentioned earlier, to set the harmonic voltage phasor at bus 26 to be equal to the harmonic voltage phasor at bus 6, we need a row in matrix $\mathbf{A}$ to include 1 as the coefficient to the harmonic voltage phasor at bus 6; and -1 as the coefficient to the harmonic voltage phasor at bus 26. Notice that, one single constraint in matrix form can capture all such equalities across all the groups. The same holds for the equality of harmonic line currents.

Remark 1: The assumption that the network topology is radial is necessary for the proposed method. For the case of a *weakly* meshed network, we may still apply our method to the radial sub-segments of the network. We may also take a meshed sub-segment of the network as a *super-node*, thus reducing the weakly meshed network topology to radial topology with a few super-nodes for applying our proposed method. However, for a truly meshed network topology, such as in some micro-grids, our method may no longer be applicable.

Remark 2: The sparsity patterns that we discussed in this section are based on the assumption that there is only one harmonic source of the same harmonic order on the distribution feeder. Of course, we can have multiple harmonic sources of *different* harmonic orders. If there are multiple harmonic sources of the same order, then our method may still work; however, as the number of harmonic sources increases, the sparsity in state variables diminishes, which might degrade the performance of the method. Addressing this issue is beyond the scope of this paper and can be studied in a future work.

IV. HSE WITH SPARSE RECOVERY

Methods from compressed sensing and sparse recovery can be used to obtain the solution of the undetermined system of equations in (3). In this regard, we can formulate the HSE problem as a Lasso optimization problem [9]:

$$\min_{\mathbf{X}(h)} \quad \frac{1}{2}\|\mathbf{Z}(h) - \mathbf{H}(h)\ \mathbf{X}(h)\|_2^2 + \lambda\|\mathbf{X}(h)\|_1. \tag{12}$$

The first term in the objective function is the least square error in state estimation. The use of ℓ_1-norm in the second term is a common approach in sparse recovery to minimize the number of non-zero state variables, where λ is a penalty factor.

Although the Lasso optimization in (12) treats $\mathbf{X}(h)$ as a sparse vector, it does not distinguish between its entries. Whilst, in our discussion in Section III, we extracted valuable information about the sparsity pattern of the specific entries in vector $\mathbf{X}(h)$ based on the physics of the system. Hence, we need to reflect that information in the the problem formulation. This raises the question on how this would be done if:

1) The location of harmonic source is *known*,
2) The location of harmonic source is *unknown*.

A. Known Location of Harmonic Source

In this scenario, we assume that the location of the harmonic source is known. We know *which entry* in the nodal harmonic current injection vector $\mathbf{I}_N(h)$ is non-zero; however, its value is still *not* known and it must be estimated by the HSE. Similarly, we know which entries in vector $\mathbf{I}_L(h)$ are non-zero; but we do *not* know their values. We also know which entries in vectors $\mathbf{I}_L(h)$ and $\mathbf{V}(h)$ are in the same group.

To incorporate the above information to the sparse recovery process, we reformulate the Lasso optimization in (12), and present it as a *constrained weighted-Lasso* optimization [14]:

$$\begin{aligned} \min_{\mathbf{X}(h)} \quad & \frac{1}{2}\|\mathbf{Z}(h) - \mathbf{H}(h)\ \mathbf{X}(h)\|_2^2 + \lambda \mathbf{W}\mathbf{X}(h) \\ \text{s.t.} \quad & \mathbf{A}\,\mathbf{X}(h) = \mathbf{0}. \end{aligned} \tag{13}$$

where $\mathbf{W}$ is a vector which contains the *weight* for each entry in vector $\mathbf{X}(h)$ to enforce the extracted sparsity patterns. If an entry in $\mathbf{X}(h)$ is known to be zero, then a large weight is used in $\mathbf{W}$ to create a large penalty to push the value toward zero. In contrast, for the entries in $\mathbf{X}(h)$ that are known to be non-zero, we only use a small weight in $\mathbf{W}$ to create a small penalty for that entry such that it does not grow.

Problem (13) is convex. We can use any convex optimization solver, such as CVX toolbox (*www.cvxr.com*), to solve it.

B. Unknown Location of Harmonic Source

As a more challenging scenario, next, we assume that the location of the harmonic source is *not* known. Hence, we do *not* know which exact entries are zero in advance.

Nevertheless, all the sparsity patterns that we extracted in Section III are still valid. Therefore, we can still solve this more challenging case by combining the method in Section IV-A with an exhaustive search. The idea is as follows:

If we *assume* a tentative location for the harmonic source, we can use the exact formulation as in (13) to solve the HSE problem by using the sparsity patterns based on such tentative assumption. Suppose we *assume* that bus k is the location of the harmonic source; and accordingly, we obtain $\mathcal{R}_k$ as the corresponding *residue* when we solve the optimization problem in (13). In other words, $\mathcal{R}_k(h)$ is the optimal objective value of the optimization problem in (13) based on the *assumption* that the harmonic source is at bus k.

Therefore, by taking each of the buses in the network as the location of the harmonic source and solving the optimization problem in (13) accordingly, we can identify the *unknown* location of the harmonic source at harmonic order h as:

$$k^{\star} = \arg\min_{k} \mathcal{R}_k(h). \tag{14}$$

TABLE I
PERFORMANCE COMPARISON OF DIFFERENT HSE METHODS FOR HARMONIC ORDER $h = 3$

Method	MSE $\mathbf{V}$	STD $\mathbf{V}$	MSE $\mathbf{I}_L$	STD $\mathbf{I}_L$
Physics-Aware Sparse HSE	0.0174	0.094	0.00275	0.0274
Sparse HSE	223.8	7.56	13.34	2.53
Method in [4]	221.1	7.368	9.79	2.512

The above problem can be solved by using an exhaustive search. This requires to solve N optimization problems of the form in (13), where N is the number of buses in the network. Once we obtain $\mathcal{R}_k(h)$ for each $k = 1, \ldots, N$, we can obtain $k^{\star}$ by simply taking the minimum of the N obtained residues.

Once $k^{\star}$ is obtained, the analysis in this section reduces to the same analysis in the first scenario in Section IV-A.

V. CASE STUDIES

We apply the proposed physics-aware sparse HSE method to the IEEE 33-bus distribution test system [15]. All the case studies are simulated in the Open Distribution System Simulator (OpenDSS) [16]. Six H-PMUs are assumed to be installed at the substation and at nodes 6, 18, 22, 25, and 33.

A. Performance Comparison

We compare the performance of the proposed physics-aware sparse HSE method with two other methods: 1) the sparse HSE but without utilizing the physics-based knowledge; and 2) the method in [4] which works based on singular value decomposition. We use the Mean Square Error (MSE) as the metric to compare the performance of different HSE methods:

$$\text{MSE} = \frac{1}{N}||\mathbf{X}_{act}(h) - \mathbf{X}_{est}(h)||_2^2, \tag{15}$$

where N is the number of unknown harmonic state variables.

Table I shows the results based on the mean and the variance of the MSE index; which are calculated separately for the unknown harmonic nodal voltages, denoted by MSE $\mathbf{V}$ and STD $\mathbf{V}$, and for the unknown harmonic line currents, denoted by MSE $\mathbf{I}_L$ and STD $\mathbf{I}_L$. The harmonic source is assumed to be of harmonic order $h = 3$. The magnitude of the injected harmonic current is assumed to vary randomly between 10% to 50% of the load at the location of the harmonic source. We assume that the location of the harmonic source is known.

As we can see, the proposed method has a drastically lower average MSE and standard deviation in comparison with the method in [4] and the sparse HSE without physics-awareness.

B. Unknown Location of Harmonic Source

Next, we assume that the location of the harmonic source is not known. Accordingly, we use the proposed method in Section IV-B. The results from the exhaustive search are shown in Fig. 2. Here, we plotted the Residue $\mathcal{R}_k(h)$ versus the candidate location of the harmonic source at buses $k = 2, \ldots, 33$. We can see that the minimum residue is obtained at $k^{\star} = 4$. This is indeed the correct location of the event bus; which confirms the effectiveness of the proposed method even when the location of the harmonic source is not known.

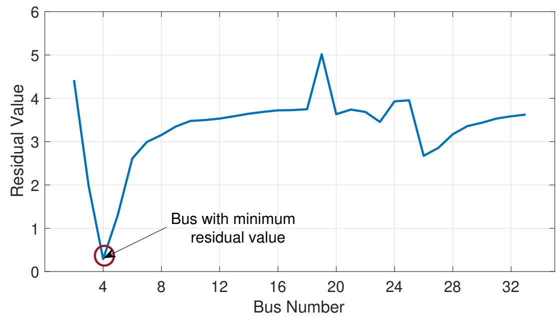

Fig. 2. Residue $\mathcal{R}_k(h)$ versus the candidate location of the harmonic source. The location of the harmonic source is *not* known in advance.

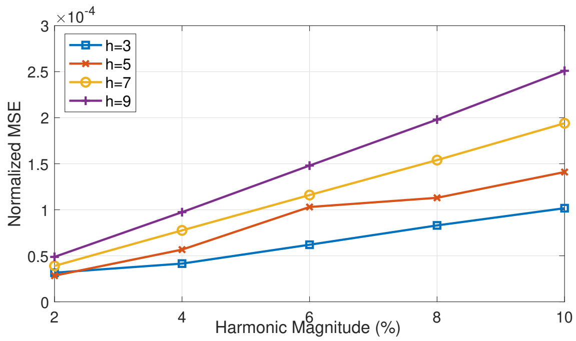

Fig. 3. Comparing the results at different harmonic orders.

C. Sensitivity Analysis: Harmonic Order

In practice, the magnitude of harmonic source may vary depending on the harmonic order. Thus, to have a fair assessment, we compare the normalized MSE for voltages for different harmonic orders. The results are shown in Fig. 3. We can see that the normalized MSE increases as we increase the harmonic order as well as the magnitude of harmonic source.

D. Sensitivity Analysis: Location of the Harmonic Source

Fig. 4 shows the HSE residue, i.e., the optimal objective value of the HSE optimization problem, for all the possible scenarios for the location of the harmonic source. We can see that, as the distance of the harmonic source from the substation grows, the performance of the HSE method gradually degrades. This is due to the fact that a longer distance for the harmonic source from the substation means a *less sparse* scenario. Also, when distance increases, the estimation error increases as well; because the observability over the nodes that are far from the sensors is lowered. As a result, as we move to bus 33, the HSE estimation error increases in general.

VI. Conclusions and Future Work

A novel physics-aware sparse HSE is formulated and solved for radial power distribution systems under low-observability conditions. The key in this design is to extract the various individual and group sparsity patterns in the harmonic nodal injection current phasors, harmonic line current phasors, and harmonic nodal voltage phasors with respect to the location of the harmonic source. Accordingly, the HSE is formulated as a constrainted weighted Lasso optimization. Our analysis covers the challenging scenario where the location of the harmonic source is not known. Multiple case studies in OpenDSS confirmed the effectiveness of the proposed method.

As a future work, one may relax the assumption on having only one harmonic source of the same order. One may also examine the case where the network topology is not radial.

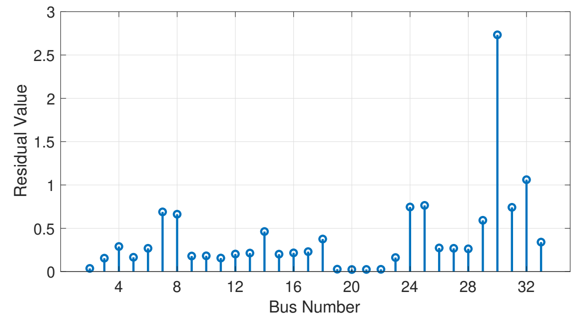

Fig. 4. HSE residue versus the location of the harmonic source, where $h = 3$.

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