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Natural convection in a vertical strip immersed in a porous medium

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Abstract

In this work, the conjugated heat transfer characteristics of a thin vertical strip of finite length, placed in a porous medium has been studied using numerical and asymptotic techniques. The nondimensional temperature distribution in the strip and the reduced Nusselt number at the top of the strip are obtained as a function of the thermal penetration parameter s, which measures the thermal region where the temperature of the strip decays to the ambient temperature of the surrounding fluid. The numerical values of this nondimensional parameter permits to classify the different physical regimes, showing different solutions: a thermally long behaviour, an intermediate transition and a short strip limit.

Keywords: Porous medium; Natural convection; Heat transfer

1. Introduction

In this work we deal with the problem of the coupled conjugate conduction-natural convective heat transfer in a vertical solid strip totally embedded in a porous medium. The book by Pop and Ingham [1] presents abundant theoretical evidence, showing the importance of the thermal interaction with different heat transfer mechanisms in practical systems. In the same direction, the book of Sundén and Heggs [2] shows that this simple geometrical configuration is found in a broad range of scientific and engineering problems associated with different industrial applications. Additional examples on this and other related topics can be obtained in the books by Ingham and Pop [3], Nield and Bejan [4] and Vafai [5]. Lock and Gunn [6] did the first theoretical study dealing with the conjugate conduction-free convection problem of a long vertical thin strip or fin embedded in a porous medium. They obtained self-similar solutions for the vertical fin geometry. Cheng and Minkowycz [7], Kuehn et al. [8] and Sparrow and Acharya [9] developed equivalent analyses for the same type of problems. Based on these studies, Pop et al. [10] obtained a set of similarity solutions for a long vertical plate projecting downward from a heated horizontal plane base at uniform temperature, for the case of the thermal conductivity-fin thickness product varying as a power of distance from a certain specified origin. Later, Pop et al. [11], improved the above analysis by developing a finite-difference numerical scheme for the case of uniform thickness and thermal conductivity of the fin. Pop and Nakayama [12] reviewed the problem of conjugate convective heat transfer from a vertical fin embedded in a fluid-saturated porous media.

The above mentioned authors mainly consider an infinitely long fin in order to formulate the thermally coupled governing equations. For instance, Pop and Nakayama [12] introduced an unknown characteristic length x_b (Eq. (14) in [12]) in order to nondimensionalize the governing equations and selected an appropriate origin of coordinates. However, the physical interpretation to choose this length scale was not enough clarified. One of the objectives of the present work is to show that this length scale, called in this paper L^*, can be easily estimated using an order of magnitude analysis of the governing equations. Furthermore, we avoid the unnecessary condition to assume an infinitely long fin by considering a vertical strip or fin of finite

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length L. Therefore, we introduce a nondimensional parameter L^*/L, which identifies all possible physical regimes for this conjugate heat transfer process.

Using order of magnitude estimates of the coupled governing equations, we identify the characteristic thermal penetration length L^*, where the temperature of an infinitely long fin would decay to the ambient temperature of the surrounding porous medium. As was previously mentioned, the ratio of L^* to the actual length L of the fin is a fundamental parameter that serves to classify the possible thermal regimes. Additional nondimensional parameters in the problem are the Rayleigh number (to be defined below) and the aspect ratio of the strip. Perturbation and numerical methods are employed, together with the Darcy and boundary layer approximations for the free convection flow, to analyze the transition from a thermally short fin ($s = L^*/L \gg 1$) to a thermally long fin ($s \ll 1$) and to ascertain the influence of the thermal properties of the strip and porous medium on the overall heat transfer rate. Finally, the analytical solutions obtained using perturbation techniques are compared with numerical results.

2. Basic equations

The physical model and a suitable coordinate system are given in Fig. 1. A vertical heated conducting strip of length L and thickness $2h \ll L$, is totally embedded in a fluid-saturated porous medium with a temperature T_∞. The upper surface of the strip is assumed to have a uniform temperature $T_0 > T_\infty$, whilst the lower surface is assumed to be adiabatic. Heat is transferred from the top of the strip to the fluid-saturated porous medium through the strip. Consider first an infinitely long strip. Owing to the heat loss to the surrounding fluid-saturated porous medium, the temperature of the strip decreases downward from its top, decaying towards the ambient temperature of the fluid in a thermal penetration region, with a characteristic length L^* which can be estimated from the balance of heat transfer to the fluid-saturated porous medium and heat conduction along the strip. Assuming that the Rayleigh number $\mathrm{Ra}^* = gK\beta(T_0 - T_\infty)L^*/\alpha_m \nu$ is very large compared with unity (where $g,\ K,\ \beta,\ \alpha_m$ and ν are the gravity acceleration, the specific permeability of the porous medium, the thermal expansion coefficient, the thermal diffusivity of the fluid-saturated porous medium and the kinematic viscosity, respectively), the flow around the strip is confined to a natural convection boundary layer of characteristic thickness $\delta^* = L^*/\mathrm{Ra}^{*1/2}$. The total heat lost by conduction to the fluid-saturated porous medium per unit time and unit width of the strip is of the order of $L^* k \Delta T/\delta^*$, while the heat conducted along the strip is of order $hk_s \Delta T/L^*$, where k and k_s are the thermal conductivity of the porous medium and the strip material, respectively ($k = \phi k_f + (1-\phi)k_m$ [4], where ϕ is the porosity, k_f and k_m are the thermal conductivities of the fluid and the porous matrix, respectively).The balance of these two fluxes yields

$$L^* = h\frac{(k_s/k)^{2/3}}{\mathrm{Ra}_h^{1/3}}, \tag{1}$$

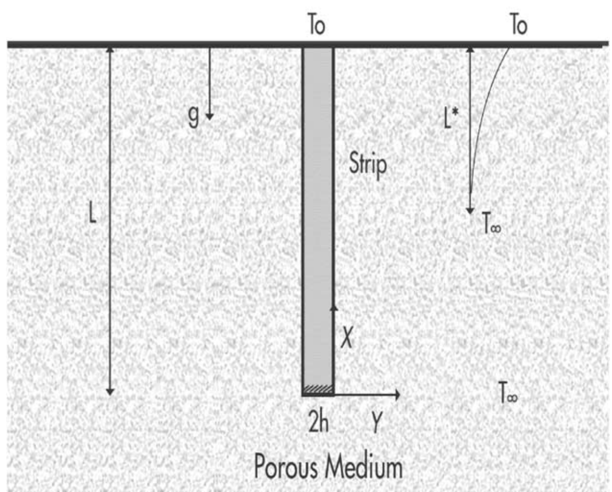

Fig. 1. Sketch of the studied problem.

where $\mathrm{Ra}_h = gK\beta(T_0 - T_\infty)h/\alpha_m \nu$ is the Rayleigh number based on the half-thickness of the strip. The boundary layer approximation is then valid for values of the ratio of thermal conductivities $k_s/k \gg L^*/h \gg 1$. For values of $L \gg L^*$, the actual length of the strip is irrelevant and L^* is the appropriate characteristic length in the heat transfer process. This regime corresponds to the thermally long strip. A parameter relating both characteristic lengths is defined by $s = L^*/L$. The thermally long strip corresponds to values of $s \ll 1$. Using the estimate of the thermal penetration length, the total heat flux transferred at the top of the strip per unit time and unit width of the strip is of order, $Q \sim hk_s(T_0 - T_\infty)/L^*$, or in nondimensional form, $\mathrm{Nu}_{\mathrm{Long}} = Q/k(T_0 - T_\infty) \sim \mathrm{Ra}^{*1/2}$. The order of magnitude of the heat flux going from the solid to the fluid in the porous medium is $k_s \Delta T_{sh}/h \sim k\Delta T \mathrm{Ra}^{*1/2}/L^*$, where ΔT_{sh} denotes the variation of the solid temperature in the transverse direction. From this relationship we obtain $\Delta T_{sh}/\Delta T \sim (k/k_s)(h/L^*)\,\mathrm{Ra}^{*1/2} \sim (h/L^*)^2$. Therefore, for values of $h \ll L^*$, the temperature variation in the transverse direction of the strip is very small compared with the overall temperature difference (thermally thin solid). This approximation is valid for values of $k_s/k \gg \mathrm{Ra}_h^{1/2}$.

In the opposite case of short strips with $h \ll L \ll L^*$, that is $s \gg 1$, the longitudinal temperature variation from the top to the bottom of the strip, ΔT_{sL} say, can be estimated from the energy balance $Q \sim hk_s \Delta T_{sL}/L \sim Lk\Delta T/\delta$, where δ is the characteristic thermal boundary layer thickness, $\delta \sim L/\mathrm{Ra}_L^{1/2}$. This balance gives $\Delta T_{sL} \sim \Delta T/s^{3/2}$. This means that the temperature of the strip is almost uniform and equal to its top temperature for values of s large compared with unity. In this regime, $k_s \Delta T_{sh}/h \sim k\Delta T \mathrm{Ra}_L^{1/2}/L$, and then $\Delta T_{sh}/\Delta T \sim (h/L^*)^{3/2}(h/L)^{1/2} \sim (h/L^*)^2 s^{1/2}$. The thermally thin approximation is valid for values of $s \ll (L^*/h)^4$, as long as $\mathrm{Ra}_L \gg 1$. In the limit of short strips, the overall Nusselt number is then $\mathrm{Nu}_{\mathrm{short}} \sim \mathrm{Ra}^{*1/2}/s^{1/2}$, for values of $s \gg 1$. The advantages of the long strip regime is clearly seen when the overall Nusselt numbers for both regimes are compared .

Using the Darcy–Boussinesq and boundary-layer approximations, the natural convection flow in the fluid-saturated porous medium is described by the following governing equations [4]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u = \frac{gK\beta}{\nu}(T - T_\infty), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

for the mass conservation, momentum and energy, respectively. The heat equation for the strip is given by

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0. \tag{5}$$

In the above equations, u, v are the velocity components along the x, y axes, T and T_s are the temperatures of the fluid-saturated porous medium and the solid plate, respectively. Eqs. (2)–(5) are to be solved with the following boundary conditions:

$$v = 0, \quad T = T_s, \quad k\frac{\partial T}{\partial y} = k_s\frac{\partial T_s}{\partial y} \quad \text{on } y = 0, \tag{6}$$

$$\left.\frac{\partial T_s}{\partial y}\right|_{y=-h} = 0, \quad T_s(L, y) = 1, \tag{7}$$

$$\left.\frac{\partial T_s}{\partial x}\right|_{x=0} = 0, \tag{8}$$

$$u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty \text{ and } x = 0 \text{ with } y \neq 0. \tag{9}$$

If the aspect ratio of the strip h/L^* is assumed to be very small compared with unity, Eq. (4) can be integrated in the transverse direction, resulting

$$h\frac{\mathrm{d}^2 T_s}{\mathrm{d}x^2} + \frac{k}{k_s}\left.\frac{\partial T}{\partial y}\right|_0 = 0. \tag{10}$$

In this case, the temperature at the strip is assumed to depend only on the longitudinal coordinate alone. In the following two sections we present the asymptotic solutions for long and short strips, respectively.

3. Long strips

For thermally long strips, $L \gg L^*$ ($s \ll 1$), the appropriate characteristic length is L^*. The following nondimensional variables are introduced for this regime

$$\sigma = \frac{L_0 - x}{L^*}, \quad Z = \frac{y}{L^*}\mathrm{Ra}^{*1/2}, \quad U = \frac{u}{u_c}, \quad V = \frac{v}{u_c}\mathrm{Ra}^{*1/2}, \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad \theta_s = \frac{T_s - T_\infty}{T_0 - T_\infty}, \tag{11}$$

where L_0 is a length to be computed later and u_c is the characteristic flow velocity, defined by $u_c = gK\beta(T_0 - T_\infty)/\nu$. The nondimensional form of the governing Eqs. (2)–(5), are now for this regime

$$\frac{\partial U}{\partial \sigma} + \frac{\partial V}{\partial Z} = 0, \tag{12}$$

$$U = -\theta, \tag{13}$$

$$U\frac{\partial \theta}{\partial \sigma} + V\frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial Z^2}, \tag{14}$$

$$\frac{\mathrm{d}^2\theta_s}{\mathrm{d}\sigma^2} + \left.\frac{\partial \theta}{\partial Z}\right|_0 = 0, \tag{15}$$

with the corresponding nondimensional boundary conditions

$$V = 0, \quad \theta = \theta_s \quad \text{at } Z = 0, \qquad U, \theta \to 0 \quad \text{for } Z, \sigma \to \infty, \qquad \theta_s = 1 \quad \text{at } \sigma = \sigma_0, \qquad \theta_s \to 0 \quad \text{for } \sigma \to \infty. \tag{16}$$

Here, $\sigma_0 = (L_0 - L)/L^*$.

This problem, as shown by [7,10], has a self-similar solution of the form

$$\theta = \theta_s(\sigma)\frac{\mathrm{d}h}{\mathrm{d}\xi}, \qquad V = -\frac{\sigma_0^{3/2}}{\sigma^2}\left[h + 2\xi\frac{\mathrm{d}h}{\mathrm{d}\xi}\right], \qquad \xi = \sigma_0^{3/2}\frac{Z}{\sigma^2}, \tag{17}$$

where θ_s is found to be $\theta_s = (\sigma_0/\sigma)^3$. The function $h(\xi)$ satisfies the ordinary differential equation

$$\frac{\mathrm{d}^3 h}{\mathrm{d}\xi^3} - 3\left(\frac{\mathrm{d}h}{\mathrm{d}\xi}\right)^2 + h\frac{\mathrm{d}^2 h}{\mathrm{d}\xi^2} = 0 \tag{18}$$

with the boundary conditions

$$h(0) = \left.\frac{\mathrm{d}h}{\mathrm{d}\xi}\right|_{\xi=0} - 1 = \left.\frac{\mathrm{d}h}{\mathrm{d}\xi}\right|_{\xi\to\infty} = 0. \tag{19}$$

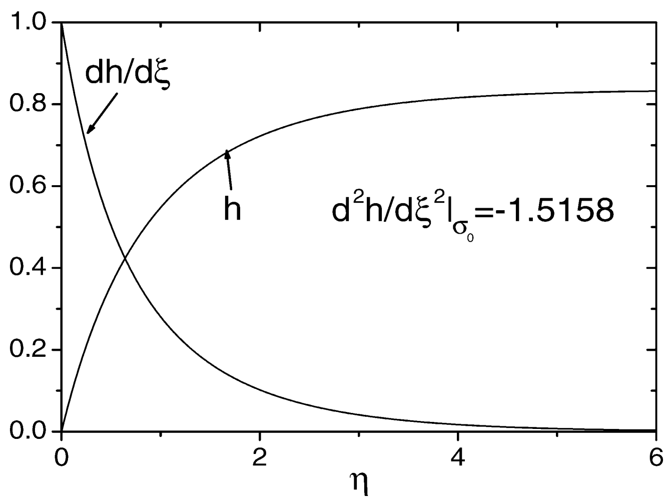

Fig. 2. Numerical solution of function $h(\xi)$, obtained by solving Eqs. (18) and (19) in the long strip regime.

The value of σ_0 can be obtained by introducing Eqs. (17) into the energy equation for the strip (15), resulting $\sigma_0 = (12/\mathrm{d}^2h/\mathrm{d}\xi^2|_{\eta=0})^{2/3} \simeq 3.9722$. The length L_0 is then $L_0 = L + 3.9722L^*$. Fig. 2 shows the function $h(\xi)$, obtained after solving numerically Eqs. (18), (19), using a central difference scheme with the aid of a quasi-linearization technique. The nondimensional temperature of the strip then is given by

$$\theta_s = \left(\frac{\sigma_0 s}{\sigma_0 s + 1 - x/L}\right)^3 \quad \text{for } s \to 0. \tag{20}$$

Finally, the nondimensional heat flux or Nusselt number at the bottom of the strip is given by

$$\frac{\mathrm{Nu}}{\mathrm{Ra}^{*1/2}} = \left|\frac{\mathrm{d}\theta_s}{\mathrm{d}\sigma}\right|_{\sigma=\sigma_0} = \frac{3}{\sigma_0} \simeq 0.75525, \quad \text{for } s \to 0. \tag{21}$$

4. Short strips

For large values of s compared with unity ($L \ll L^*$), the appropriate characteristic length is L. The heat transfer problem in this regime can be studied using the nondimensional variables

$$\chi = \frac{x}{L}, \qquad \theta_s = \frac{T_s(x) - T_\infty}{T_0 - T_\infty}, \qquad \eta = \frac{\mathrm{Ra}_L^{1/2}\, y}{L^{1/2}x^{1/2}}, \tag{22}$$

and

$$\theta = \frac{T(x,y) - T_\infty}{T_0 - T_\infty} = \frac{u}{u_c} = \frac{\partial f}{\partial \eta}, \qquad V = -\frac{v x^{1/2}\,\mathrm{Ra}_L}{L^{1/2}u_c}\left[\chi\frac{\partial f}{\partial \chi} + \frac{f}{2} - \frac{\eta}{2}\frac{\partial f}{\partial \eta}\right], \tag{23}$$

where Ra_L is the Rayleigh number based on the length of the strip. In nondimensional variables, Eq. (10) becomes

$$s^{3/2}\frac{\mathrm{d}^2\theta_s}{\mathrm{d}\chi^2} = -\frac{1}{\chi^{1/2}}\left.\frac{\partial^2 f}{\partial \eta^2}\right|_{\eta=0}, \tag{24}$$

with the boundary conditions at the top and bottom of the strip given by

$$\theta_s = 1 \quad \text{at } \chi = 1 \quad \text{and} \quad \frac{\mathrm{d}\theta_s}{\mathrm{d}\chi} = 0 \quad \text{at } \chi = 0. \tag{25}$$

The nondimensional governing equations for the fluid immersed in the porous medium reduces to

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{f}{2}\frac{\partial^2 f}{\partial \eta^2} = \chi\left[\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \chi \partial \eta} - \frac{\partial f}{\partial \chi}\frac{\partial^2 f}{\partial \eta^2}\right]. \tag{26}$$

The associated nondimensional boundary conditions are then

$$f = \frac{\partial f}{\partial \eta} - \theta_s \quad \text{at } \eta = 0 \quad \text{and} \quad \frac{\partial f}{\partial \eta} \to 0 \quad \text{for } \eta \to \infty, \tag{27}$$

plus conditions of regularity at the origin of the boundary layer $\chi = 0$.

An asymptotic solution of problem (24)–(27) for large values of s can be sought as a regular expansion in powers of $s^{-3/2}$, which is the small parameter dictated by Eq. (24). The solution can then be written as

$$\begin{Bmatrix} \theta_s \\ f \end{Bmatrix} = \begin{Bmatrix} \theta_{s0} \\ f_0(\eta) \end{Bmatrix} + \sum_{j=1}^{\infty} \frac{1}{s^{3j/2}} \begin{Bmatrix} \theta_{sj}(\chi) \\ f_j(\chi,\eta) \end{Bmatrix}. \tag{28}$$

The leading term for the nondimensional temperature in the strip is clearly $\theta_{s0} = 1$. Carrying expansions given in Eqs. (28) into the nondimensional Eqs. (24) and (26) with the associated boundary conditions, and keeping terms up to order s^{-3}, the following set of equations are obtained.

For the solid:

$$\frac{\mathrm{d}^2\theta_{sj}}{\mathrm{d}\chi^2} = -\frac{1}{\chi^{1/2}}\left.\frac{\partial^2 f_{j-1}}{\partial \eta^2}\right|_{\eta=0} \quad \text{for } j \geqslant 1, \tag{29}$$

with the boundary conditions

$$\left.\frac{\mathrm{d}\theta_{sj}}{\mathrm{d}\chi}\right|_{\chi=0} = \theta_{sj}(1) = 0, \quad \text{for } j \geqslant 1. \tag{30}$$

For the fluid immersed in the porous medium:

$$\frac{d^3 f_0}{d\eta^3} + \frac{1}{2} f_0 \frac{d^2 f_0}{d\eta^2} = 0, \tag{31}$$

$$\frac{\partial^3 f_1}{\partial\eta^3} + \frac{1}{2}\left(f_0 \frac{\partial^2 f_1}{\partial\eta^2} + \frac{d^2 f_0}{d\eta^2} f_1\right) - \chi\left[\frac{d f_0}{d\eta}\frac{\partial^2 f_1}{\partial\chi\partial\eta} - \frac{d^2 f_0}{d\eta^2}\frac{\partial f_1}{\partial\chi}\right] = 0, \tag{32}$$

etc., with the boundary conditions valid for $j \geqslant 0$

$$f_j = \frac{\partial f_j}{\partial\eta} - \theta_{sj} = 0 \quad \text{at } \eta = 0 \tag{33}$$

and

$$\frac{\partial f_j}{\partial\eta} = 0 \quad \text{for } \eta \to \infty. \tag{34}$$

Eq. (31), with the boundary conditions (33) and (34) for $j = 0$, corresponds to the classical problem of convection in a porous medium adjacent to a heated isothermal vertical plate [7], giving $d^2 f_0/d\eta^2|_0 = -G_0 = -0.444$. The first order correction for the nondimensional temperature of the strip given by Eq. (29), can be obtained after integrating it twice to yield

$$\theta_{s1} = \sum_{n=0,3/2} a_n\chi^n = -\frac{4G_0}{3}\left(1 - \chi^{3/2}\right). \tag{35}$$

Here $a_0 = -a_{3/2} = -4G_0/3$. The solution to the linear Eq. (32) with the corresponding boundary conditions must be of the form $f_1(\chi,\eta) = \sum_{n=0,3/2} a_n\chi^n g_n(\eta)$, where $g_n(\eta)$ satisfies the ordinary differential equations

$$\frac{d^3 g_n}{d\eta^3} + \frac{1}{2} f_0 \frac{d^2 g_n}{d\eta^2} - n\frac{d f_0}{d\eta}\frac{d g_n}{d\eta} + \left(\frac{1}{2} - n\right)\frac{d^2 f_0}{d\eta^2} g_n = 0, \quad \text{for } n = 0, 3/2. \tag{36}$$

The associated boundary conditions for g_n are

$$g_n(0) = \left.\frac{d g_n}{d\eta}\right|_{\eta=0} - 1 = \left.\frac{d g_n}{d\eta}\right|_{\eta\to\infty} = 0. \tag{37}$$

The solution of Eqs. (36) and (37), gives the values $G_1(n) = -d^2 g_n/d\eta^2|_0$. It can be easily shown, using the invariance properties of the boundary layer equations, that $G_1(0) = 3/2G_0 = 0.666$. The obtained numerical value of $G_1(3/2) = 0.937$. Following the same procedure, the second order correction of the nondimensional temperature in the strip is given by

$$\theta_{s2} = \sum_{n=0,3/2} \frac{a_n G_1(n)}{(n+1/2)(n+3/2)}\left(\chi^{n+1/2} - 1\right). \tag{38}$$

Summarizing, the nondimensional temperature of the plate for large values of the parameter s, up to terms of order s^{-3}, can be written as

$$\theta_s = 1 - \frac{4}{3}\frac{G_0}{s^{3/2}}\left(1 - \chi^{3/2}\right) + \frac{2}{9}\frac{G_0}{s^3}\left[12G_0\left(1 - \chi^{3/2}\right) - G_1(3/2)\left(1 - \chi^3\right)\right] + O\left(s^{-9/2}\right). \tag{39}$$

The reduced Nusselt number, $\mathrm{Nu}/\mathrm{Ra}^{*1/2} = s\, d\theta_s/d\chi|_1$ is then given by

$$\frac{\mathrm{Nu}}{\mathrm{Ra}^{*1/2}} = \frac{2G_0}{s^{1/2}}\left[1 - \frac{6G_0 - G_1(3/2)}{3s^{3/2}}\right] + O\left(s^{-7/2}\right), \text{ for } s \to \infty. \tag{40}$$

The thermal properties of the material, appearing in the definition of L^* play no role in the heat transfer at the leading order, because $\mathrm{Ra}^{*1/2}/s^{1/2} = \mathrm{Ra}_L^{1/2}$ and is independent of L^*.

5. Numerical results

Eqs. (24) to (27) have been numerically solved for different values of parameter s in the thermally short strip regime. The boundary value problem, represented by Eq. (24) has been transformed to an initial value problem by introducing the new nondimensional longitudinal coordinate $\zeta = \chi/s$. Therefore, Eqs. (24) and (26), take the form

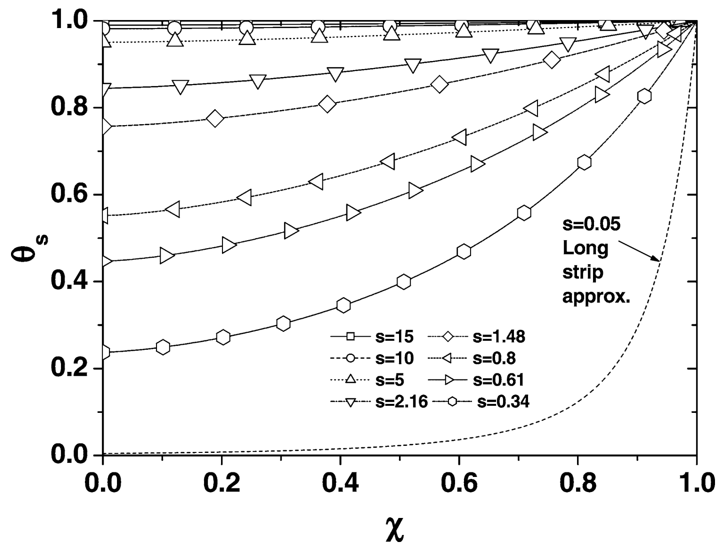

Fig. 3. Numerical results for the nondimensional temperature in the strip as a function of the normalized nondimensional longitudinal coordinate χ, for different values of s. For a value of $s = 0.05$, the asymptotic solution for long strips, given by Eq. (20), is also plotted.

$$\frac{\mathrm{d}^2\theta_s}{\mathrm{d}\zeta^2} = -\frac{1}{\zeta^{1/2}}\frac{\partial^2 f}{\partial\eta^2}\bigg|_{\eta=0}, \tag{41}$$

$$\frac{\partial^3 f}{\partial\eta^3} + \frac{f}{2}\frac{\partial^2 f}{\partial\eta^2} = \zeta\left[\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\zeta\partial\eta} - \frac{\partial f}{\partial\zeta}\frac{\partial^2 f}{\partial\eta^2}\right], \tag{42}$$

with the boundary conditions at the top and bottom of the strip given by

$$\theta_s = 1 \quad \text{at } \zeta = \zeta_f \quad \text{and} \quad \frac{\mathrm{d}\theta_s}{\mathrm{d}\zeta} = 0 \quad \text{at } \zeta = 0, \tag{43}$$

where $\zeta_f = 1/s$. Close to the bottom of the strip, $\zeta \to 0$, θ_s behaves like

$$\theta_s = a + \frac{4}{3}a^{3/2}G_0\zeta^{3/2} + \mathrm{O}\big(\zeta^{7/2}\big), \tag{44}$$

where $a \leqslant 1$ is a constant to be given a priori. For a given value of a, a value of ζ_f and s is obtained. For values of $s > 1$, Eq. (39) is used in order to have a preliminary nondimensional temperature profile at the strip. The nondimensional heat flux $\partial^2 f/\partial\eta^2|_0$ as a function of ζ is obtained by integrating Eq. (42) and the corresponding boundary conditions. This equation has been decomposed in one first order and one second order equations and solved using central difference discretization. Once the nondimensional heat flux at each ζ position has been obtained, the new nondimensional temperature profile in the strip is obtained after solving Eq. (41), with the starting behavior (44), using a fourth-order Runge–Kutta technique. The process is repeated until a convergence criterion is fulfilled. Fig. 3 shows the numerical results of the nondimensional temperature profiles for the strip as a function of the nondimensional longitudinal coordinate, for different values of s. In this figure, the nondimensional temperature of the strip has been plotted by using the long strip approximation for a value of $s = 0.05$. The reduced Nusselt number $\mathrm{Nu}/\mathrm{Ra}^{*1/2}$ is plotted in Fig. 4 as a function of parameter s, covering the full transition from the short to long strips. The numerical results are plotted with open circles. The one and two terms asymptotic solutions given by Eq. (40), obtained for large values of s (short strips) are plotted, together with the asymptotic solution for long strips ($s \to 0$), given by Eq. (21). This figure shows that the short strip approximation is practically valid for values of $s > 4$, whereas the long strip approximation holds for values of $s < 0.4$.

6. Conclusions

In this paper, we have extended previous results for the conjugated heat transfer in a vertical strip with a prescribed temperature at its top and immersed in a fluid-saturated porous medium, using analytical and numerical techniques. For large values of the Rayleigh number, the problem depends on one nondimensional parameter: $s = L^*/L$. We show that the existing

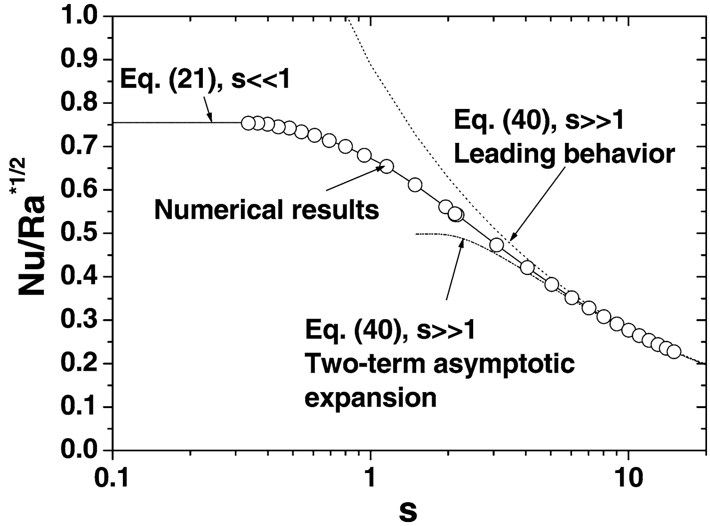

Fig. 4. Reduced Nusselt number, $\mathrm{Nu}/\mathrm{Ra}^{*^{1/2}}$ as a function of s. The open circles correspond to the numerical solution and the broken lines correspond to asymptotic solutions for long and short strips.

regimes are well classified by the assumed values of the nondimensional parameter s, ranging from the limit of a thermally long strip to the limit of a thermally short strip. In addition, the temperature of the strip, which depends on the thermal properties of the strip and the porous medium as well as on the thickness of the strip, decreases to the temperature of the porous medium in a thermal penetration length L^*. This is defined as the length needed to reach the ambient temperature of the fluid-saturated porous medium. Due to the finite thermal conductivity of the strip material, the heat transfer by conduction along the strip is a relevant mechanism that modifies the previous estimations of the reduced Nusselt number with prescribed boundary conditions. In particular, we identify that the limit of $s \ll 1$, corresponds to the case of an infinitely long strip [12]. In this case, the problem has a self-similar closed form solution for small values of s, giving an overall Nusselt number strongly dependent on the thermal conductivity of the strip material and its thickness. For long strips, the actual length of the strip plays no role on the heat transfer process, resulting heat flux at the top of the strip given by

$$Q = 0.75525 k^{2/3} k_s^{1/3} (T_0 - T_\infty)^{4/3} \left[\frac{g K \beta h}{\alpha_m \nu} \right]^{1/3}. \tag{45}$$

In the opposite case of large values of s (thermally short strip), the reduced Nusselt number depends weakly on the thermal properties of the strip material, as given by the first term in Eq. (40). The leading order behavior is in fact independent on the thermal conductivity of the strip. A two-term asymptotic solution is derived for the limit of $s \to \infty$. We have shown that the short strip approximation is valid for values of $s > 4$, whereas the long strip approximation [7,10] holds for values of $s < 0.4$. In this work, the full transition from short strip (low heat transfer rates) to long strip (large heat transfer rates) has been treated using numerical and asymptotic techniques.

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References

[1] I. Pop, D.B. Ingham, Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids in Porous Media, Pergamon, Oxford, 2001.
[2] B. Sundén, P.J. Heggs (Eds.), Recent Advances in Analysis of Heat Transfer for Fin Type Surfaces, WIT Press, Southampton, Boston, 2000.
[3] D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, Pergamon, Oxford, 1998.
[4] D.A. Nield, A. Bejan, Convection in Porous Media, 2nd Edition, Springer, New York, 1999.
[5] K. Vafai (Ed.), Handbook of Porous Media, Marcel Dekker, New York, 2000.

[6] G.S. Lock, J.C. Gunn, Laminar free convection from a downward-projecting fin, J. Heat Transfer 90 (1968) 63–70.
[7] P. Cheng, W.J. Minkowycz, Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike, J. Geophys. Res. 82 (1977) 2040–2044.
[8] T.H. Kuehn, S.S. Kwon, A.K. Tolpadi, Similarity solution for conjugate natural convection heat transfer from a long vertical fin, Int. J. Heat Mass Transfer 26 (1983) 1718–1721.
[9] E.M. Sparrow, S. Acharya, A natural convection fin with a solution-determined nonmonotonically varying heat transfer coefficient, J. Heat Transfer 103 (1981) 218–225.
[10] I. Pop, J.K. Sunada, P. Cheng, W.J. Minkowycz, Conjugate free convection from long vertical plate fins embbeded in a porous medium at high Rayleigh numbers, Int. J. Heat Mass Transfer 28 (1985) 1629–1636.
[11] I. Pop, D.B. Ingham, P.J. Heggs, D. Gardner, Conjugate heat transfer from a downward projecting fin immersed in a porous medium, in: Proc. 8th Int. Heat Transfer Conference, San Francisco, CA, 1986.
[12] I. Pop, A. Nakayama, Conjugate free and mixed convection heat transfer from a vertical fin embbeded in a porous medium, in: B. Sundén, P.J. Heggs (Eds.), Recent Advances in Analysis of Heat Transfer for Fin Type Surfaces, WIT Press, Southampton, Boston, 2000, Chapter 4.