

# **AIDED NAVIGATION**

## **GPS with High Rate Sensors**

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Jay A. Farrell



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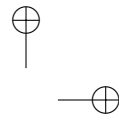
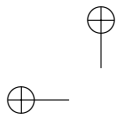
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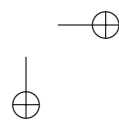
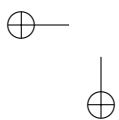
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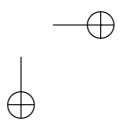
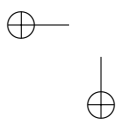
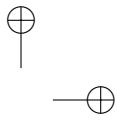
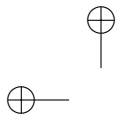
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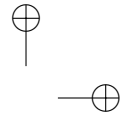
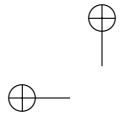
*To my students and colleagues, without whom  
this book would not have been possible.*



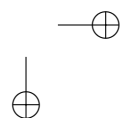
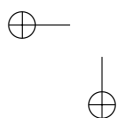


# Contents

<b>I</b>	<b>Theory</b>	<b>1</b>
	<b>Part I Overview</b>	<b>3</b>
<b>1</b>	<b>Introduction</b>	<b>5</b>
1.1	Method Overview . . . . .	7
1.1.1	Methodology Example . . . . .	8
1.1.2	Methodology Summary . . . . .	10
1.2	Overview of Part I: Theory . . . . .	12
1.2.1	Reference Frames . . . . .	12
1.2.2	Deterministic Systems . . . . .	13
1.2.3	Stochastic Processes . . . . .	13
1.2.4	Optimal State Estimation . . . . .	14
1.2.5	Performance Analysis . . . . .	15
1.2.6	Aided Navigation System Design and Analysis . . . . .	15
1.3	Overview of Part II: Applications . . . . .	16
1.3.1	GPS . . . . .	16
1.3.2	Aided Navigation Systems . . . . .	16
1.4	Overview of Appendices . . . . .	17
<b>2</b>	<b>Reference Frames</b>	<b>19</b>
2.1	Reference Frame Properties . . . . .	21
2.2	Reference Frame Definitions . . . . .	23
2.2.1	Inertial Frame . . . . .	23
2.2.2	Earth Centered Earth Fixed (ECEF) Frames . . . . .	24
2.2.3	Geographic Frame . . . . .	24
2.2.4	Geocentric Frame . . . . .	24
2.2.5	Local Geodetic or Tangent Plane . . . . .	25
2.2.6	Body Frame . . . . .	26
2.2.7	Platform Frame . . . . .	27
2.2.8	Instrument Frames . . . . .	27
2.2.9	Summary . . . . .	27
2.3	ECEF Coordinate Systems . . . . .	28



2.3.1	ECEF Rectangular Coordinates . . . . .	29
2.3.2	The Earth Geoid and Gravity Model . . . . .	29
2.3.3	ECEF Transformations . . . . .	33
2.4	Reference Frame Transformations . . . . .	35
2.4.1	The Direction Cosine Matrix . . . . .	35
2.4.2	Point Transformation . . . . .	39
2.4.3	Vector Transformation . . . . .	39
2.4.4	Matrix Transformation . . . . .	40
2.5	Specific Vector Transformations . . . . .	41
2.5.1	Plane Rotations . . . . .	41
2.5.2	Transformation: ECEF to Tangent Plane . . . . .	42
2.5.3	Transformation: ECEF to Geographic . . . . .	44
2.5.4	Transformation: Vehicle to Navigation Frame . . . . .	46
2.5.5	Transformation: Orthogonal Small Angle . . . . .	50
2.6	Rotating Reference Frames . . . . .	51
2.6.1	Direction Cosine Kinematics . . . . .	51
2.6.2	Derivative Calculations in Rotation Frames . . . . .	53
2.7	Calculation of the Direction Cosine . . . . .	54
2.7.1	Direction Cosine Derivatives . . . . .	55
2.7.2	Euler Angle Derivatives . . . . .	56
2.8	References and Further Reading . . . . .	58
2.9	Exercises . . . . .	58
<b>3</b>	<b>Deterministic Systems</b>	<b>63</b>
3.1	Continuous-Time Systems Models . . . . .	63
3.1.1	Ordinary Differential Equations . . . . .	64
3.1.2	Transfer Functions . . . . .	65
3.1.3	State Space . . . . .	66
3.2	State Augmentation . . . . .	69
3.3	State Space Linearization . . . . .	72
3.4	Discrete-Time State Space Notation . . . . .	74
3.5	State Space Analysis . . . . .	74
3.5.1	Similarity Transformation . . . . .	75
3.5.2	State Space to Transfer Function . . . . .	76
3.5.3	State Transition Matrix Properties . . . . .	79
3.5.4	Linear Time-Invariant Systems . . . . .	80
3.5.5	Discrete-Time Equivalent Models . . . . .	81
3.6	State Estimation . . . . .	82
3.6.1	Observability . . . . .	85
3.6.2	Estimator Design by Pole Placement . . . . .	87
3.6.3	Observable Subspace . . . . .	92
3.7	References and Further Reading . . . . .	95
3.8	Exercises . . . . .	95





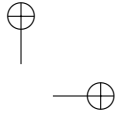
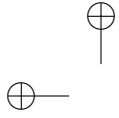
<b>4</b>	<b>Stochastic Processes</b>	<b>105</b>
4.1	Basic Stochastic Process Concepts . . . . .	105
4.1.1	Examples . . . . .	106
4.1.2	Plan of Study . . . . .	110
4.2	Scalar Random Variables . . . . .	110
4.2.1	Basic Properties . . . . .	110
4.2.2	Gaussian Distributions . . . . .	112
4.2.3	Transformations of Scalar Random Variables . . . . .	113
4.3	Multiple Random Variables . . . . .	115
4.3.1	Basic Properties . . . . .	115
4.3.2	Statistics and Statistical Properties . . . . .	117
4.3.3	Vector Gaussian Random Variables . . . . .	120
4.3.4	Transformations of Vector Random Variables . . . . .	120
4.4	Stochastic Processes . . . . .	121
4.4.1	Statistics and Statistical Properties . . . . .	121
4.4.2	White and Colored Noise . . . . .	123
4.5	Linear Systems with Random Inputs . . . . .	125
4.6	State Models for Stochastic Processes . . . . .	130
4.6.1	Standard Model . . . . .	131
4.6.2	Stochastic Systems and State Augmentation . . . . .	132
4.6.3	Gauss-Markov Processes . . . . .	133
4.6.4	Time-propagation of the Mean . . . . .	139
4.6.5	Time-propagation of the Variance . . . . .	139
4.7	Discrete-time Equivalent Models . . . . .	140
4.7.1	Calculation of $\Phi_k$ from $\mathbf{F}(t)$ . . . . .	140
4.7.2	Calculation of $\mathbf{Qd}_k$ from $\mathbf{Q}(t)$ . . . . .	141
4.8	Linear State Estimation . . . . .	144
4.9	Detailed Examples . . . . .	146
4.9.1	System Performance Metrics . . . . .	146
4.9.2	Instrument Specifications . . . . .	154
4.9.3	One Dimensional INS . . . . .	157
4.9.4	One Dimensional Position Aided INS . . . . .	159
4.10	Complementary Filtering . . . . .	161
4.11	References and Further Reading . . . . .	162
4.12	Exercises . . . . .	163
<b>5</b>	<b>Optimal State Estimation</b>	<b>169</b>
5.1	State Estimation: Review . . . . .	170
5.2	Minimum Variance Gain Derivation . . . . .	172
5.2.1	Kalman Gain Derivation . . . . .	172
5.2.2	Kalman Gain: Posterior Covariance . . . . .	173
5.2.3	Summary . . . . .	173
5.3	From WLS to the Kalman Filter . . . . .	174
5.3.1	Weighted Least Squares (WLS) . . . . .	174

5.3.2	Weighted Least Squares Solution . . . . .	175
5.3.3	Recursive Least Squares (RLS) . . . . .	179
5.3.4	Kalman Filtering . . . . .	183
5.4	Kalman Filter Derivation Summary . . . . .	184
5.4.1	Equivalent Measurement Updates . . . . .	186
5.4.2	Equivalent Covariance Measurement Updates . . . . .	187
5.4.3	Kalman Filter Examples . . . . .	187
5.5	Kalman Filter Properties . . . . .	189
5.6	Implementation Issues . . . . .	191
5.6.1	Scalar Measurement Processing . . . . .	191
5.6.2	Correlated Measurements . . . . .	193
5.6.3	Bad or Missing Data . . . . .	194
5.7	Implementation Sequence . . . . .	195
5.8	Asynchronous Measurements . . . . .	195
5.9	Numeric Issues . . . . .	196
5.9.1	Covariance Matrix Symmetry . . . . .	196
5.9.2	Covariance Matrix Positive Definiteness . . . . .	197
5.10	Suboptimal Filtering . . . . .	197
5.10.1	Deleting States . . . . .	198
5.10.2	Schmidt-Kalman Filtering . . . . .	199
5.10.3	Decoupling . . . . .	202
5.10.4	Off-line Gain Calculation . . . . .	202
5.10.5	Nonlinear Filtering . . . . .	203
5.11	References and Further Reading . . . . .	211
5.12	Exercises . . . . .	211
<b>6</b>	<b>Performance Analysis</b>	<b>217</b>
6.1	Covariance Analysis . . . . .	217
6.2	Monte Carlo Analysis . . . . .	224
6.3	Error Budgeting . . . . .	224
6.4	Covariance Divergence . . . . .	230
6.5	References and Further Reading . . . . .	232
6.6	Exercise . . . . .	233
<b>7</b>	<b>Navigation System Design</b>	<b>235</b>
7.1	Methodology Summary . . . . .	235
7.2	Methodology: Detailed Example . . . . .	237
7.2.1	Augmented Kinematic Model . . . . .	238
7.2.2	Navigation Mechanization Equations . . . . .	238
7.2.3	Sensor Models . . . . .	239
7.2.4	Error Models . . . . .	240
7.2.5	State Estimator Design . . . . .	240
7.2.6	Covariance Analysis . . . . .	244
7.3	Complementary Filtering . . . . .	247

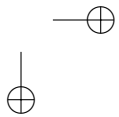
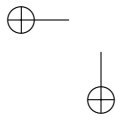
7.3.1	Frequency Domain Approach . . . . .	248
7.3.2	Kalman Filter Approach . . . . .	250
7.4	An Alternative Approach . . . . .	251
7.4.1	Total State: Kinematic Model . . . . .	252
7.4.2	Total State: Time Update . . . . .	252
7.4.3	Total State: Measurement Update . . . . .	253
7.5	Approach Comparison . . . . .	254
7.6	A Caution . . . . .	255
7.7	References and Further Reading . . . . .	256
7.8	Exercises . . . . .	256
 <b>II Application</b>		<b>259</b>
<b>Part II Overview</b>		<b>261</b>
<b>8</b>	<b>Global Positioning System</b>	<b>263</b>
8.1	GPS Overview . . . . .	264
8.1.1	GPS System . . . . .	264
8.1.2	Original GPS Signal . . . . .	265
8.2	GPS Pseudorange . . . . .	266
8.2.1	GPS Pseudorange Notation . . . . .	267
8.2.2	GPS Pseudorange Solution . . . . .	269
8.2.3	Satellite Azimuth and Elevation . . . . .	273
8.3	GPS Receiver Overview . . . . .	276
8.3.1	Carrier Phase Observables . . . . .	277
8.3.2	Delta Pseudorange Observable . . . . .	278
8.4	GPS URE Characteristics . . . . .	280
8.4.1	Clocks . . . . .	281
8.4.2	Satellite Clock Bias, $c\delta t^s$ . . . . .	282
8.4.3	Receiver Clock Error, $\Delta\tau_r$ . . . . .	283
8.4.4	Atmospheric Delay, $c\delta t_a^s$ . . . . .	287
8.4.5	Ephemeris Errors, $E^s$ . . . . .	292
8.4.6	Selective Availability, $SA^s$ . . . . .	293
8.4.7	Multipath, $M_\rho^s, M_\phi^s$ . . . . .	294
8.4.8	Receiver Noise, $\eta_\rho^i, \eta_\phi^i$ . . . . .	295
8.4.9	Carrier Tracking and Integer Ambiguity, $N^i$ . . . . .	295
8.4.10	Summary . . . . .	301
8.5	Geometric Dilution of Precision . . . . .	302
8.6	Two Frequency Receivers . . . . .	306
8.6.1	Wide and Narrow Lane Observables . . . . .	309
8.7	Carrier-Smoothed Code . . . . .	310
8.8	Differential GPS . . . . .	312
8.8.1	Relative DGPS . . . . .	313

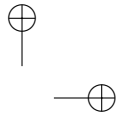
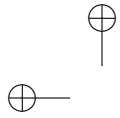
8.8.2	Differential GPS . . . . .	317
8.8.3	Double Differences . . . . .	323
8.9	Integer Ambiguity Resolution . . . . .	325
8.9.1	Decreasing the Search Space . . . . .	328
8.9.2	Selection of Optimal Integers . . . . .	329
8.9.3	Modernized GPS Signal . . . . .	331
8.10	GPS Summary . . . . .	332
8.11	References and Further Reading . . . . .	333
<b>9</b>	<b>GPS Aided Encoder-Based Dead-Reckoning</b>	<b>335</b>
9.1	Encoder Model . . . . .	336
9.2	Kinematic Model . . . . .	338
9.3	Encoder Navigation Equations . . . . .	340
9.3.1	Continuous-Time: Theory . . . . .	340
9.3.2	Discrete-Time: Implementation . . . . .	341
9.4	Error State Dynamic Model . . . . .	341
9.5	GPS Aiding . . . . .	342
9.5.1	Receiver Clock Modeling . . . . .	343
9.5.2	Measurement Differencing . . . . .	344
9.5.3	Comparison . . . . .	345
9.6	Performance Analysis . . . . .	346
9.6.1	Observability . . . . .	346
9.6.2	Covariance Analysis . . . . .	348
9.7	General 3-d Problem . . . . .	351
<b>10</b>	<b>AHRS</b>	<b>353</b>
10.1	Kinematic Model . . . . .	354
10.2	Sensor Models . . . . .	355
10.3	Initialization . . . . .	356
10.3.1	State Initialization: Approach 1 . . . . .	356
10.3.2	State Initialization: Approach 2 . . . . .	358
10.4	AHRS Mechanization Equations . . . . .	358
10.5	Error Models . . . . .	359
10.5.1	Measurement Error Model . . . . .	360
10.5.2	Attitude Error Dynamics . . . . .	363
10.5.3	AHRS State Space Error Model . . . . .	365
10.5.4	Measurement Noise Covariance . . . . .	366
10.5.5	Initial Error Covariance Matrix . . . . .	367
10.6	AHRS Approach Summary . . . . .	368
10.7	Observability and Performance Analysis . . . . .	369
10.8	Pitch and Roll Application . . . . .	370
10.9	References and Further Reading . . . . .	377

<b>11 Aided Inertial Navigation</b>	<b>379</b>
11.1 Gravitation and Specific Force . . . . .	379
11.1.1 Gravitation . . . . .	379
11.1.2 Specific Force . . . . .	380
11.1.3 Accelerometers . . . . .	381
11.1.4 Gravity Error . . . . .	384
11.2 INS Kinematic Equations . . . . .	386
11.2.1 Inertial Frame . . . . .	388
11.2.2 ECEF Frame . . . . .	388
11.2.3 Tangent Frame . . . . .	389
11.2.4 Geographic Frame . . . . .	389
11.3 INS Mechanization Equations . . . . .	390
11.4 INS Error State Dynamic Equations . . . . .	392
11.4.1 Position Error Linearization . . . . .	393
11.4.2 Attitude Error Linearization . . . . .	393
11.4.3 Velocity Error Linearization . . . . .	395
11.5 INS Error Characteristics . . . . .	396
11.5.1 Simplified Error Models . . . . .	397
11.5.2 Full Error Model . . . . .	399
11.6 Augmented State Equations . . . . .	406
11.6.1 Instrument Error Overview . . . . .	407
11.6.2 Accelerometer Error Modeling . . . . .	408
11.6.3 Gyro Error Modeling . . . . .	411
11.6.4 Error Characteristics . . . . .	414
11.7 Initialization . . . . .	414
11.7.1 Self-Alignment Techniques . . . . .	416
11.8 Aiding Measurements . . . . .	421
11.8.1 Position Aiding . . . . .	421
11.8.2 GPS Pseudorange Aiding . . . . .	422
11.9 Observability Analysis . . . . .	427
11.9.1 Stationary, Level, Known Biases . . . . .	428
11.9.2 Stationary, Level, Unknown Biases . . . . .	429
11.10References and Further Reading . . . . .	430
<b>12 LBL and Doppler Aided INS</b>	<b>431</b>
12.1 Kinematics . . . . .	432
12.1.1 Notation . . . . .	432
12.1.2 System Kinematics . . . . .	433
12.2 Sensors . . . . .	433
12.2.1 Inertial Measurement Unit . . . . .	434
12.2.2 Attitude and Yaw Sensor . . . . .	435
12.2.3 Doppler Velocity Log . . . . .	436
12.2.4 Pressure Sensor . . . . .	436
12.2.5 Long Baseline Transceiver . . . . .	436



12.3	Mechanization and IMU Processing . . . . .	438
12.3.1	Mechanization Equations . . . . .	438
12.3.2	IMU Processing . . . . .	438
12.4	Error State Dynamic Model . . . . .	439
12.4.1	Position Error Model . . . . .	439
12.4.2	Velocity Error Model . . . . .	440
12.4.3	Attitude Error Model . . . . .	441
12.4.4	Calibration Parameter Error Models . . . . .	442
12.4.5	Error Model Summary . . . . .	442
12.5	Aiding Measurement Models . . . . .	443
12.5.1	Attitude and Yaw Prediction . . . . .	443
12.5.2	Doppler Prediction . . . . .	444
12.5.3	Depth Prediction . . . . .	445
12.5.4	LBL Prediction . . . . .	445
12.6	EKF Sensor Integration . . . . .	446
12.6.1	Measurement Updates for $t \in [t_0, t_4]$ . . . . .	447
12.6.2	Measurement Updates for $t \notin [t_0, t_4]$ . . . . .	448
12.6.3	Covariance Propagation . . . . .	449
12.7	Observability . . . . .	449
12.8	Simulation Performance . . . . .	452
<b>A</b>	<b>Notation</b> . . . . .	<b>455</b>
A.1	Notation . . . . .	455
A.2	Useful Constants . . . . .	457
A.3	Acronyms . . . . .	457
A.4	Other Notation . . . . .	458
<b>B</b>	<b>Linear Algebra Review</b> . . . . .	<b>459</b>
B.1	Definitions . . . . .	459
B.2	Matrix and Vector Operations . . . . .	461
B.3	Independence and Determinants . . . . .	464
B.4	Matrix Inversion . . . . .	465
B.5	Matrix Inversion Lemma . . . . .	466
B.6	Eigenvalues and Eigenvectors . . . . .	467
B.7	Positive Definite Matrices . . . . .	468
B.8	Singular Value Decomposition . . . . .	469
B.9	Orthogonalization . . . . .	469
B.10	LU Decomposition . . . . .	470
B.11	UD Decomposition . . . . .	472
B.12	Matrix Exponential . . . . .	472
B.12.1	Power Series . . . . .	472
B.12.2	Laplace Transform . . . . .	473
B.13	Matrix Calculus . . . . .	474
B.13.1	Derivatives with Respect to Scalars . . . . .	474





CONTENTS

B.13.2 Derivatives with Respect to Vectors . . . . . 474

B.13.3 Derivatives with Respect to Matrices . . . . . 475

B.14 Numeric Zero Finding & Optimization . . . . . 475

    B.14.1 Numerical Zero Finding . . . . . 475

    B.14.2 Numeric Optimization . . . . . 477

B.15 References and Further Reading . . . . . 480

B.16 Exercises . . . . . 480

**C Calculation of GPS Satellite Position & Velocity 485**

C.1 Satellite Clock Corrections . . . . . 487

C.2 Satellite Position Calculations . . . . . 488

C.3 Reference Frame Consistency . . . . . 494

C.4 Satellite Velocity . . . . . 495

    C.4.1 Equations from Ephemeris . . . . . 495

    C.4.2 Practical Issues . . . . . 496

C.5 Ionospheric Model . . . . . 497

C.6 References and Further Reading . . . . . 500

**D Quaternions 501**

D.1 Quaternions Basics . . . . . 501

D.2 Rotations . . . . . 503

    D.2.1 Direction Cosine to Quaternion . . . . . 504

    D.2.2 Quaternions to Euler Angles . . . . . 505

D.3 Quaternion Derivative . . . . . 505

    D.3.1 General Derivation . . . . . 505

    D.3.2 Body to Navigation Frame Result . . . . . 506

D.4 Summary . . . . . 507

D.5 Quaternion Integration . . . . . 508

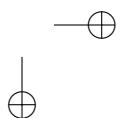
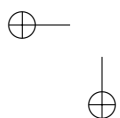
D.6 Attitude Representation Comparison . . . . . 510

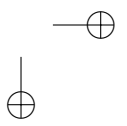
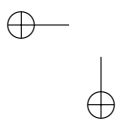
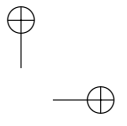
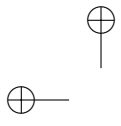
D.7 References and Further Reading . . . . . 511

D.8 Exercises . . . . . 511

**Bibliography 515**

**Index 527**









# Preface

Technological innovations over recent decades have enabled an ever increasing array of inexpensive, high accuracy, aided navigation systems suitable for student projects, research test-bed usage, and commercial and military applications. These innovations include: small, low cost, low power sensors; GPS technology; and, high performance, low cost, computational equipment for embedded processing.

Aided navigation involves two categories of sensors. The output signals from sensors in the first category are integrated using a kinematic model of the system. The result of this integration provides a reference trajectory. Example kinematic input sensors include inertial measurement units, Doppler radar or sonar, and wheel encoders. Elements of the second category of sensors are used to estimate the error between this reference trajectory and trajectory of the vehicle. In the estimation process, these aiding sensors may also determine various calibration parameters to improve the future performance of the system. The advent of Global Navigation Satellite Systems (GNSS) has provided an accurate and reliable aiding measurement source for navigation systems in certain outdoor applications. A prototypical example of a GNSS sensor suitable for aided operations is the Global Positioning System (GPS).





Aided navigation is motivated when an application, such as an autonomous vehicle, requires accurate high bandwidth information about the vehicle state reliably at a high sample rate. The aiding sensors may satisfy the accuracy requirement, but not the reliability, bandwidth, and high sample rate specifications. The reference trajectory computed from the kinematic input sensors may satisfy the reliability, bandwidth, and high sample rate specifications; however, due to its integrative nature, errors that accumulate over time can cause the unaided system to eventually fail to meet the accuracy specification.

The aiding sensors and the reference trajectory computed from the kinematic input sensors have complementary characteristics; therefore, it is natural to consider their implementation within an integrated approach.

## Objective

My main objective in writing this book is to provide a self-contained reference on aided navigation system design that is appropriate for use in a classroom setting. The methodology presented herein is an industry standard approach. It has been used by many persons in various applications with a high-level of success.

The text is written with the expectation that the reader has the standard background of a senior in a bachelor-of-science (BS) engineering program.



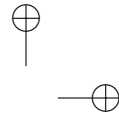
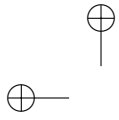
I have taken special care in presenting the material at the BS level. For example, the topic of optimal filtering is derived both in the least squares context and in the standard stochastic processes context. Examples are included throughout the text to relate theoretical concepts back to applications, to motivate the importance of specific concepts, and to illustrate the tradeoffs between alternative techniques. In addition, detailed derivations are included. The intended audience for the book includes engineers, students, researchers, scientists, and project managers who may be interested in either designing or using an aided navigation system in a given application.

## Outline

The text presents a systematic method for designing and analyzing aided navigation systems along with the essential theory to support that methodology. The book is divided into two parts.

Part I contains seven chapters. Chapter 1 motivates the design and analysis methodology. This chapter introduces simple examples and comparisons to illustrate the method and to stimulate interest in the theoretical information presented in Chapters 2–6. Chapter 2 defines frames-of-reference, transformations between frames-of-reference, and methods for maintaining the transformations between rotating frames-of-reference. Chapters 3 and 4 focus on model development. An accurate mathematical model is a necessary precondition to being able to make accurate quantitative statements about navigation system performance during the design process. Chapter 3 presents various concepts from (deterministic) systems theory. Chapter 4 introduces certain necessary concepts from the theory of stochastic processes. Chapter 5 introduces and derives optimal state estimation methods (i.e., Kalman filtering). Knowledge of optimal and sub-optimal state estimation are essential to the implementation and performance analysis of navigation systems. Chapter 6 discusses methods for performance analysis. Chapter 7 returns to the methodology suggested in Chapter 1; however, now with the tools provided in Chapters 2–6 the same examples can be rigorously and quantitatively analyzed. Examples are used extensively throughout Chapters 1–7.

Part II of the text provides in-depth discussion of several specific navigation applications. Chapter 8 provides a self-contained description of the basic GPS solution, differential GPS, Doppler processing, and various carrier phase processing techniques. This presentation of the various GPS techniques using unified notation greatly facilitates the understanding of the techniques and their relative tradeoffs. For clarity of the presentation and comparison of methods within Chapter 8, all techniques presented in that chapter consider only point-wise data processing (i.e., no Kalman fil-



tering). GPS is discussed herein as a prototype of the various GNSS which are now or soon will be available. Chapters 9–12 each focus on a specific aided navigation application. Each chapter derives the kinematic model and presents the navigation mechanization equations based on the kinematic model; derives and analyzes the dynamic model for the navigation error state; and, presents equations for predicting the aiding measurements and for modeling the residual aiding measurements. Different forms of performance analysis are included in each of the chapters including covariance analysis, observability analysis, and data analysis from application or simulation. Chapter 9 discusses aided encoder-based dead-reckoning. Chapter 10 discusses an attitude and heading reference system that uses gyros as inputs to the kinematic model and accelerometers as gravity sensors for error correction. Chapter 11 discusses aided inertial navigation. Chapter 12 discusses a specific application of an aided inertial system. Together these chapters provide examples of methods that the reader can modify to fit their particular application needs.

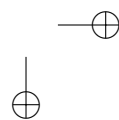
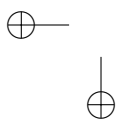
Four appendices are included. Appendix A defines the notation and constants used in the text. Appendix B reviews various linear algebra concepts that are used in the main body of the text. Appendix C presents material from the GPS interface specification that is necessary to process GPS pseudorange, phase, and Doppler measurements. Example calculations are also included. Appendix D presents a short tutorial on quaternions. This appendix discusses their definition, operations, and kinematic model. Example calculations are included in each appendix.

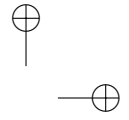
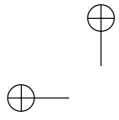
## Motivation

This book is motivated in part by the reader response to my prior book [48]. That response requested two major changes. First, that the material be restructured to support the book's use in a classroom setting. Second, that the subject matter be widened to the broader topic of aided navigation. Therefore, while the present book draws a significant amount of its source material from [48], the objectives of this book are distinct enough and the amount of new material is significant enough to necessitate the new title.

The present book discusses the general topic of aided navigation system design. Chapters 1 and 7 present the aided navigation system design methodology. Chapters 2–6 present the theoretical material required to understand and implement that method. Chapter 8 provides a detailed discussion of GPS. Chapters 9–12 provide detailed discussion of the aided navigation system design and analysis methodology.

This book is appropriate for use as a textbook in a senior or first year graduate level course. As such, the book includes extensive use of examples and end-of-chapter problems throughout Part I. As a textbook, the





lectures could cover Part I, with a student end-of-course project to design and implement an aided navigation system. The systems in Chapters 9, 10, and 11 are particularly appropriate for senior design projects.

Many engineering programs require senior-level students to complete a substantial design project. A challenge for faculty involved in directing such projects is leading the student team through the quantitative definition of the specification and analysis of system performance relative to the specifications. A main reason that the aided navigation system design and analysis methodology presented herein is widely used in commercial and military applications is that it is easily amenable to such quantitative analysis of performance relative to specifications. Performance analysis is discussed in Chapter 6.

## Book Website

Associated with this book is the publisher hosted website. The website contains various resources related to this book:

- The source code used to create examples using MATLAB;
- Data sets to support examples;
- Data sets that are requested by readers and that are reasonable for the author to provide;
- An errata list;
- Clarifications as requested by readers.

The official web site for this text can be found at

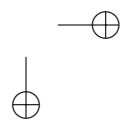
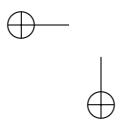
[www.mhprofessional.com](http://www.mhprofessional.com)

then searching by isbn, title, or author. The book web site that results will contain a download section containing the above material.

## Errata

While I have worked to ensure that this book is free from errors, previous experience has taught me that this is unlikely to occur. An errata list will be made available through the publisher website and through my university homepage, currently

[www.ee.ucr.edu/~farrell](http://www.ee.ucr.edu/~farrell)



Readers who detect errors not already on the errata list are encouraged to report them electronically to the author. My e-mail address is currently

farrell@ee.ucr.edu

Identification of errors, or suggestions of either additional material (e.g., examples, exercises, or topics), is greatly appreciated.

## Acknowledgements

First, I appreciate the many readers of [48] who communicated to me interesting ideas, corrections, and useful suggestions. Those communications helped to motivate and define this book. Second, I am grateful to acknowledge the help and collaboration of my students and colleagues in the development and proofreading of this book. The first set of reviewers included Anning Chen, Licheng Luo, Angello Pozo, and Arvind Ramanandan. The second set of reviewers included Jinrong Cheng, Wenjie Dong, Yu Lu, Paul Miller, Rolf Rysdyk, and Peng Xie. In addition, Wenjie Dong collaborated in the writing of Section 4.9.1; Arvind Ramanandan collaborated in the writing of Section 4.9.2; Yunchun Yang provided information essential to the writing of Chapter 10; Yu Lu collaborated in the writing of Section C.4 and provided source material helpful in the writing of Chapter 9; and, collaboration with Paul Miller was essential to the writing of Chapter 12. Of course, I take responsibility for any errors or omissions in the final presentation of all the material contained in the book. Third, I appreciate the help and support of the production team, mainly Wendy Rinaldi and Jean Bodeaux. Finally, I appreciate the patience of my family throughout this project.

This book was typeset using L<sup>A</sup>T<sub>E</sub>X. The examples and graphs for figures were implemented in MATLAB and SIMULINK. Drawings for figures were created using Macromedia FreeHand MX.

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