Quantum uncollapsing: theory and experiment

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Support:

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Quantum eraser

Quantum eraser proposal by Scully and Drühl (PRA, 1982)

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Here only virtual information is erased. Can we really measure (extract information) and then uncollapse quantum state?
The problem

It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo weak (partial) quantum measurement?  
Yes! (but with a finite probability)

If uncollapsing is successful, an unknown state is fully restored

Information is erased by another measurement with “exactly contradicting” result
Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to uncollapse? One more measurement!

(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, PRL-2006)
First example: double-dot qubit with no tunneling, measured by QPC

\[ \hat{H}_{QB} = \left( \epsilon / 2 \right) (c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1) \]

Assume “frozen” qubit: \( \epsilon = H = 0 \)

Bayesian evolution due to measurement (Korotkov-1998)

1) Diagonal matrix elements of the density matrix evolve according to the classical Bayes rule

2) Non-diagonal matrix elements evolve so that the degree of purity \( \rho_{ij}/[\rho_{ii}\rho_{jj}]^{1/2} \) is conserved

\[
\rho_{11}(\tau) = \frac{\rho_{11}(0) \exp\left[-(I - I_1)^2 / 2D\right]}{\rho_{11}(0) \exp\left[-(I - I_1)^2 / 2D\right] + \rho_{22}(0) \exp\left[-(I - I_2)^2 / 2D\right]} \\
\rho_{12}(\tau) / [\rho_{12}(\tau) \rho_{22}(\tau)]^{1/2} = \frac{\rho_{12}(0)}{[\rho_{12}(0) \rho_{22}(0)]^{1/2}} \quad , \quad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)
\]

where \( I = \frac{1}{\tau} \int_0^\tau I(t) \, dt \), \( D = S_I / 2\tau \), \( \Delta I = I_1 - I_2 \) - response

\( S_I = 2eI(1 - T) \) - shot noise
Uncollapsing for DQD-QPC system

Measurement result:
\[ r(t) = \frac{\Delta I}{S_I} \int_0^t I(t') \, dt' - I_0 t \]

If \( r = 0 \), then no information and no evolution!

Simple strategy: continue measuring until result \( r(t) \) becomes zero. Then any initial state is fully restored.
(same for an entangled qubit)

It may happen though that \( r = 0 \) never crossed; then undoing procedure is unsuccessful.

Probability of success:
\[ P_s = \frac{e^{-|r_0|}}{e^{r_0} \rho_{11}(0) + e^{-r_0} \rho_{22}(0)} \]

Averaged probability of success (over result \( r_0 \)):
\[ P_{av} = 1 - \text{erf}[\sqrt{t / 2T_m}] \]
\[ T_m = 2S_I / (\Delta I)^2 \]

(does not depend on initial state)
Second example: uncollapsing of a superconducting phase qubit

1) Start with an unknown state
2) Partial measurement of strength $p$
3) $\pi$-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the same strength $p$
5) $\pi$-pulse

This is what was demonstrated experimentally (in more detail later)
General theory of uncollapsing

Measurement operator $M_r$: $\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$ (POVM formalism)

Undoing measurement operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}, \; p_i \text{ are eigenvalues of } M_r^\dagger M_r$$

Probability of success: $P_S \leq \frac{\min P_r}{P_r(\rho_{in})}$

$P_r(\rho_{in})$ – probability of result $r$ for initial state $\rho_{in}$,

$\min P_r$ – probability of result $r$ minimized over all possible initial states

Averaged (over $r$) probability of success: $P_{av} \leq \sum_r \min P_r$

(independent of initial state, otherwise get information)

(similar to Koashi-Ueda, PRL, 1999)
Third example: evolving charge qubit

\[
\hat{H}_{QB} = (\epsilon / 2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)
\]

(now non-zero \(H\) and \(\epsilon\), qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

Fourth example: general uncollapsing for entangled charge qubits

1) unitary transformation of \(N\) qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state \(|11..1\rangle\))
3) repeat \(2^N\) times, sequentially transforming the basis vectors of the diagonalized measurement operator into \(|11..1\rangle\)

In all four examples the success probability \(P_S\) reaches the upper bound (optimal uncollapsing)
Partial collapse of superconducting phase qubit

How does a coherent state evolve in time before tunneling event?
(What happens when nothing happens?)

Main idea:

\[ \psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi(t) = \begin{cases} |\text{out}\rangle, \text{if tunneled} \\ \frac{\alpha |0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi} |1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2} e^{-\Gamma t}}, \text{if not tunneled} \end{cases} \]

amplitude of state $|0\rangle$ grows without physical interaction

continuous null-result collapse
(similar to optics, Dalibard-Castin-Molmer, PRL-1992)


(better theory: Pryadko & A.K., 2007)
Superconducting phase qubit at UCSB

Courtesy of Nadav Katz (UCSB)

\[ |0\rangle \quad \text{Flux bias} \quad I_{dc} + I_z \]

\[ |1\rangle \quad \text{Qubit} \quad I_{\mu W} \]

\[ V_S \quad \text{SQUID} \quad I_s \]

Repeat 1000x
prob. 0,1

Courtesy of Nadav Katz (UCSB)
Experimental technique for partial collapse

Nadav Katz et al. (John Martinis’ group)

Protocol:
1) State preparation by applying microwave pulse (via Rabi oscillations)
2) Partial measurement by lowering barrier for time $t$
3) State tomography (microwave + full measurement)

Measurement strength $p = 1 - \exp(-\Gamma t)$ is actually controlled by $\Gamma$, not by $t$

$p=0$: no measurement
$p=1$: orthodox collapse
Experimental tomography data

\[ \psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \]

\[ \theta_x \]
\[ \theta_y \]
\[ \pi \]
\[ \pi/2 \]

\[ p = 0 \]
\[ p = 0.06 \]
\[ p = 0.14 \]
\[ p = 0.23 \]
\[ p = 0.32 \]
\[ p = 0.43 \]
\[ p = 0.56 \]
\[ p = 0.70 \]
\[ p = 0.83 \]

Nadav Katz et al. (UCSB)

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Partial collapse: experimental results

- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)

\[ \eta_0 > 0.8 \]
Uncollapsing of a phase qubit state

1) Start with an unknown state
2) Partial measurement of strength $p$
3) $\pi$-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the same strength $p$
5) $\pi$-pulse

If no tunneling for both measurements, then initial state is fully restored!

\[
\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}}
\]

\[
e^{i\phi} \frac{\alpha e^{-\Gamma t/2} |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} = e^{i\phi} (\alpha |0\rangle + \beta |1\rangle)
\]

Phase is also restored (spin echo)
Experiment on wavefunction uncollapsing

Uncollapse protocol:
- partial collapse
- $\pi$-pulse
- partial collapse (same strength)

State tomography with $X$, $Y$, and $Z$ pulses

Background $P_B$ should be subtracted to find qubit density matrix
Experimental results on Bloch sphere

N. Katz et al.

| Initial state | $|1\rangle$ | $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ | $\frac{|0\rangle+i|1\rangle}{\sqrt{2}}$ | $|0\rangle$ |
|---------------|-------------|-----------------------------------|-----------------------------------|------------|
| Partial collapse | ![Partial collapse image] | ![Partial collapse image] | ![Partial collapse image] | ![Partial collapse image] |
| Uncollapsed | ![Uncollapsed image] | ![Uncollapsed image] | ![Uncollapsed image] | ![Uncollapsed image] |

Collapse strength: $0.05 < p < 0.7$

uncollapsing works well!

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Same with polar angle dependence (another experimental run)

Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution
Quantum process tomography

Why getting worse at $p>0.6$?
Energy relaxation $p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1$
Selection affected when $1-p \sim p_r$

Overall: uncollapsing is well-confirmed experimentally
Conclusions

- Partial (weak, etc.) quantum measurement can be undone, though with some probability $P_S$, which decreases with increasing strength of measurement ($P_S=0$ for orthodox case)

- Arbitrary initial state is uncollapsed exactly in the case of success (need a detector with perfect quantum efficiency)

- Uncollapsing for a superconducting phase qubit has been demonstrated, extending the previous experiment on partial collapse

- Solid-state experiments on non-projective quantum measurement are now competitive with (sometimes ahead of) optical experiments (also, recent expt. on persistent Rabi oscillations)