Non-projective measurement of solid-state qubits: collapse and uncollapse (what is “inside” collapse)

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Outline:
• Long introduction (collapse, solid-state qubits)
• Bayesian formalism for quantum measurement
• Some experimental predictions
• Recent experiments on partial collapse and uncollapse

Ackn.:
R. Ruskov, A. Jordan (theory)
N. Katz, J. Martinis, P. Bertet (expt.)

Funding:
Niels Bohr:
“If you are not confused by quantum physics then you haven’t really understood it”

Richard Feynman:
“I think I can safely say that nobody understands quantum mechanics”
Quantum mechanics =
Schrödinger equation + collapse postulate

1) Probability of measurement result \( p_r = \left| \langle \psi | \psi_r \rangle \right|^2 \)

2) Wavefunction after measurement = \( \psi_r \)

- State collapse follows from common sense
- Does not follow from Schr. Eq. (contradicts; Schr. cat, random vs. deterministic)

What is “inside” collapse?
(What if measurement is continuous, as typical for solid-state experiments?)
Einstein-Podolsky-Rosen (EPR) paradox

Phys. Rev., 1935

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system.

\[ \psi(x_1, x_2) = \sum_n \psi_n(x_2) u_n(x_1) \] (nowadays we call it entangled state)

\[ \psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar}(x_1 - x_2)p\right) dp \sim \delta(x_1 - x_2) \]

Measurement of particle 1 cannot affect particle 2, while QM says it affects (contradicts causality)

=> Quantum mechanics is incomplete

Bohr’s reply (Phys. Rev., 1935) (seven pages, one formula: \( \Delta p \Delta q \sim h \))

It is shown that a certain “criterion of physical reality” formulated … by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena.

Crudely: No need to understand QM, just use the result
Bell’s inequality (John Bell, 1964)

\[ \psi = \frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2) \]

Perfect anticorrelation of results for same meas. directions, \( \vec{a} = \vec{b} \)

Is it possible to explain the QM result assuming local realism and hidden variables (without superluminal collapse)? \textbf{No!!!}

\[ A(\vec{a}, \lambda) = \pm 1, \quad B(\vec{b}, \lambda) = \pm 1 \] (deterministic result with hidden variable \( \lambda \))

Then: \[ |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \leq 1 + P(\vec{b}, \vec{c}) \]

where \( P \equiv P(++) + P(--) - P(+-) - P(-+) \)

QM: \[ P(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} \] For 0°, 90°, and 45°: \( 0.71 \neq 1 - 0.71 \) \textit{violation!}

\textbf{Experiment} (Aspect et al., 1982; photons instead of spins, CHSH):
yes, “spooky action-at-a-distance”
What about causality?

Actually, not too bad: you cannot transmit your own information choosing a particular measurement direction $a$

Result of the other measurement does not depend on direction $a$

Randomness saves causality

Collapse is still instantaneous: OK, just our recipe, not an “objective reality”, not a “physical” process

Consequence of causality: **No-cloning theorem**

Wootters-Zurek, 1982; Dieks, 1982; Yurke

You cannot copy an unknown quantum state

Proof: Otherwise get information on direction $a$ (and causality violated)

Application: quantum cryptography

Information is an important concept in quantum mechanics
Quantum measurement in solid-state systems

No violation of locality – too small distances

However, interesting issue of continuous measurement (weak coupling, noise ⇒ gradual collapse)

Starting point:

What happens to a solid-state qubit (two-level system) during its continuous measurement by a detector?
Superconducting “charge” qubit


Vion et al. (Devoret’s group); Science, 2002
Q-factor of coherent (Rabi) oscillations = 25,000

\[ \hat{H} = \frac{(2e)^2}{2C} (\hat{n} - n_g)^2 - \frac{E_J}{2} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \]

Quantum coherent (Rabi) oscillations

n: number of Cooper pairs on the island

2e \downarrow \quad “island”

V. V. Klimov, Sergey Korotkov, University of California, Riverside

Alexander Korotkov
More of superconducting charge qubits

Cooper-pair box measured by single-electron transistor (rf-SET)

Setup can be used for continuous measurements

All results are averaged over many measurements (not “single-shot”)

Duty, Gunnarsson, Bladh, Delsing, PRB 2004

Guillaume et al. (Echternach’s group), PRB 2004
Some other superconducting qubits

**Flux qubit**
Mooij et al. (Delft)

**Phase qubit**
J. Martinis et al. (UCSB and NIST)

**Charge qubit with circuit QED**
R. Schoelkopf et al. (Yale)
Some other superconducting qubits

**Flux qubit**

J. Clarke et al. (Berkeley)

**“Quantronium” qubit**

I. Siddiqi, R. Schoelkopf, M. Devoret, et al. (Yale)
Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003

Detector is not separated from qubit, also possible to use a separate detector

Rabi oscillations
Some other semiconductor qubits

**Spin qubit**
C. Marcus et al. (Harvard)

**Spin qubit**
L. Kouwenhoven et al. (Delft)

**Double-dot qubit**
Gorman, Hasko, Williams (Cambridge)
“Which-path detector” experiment


Theory: Aleiner, Wingreen, and Meir, PRL 1997

Dephasing rate:

$$\Gamma = \frac{e V (\Delta T)^2}{h T (1 - T)} = \frac{(\Delta I)^2}{4 S_I}$$

$$\Delta I$$ – detector response, $$S_I$$ – shot noise

The larger noise, the smaller dephasing!!!

$$(\Delta I)^2 / 4 S_I \sim \text{rate of “information flow”}$$
The system we consider: qubit + detector

\[ H = H_{QB} + H_{DET} + H_{INT} \]

\[ H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1) \]

\[ \Omega = (4H^2 + \varepsilon^2)^{1/2}/\hbar \]

Two levels of average detector current: \( I_1 \) for qubit state \(|1\rangle\), \( I_2 \) for \(|2\rangle\)

Response: \( \Delta I = I_1 - I_2 \)

Detector noise: white, spectral density \( S_I \)

**DQD and QPC** (setup due to Gurvitz, 1997)

\[ H_{DET} = \sum_l E_l a_l^+a_l + \sum_r E_r a_r^+a_r + \sum_{l,r} T(a_r^+a_l + a_l^+a_r) \]

\[ H_{INT} = \sum_{l,r} \Delta T (c_1^+c_1 - c_2^+c_2)(a_r^+a_l + a_l^+a_r) \]

\[ S_I = 2eI \]
What happens to a qubit state during measurement?

Start with density matrix evolution due to measurement only \((H=\varepsilon=0)\)

- **“Orthodox” answer**
  \[
  \begin{pmatrix}
  1 & 1 \\
  2 & 2 \\
  1 & 1 \\
  2 & 2 \\
  \end{pmatrix} \rightarrow
  \begin{pmatrix}
  1 & 0 \\
  0 & 0 \\
  0 & 0 \\
  1 & 1 \\
  \end{pmatrix}
  \]

- **“Conventional” (decoherence) answer**
  \[
  \begin{pmatrix}
  1 & 1 \\
  2 & 2 \\
  1 & 1 \\
  2 & 2 \\
  \end{pmatrix} \rightarrow
  \begin{pmatrix}
  \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\
  \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \\
  \end{pmatrix} \rightarrow
  \begin{pmatrix}
  \frac{1}{2} & 0 \\
  0 & \frac{1}{2} \\
  \end{pmatrix}
  \]

- |1> or |2>, depending on the result
- no measurement result! (ensemble averaged)

Orthodox and decoherence answers contradict each other!

<table>
<thead>
<tr>
<th>applicable for:</th>
<th>single quant. system</th>
<th>continuous meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Orthodox</strong></td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td><strong>Decoherence (ensemble)</strong></td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Bayesian, POVM, quant. traject., etc.</strong></td>
<td>yes</td>
<td>yes</td>
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Bayesian (POVM, etc.) formalism describes gradual collapse of a single quantum system, **taking into account noisy detector output** \(I(t)\).
Bayesian formalism for DQD-QPC system

Qubit evolution due to measurement (quantum back-action):

\[ |\psi(t)\rangle = \alpha(t) |1\rangle + \beta(t) |2\rangle \quad \text{or} \quad \rho_{ij}(t) \]

1) \(|\alpha(t)|^2\) and \(|\beta(t)|^2\) evolve as probabilities, i.e. according to the Bayes rule (same for \(\rho_{ii}\))

2) phases of \(\alpha(t)\) and \(\beta(t)\) do not change (no decoherence!), \(\rho_{ij}/(\rho_{ii}\rho_{jj})^{1/2} = \text{const}\)

Bayes rule (1763, Laplace-1812):

\[ P(A_i | \text{res}) = \frac{P(A_i) P(\text{res} | A_i)}{\sum_k P(A_k) P(\text{res} | A_k)} \]

So simple because:

1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)
Bayesian formalism for a single qubit

\[ \hat{H}_{QB} = \frac{\varepsilon}{2} (c_1^\dagger c_2 - c_2^\dagger c_1) + H(c_1^\dagger c_2 + c_2^\dagger c_1) \]

\[ |1\rangle \rightarrow I_1, \quad |2\rangle \rightarrow I_2, \quad \Delta I = I_1 - I_2, \quad I_0 = (I_1 + I_2)/2 \]

\[ S_I - \text{detector noise} \]

\[ \dot{\rho}_{11} = - \dot{\rho}_{22} = -2 (H / \hbar) \text{Im} \rho_{12} + \rho_{11} \rho_{22} (2 \Delta I / S_I) [I(t) - I_0] \]

\[ \dot{\rho}_{12} = i (\varepsilon / \hbar) \rho_{12} + i (H / \hbar) (\rho_{11} - \rho_{22}) + \rho_{12} (\rho_{11} - \rho_{22}) (\Delta I / S_I) [I(t) - I_0] - \gamma \rho_{12} \]

\( \gamma = \Gamma - (\Delta I)^2 / 4 S_I, \quad \Gamma - \text{ensemble decoherence} \)

\( \eta = 1 - \gamma / \Gamma = (\Delta I)^2 / 4 S_I \Gamma \quad - \text{detector ideality (efficiency)}, \eta \leq 100\% \)

Ideal detector (\( \eta = 1 \), as QPC) does not decohere a qubit, then random evolution of qubit wavefunction can be monitored

Averaging over result \( I(t) \) leads to conventional master equation:

\[ d\rho_{11}/dt = -d\rho_{22}/dt = -2 (H / \hbar) \text{Im} \rho_{12} \]

\[ d\rho_{12}/dt = i (\varepsilon / \hbar) \rho_{12} + i (H / \hbar) (\rho_{11} - \rho_{22}) - \Gamma \rho_{12} \]

Ensemble averaging includes averaging over measurement result!
Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved
  \[ eV \gg \hbar\Omega, \quad eV \gg \hbar\Gamma, \quad \hbar/eV \ll (1/\Omega, 1/\Gamma) \]
  (no coherence in the detector, classical output, Markovian approximation)

- Simpler if weak response, \(|\Delta l| \ll l_0\), (coupling \(C \sim \Gamma/\Omega \) is arbitrary)

Derivations:

1) “logical”: via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) “microscopic”: Schr. eq. + collapse of the detector (A.K., 2000)

3) from “quantum trajectory” formalism developed for quantum optics
   (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

5) from Keldysh formalism (Wei-Nazarov, 2007)
Fundamental limit for ensemble decoherence

\[ \Gamma = (\Delta I)^2/4S_I + \gamma \]

ensemble decoherence rate  
single-qubit decoherence  
\sim rate of information acquisition [bit/s]

\[ \gamma \geq 0 \Rightarrow \Gamma \geq (\Delta I)^2/4S_I \]

detector ideality (quantum efficiency)
\[ \eta \leq 100\% \]

Translated into energy sensitivity:
\[ (\mathcal{E}_O \mathcal{E}_{BA})^{1/2} \geq \hbar/2 \]

where \( \mathcal{E}_O \) is output-noise-limited sensitivity [J/Hz]  
and \( \mathcal{E}_{BA} \) is back-action-limited sensitivity [J/Hz]

Sensitivity limitation is known since 1980s (Clarke, Tesche, Likharev, etc.);  
Measurement vs. decoherence

Widely accepted point of view:

\[ \text{measurement} = \text{decoherence (environment)} \]

Is it true?

- **Yes**, if not interested in information from detector (ensemble-averaged evolution)
- **No**, if take into account measurement result (single quantum system)
Experimental predictions and proposals from Bayesian formalism

- Direct experimental verification (1998)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Simple quantum feedback of a qubit (2004)
- Violation of Leggett-Garg inequality (2005)
- Partial collapse of a phase qubit (2005)
- Undoing of a weak measurement (2006)
Persistent Rabi oscillations

- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously

Phase of Rabi oscillations fluctuates (dephasing)

Direct experiment is difficult (good quantum efficiency, bandwidth, control)

A.K., 1998
Measured spectrum of Rabi oscillations

What is the spectral density $S_I(\omega)$ of detector current?

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

Assume classical output, $eV \gg \hbar \Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1} (\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio $\leq 4\eta \leq 4$

(result can be obtained using various methods, not only Bayesian method)

Expt. confirmed (Saclay)

A.K., LT’99
A.K.-Averin, 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, PRL-2004
Bell-type (Leggett-Garg-type) inequalities for continuous measurement of a qubit

**Assumptions of macrorealism (similar to Leggett-Garg'85):**

\[ I(t) = I_0 + \left( \frac{\Delta I}{2} \right) Q(t) + \xi(t) \]

\[ |Q(t)| \leq 1, \quad \langle \xi(t) Q(t + \tau) \rangle = 0 \]

Then for correlation function

\[ K(\tau) = \langle I(t) I(t + \tau) \rangle \]

\[ K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \leq \left( \frac{\Delta I}{2} \right)^2 \]

and for area under spectral peak

\[ \int [S_I(f) - S_0] df \leq \left( \frac{8}{\pi^2} \right) \left( \frac{\Delta I}{2} \right)^2 \]

Quantum result: 

\[ \frac{3}{2} \left( \frac{\Delta I}{2} \right)^2 \times \frac{3}{2} \]

Violation: 

\[ \frac{\pi^2}{8} \]

Experimentally measurable violation of classical bound
Recent experiment (Saclay group, unpub.)

- superconducting charge qubit (transmon) in circuit QED setup
- driven Rabi oscillations
  - perfect spectral peaks
  - LGI violation

courtesy of Patrice Bertet

A. Palacios-Laloy et al. (unpublished)
Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!

Goal: persistent Rabi oscillations with perfect phase

Idea: monitor the Rabi phase $\phi$ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F_\times \Delta \phi$

To monitor phase $\phi$ we plug detector output $I(t)$ into Bayesian equations

Ruskov & A.K., 2001
Performance of Bayesian feedback

Feedback fidelity vs. feedback strength

For ideal detector and wide bandwidth, feedback fidelity can be close to 100%

\[ D = \exp(-C/32F) \]

\[ C = \hbar(\Delta I)^2 / S_I H \] - coupling

\[ F \] - feedback strength

\[ D = 2\langle \text{Tr}\rho_{\text{desired}}\rho \rangle - 1 \]

\[ C_{\text{env}} / C_{\text{det}} = 0, 0.1, 0.5 \]

\[ C = C_{\text{det}} = 1 \]

\[ \tau_a = 0 \]

Feedback fidelity vs. detector efficiency

\[ \eta \ll 1 \Rightarrow D_{\text{max}} \approx 1.25\sqrt{\eta} \]

\[ \eta \approx 1 \Rightarrow D_{\text{max}} \approx (1 + \eta) / 2 \]

Zhang, Ruskov, A.K., 2005

Experimental difficulties:

- need real-time solution of Bayesian eqs.
- wide bandwidth (\( \gg \Omega \)) of the output \( I(t) \)
Simple quantum feedback of a solid-state qubit

(A.K., 2005)

\[ H = H_0 [1 - F \times \phi_m(t)] \]

Goal: maintain coherent (Rabi) oscillations for arbitrarily long time

Idea: use two quadrature components of the detector current \( I(t) \)

to monitor approximately the phase of qubit oscillations
(a very natural way for usual classical feedback!)

\[
X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t - t')/\tau] \, dt'
\]
\[
Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t - t')/\tau] \, dt'
\]

\[ \phi_m = -\arctan \left( \frac{Y}{X} \right) \]

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: simplicity and relatively narrow bandwidth \( 1/\tau \sim \Gamma_d \ll \Omega \)

Essentially classical feedback. Does it really work?
Fidelity of simple quantum feedback

\[ D_{\text{max}} \approx 90\% \]
\[ D \equiv 2F_Q - 1 \]
\[ F_Q \equiv \langle \text{Tr} \rho(t) \rho_{\text{des}}(t) \rangle \]

Robust to imperfections (inefficient detector, frequency mismatch, qubit asymmetry)

How to verify feedback operation experimentally?
Simple: just check that in-phase quadrature \( \langle X \rangle \) of the detector current is positive \( D = \langle X \rangle (4/\tau \Delta I) \)
\[ \langle X \rangle = 0 \] for any non-feedback Hamiltonian control of the qubit

Simple enough for real experiment!
Quantum feedback in optics


Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

J.M. Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.

First detailed theory:
Quantum feedback in optics


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PRL 94, 203002 (2005) also withdrawn

First detailed theory:
H.M. Wiseman and G. J. Milburn,

Recent experiment:
Cook, Martin, Geremia,
Nature 446, 774 (2007)
(coherent state discrimination)
Two-qubit entanglement by measurement

Symmetric setup, no qubit interaction

Two evolution scenarios:

Collapse into |\text{Bell}\rangle state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

Ruskov & A.K., 2002

Peak/noise $= (32/3) \eta$
**Quadratic quantum detection**

Mao, Averin, Ruskov, & A.K., PRL-2004

Nonlinear detector:
- Spectral peaks at $\Omega$, $2\Omega$ and 0

Quadratic detector:
- Peak only at $2\Omega$, peak/noise = $4\eta$

Three evolution scenarios:
1) Collapsing into $|\uparrow\downarrow - \downarrow\uparrow\rangle$, current $I_{\uparrow\downarrow}$, flat spectrum
2) Collapsing into $|\uparrow\uparrow - \downarrow\downarrow\rangle$, current $I_{\uparrow\uparrow}$, flat spectrum; 3) Collapsing into remaining subspace, current $(I_{\uparrow\downarrow} + I_{\uparrow\uparrow})/2$, spectral peak at $2\Omega$

Entangled states distinguished by average detector current

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QND squeezing of a nanomechanical resonator

Ruskov, Schwab, & A.K., PRB-2005

Experimental status:
\[ \frac{\omega_0}{2\pi} \sim 1 \text{ GHz} \quad (\hbar \omega_0 \sim 80 \text{ mK}), \quad \text{Roukes’ group, 2003} \]
\[ \Delta x / \Delta x_0 \sim 5 \quad [\text{SQL} \Delta x_0=(\hbar/2m\omega_0)^{1/2}], \quad \text{Schwab’s group, 2004} \]

\[ S_{\text{max}} = \frac{3}{4} \left[ \frac{\sqrt{\eta} C_0 Q}{\coth (\hbar \omega_0 / 2T)} \right]^{1/3} \]

\( C_0 \) – coupling with detector, \( \eta \) – detector efficiency, 
\( T \) – temperature, \( Q \) – resonator Q-factor

Potential application: ultrasensitive force measurements

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Undoing a weak measurement of a qubit ("uncollapse")

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement? (To restore a "precious" qubit accidentally measured)

Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored
Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

Our idea of uncollapsing is quite different: we really extract quantum information and then erase it.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)
Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is non-unitary (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!

\[ |0\rangle \times |1\rangle = |0\rangle |0\rangle \]

need ideal (quantum-limited) detector

(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)
Evolution of a charge qubit

\[ H = 0 \]

\[
\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]
\]

\[
\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}
\]

where measurement result \( r(t) \) is

\[
r(t) = \frac{\Delta I}{S_i} \left[ \int_0^t I(t') \, dt' - I_0 t \right]
\]

If \( r = 0 \), then no information and no evolution!

Jordan-Korotkov-Büttiker, PRL-06
Uncollapsing for DQD-QPC system

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First “accidental” measurement

Undoing measurement

Simple strategy: continue measuring until result $r(t)$ becomes zero! Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never happens; then undoing procedure is unsuccessful.

Probability of success:

$$P_s = \frac{e^{-|r_0|}}{e^{|r_0|} \rho_{11}(0) + e^{-|r_0|} \rho_{22}(0)}$$
General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.85)

Measurement operator $M_r$:
$$\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$$

Probability: $P_r = \text{Tr}(M_r \rho M_r)$

Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator:
$$C \times M_r^{-1}$$
(to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}, \quad p_i \text{ – eigenvalues of } M_r^\dagger M_r$$

Probability of success:
$$P_S \leq \min \frac{P_r}{P_r(\rho_{\text{in}})}$$

$P_r(\rho_{\text{in}})$ – probability of result $r$ for initial state $\rho_{\text{in}}$,

$\min P_r$ – probability of result $r$ minimized over all possible initial states

Averaged (over $r$) probability of success:
$$P_{av} \leq \sum_r \min P_r$$

(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)
Partial collapse of a “phase” qubit

How does a coherent state evolve in time before tunneling event?
(What happens when nothing happens?)

Qubit “ages” in contrast to a radioactive atom!

Main idea:

\[ \psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi(t) = \begin{cases} |\text{out}\rangle, & \text{if tunneled} \\ \frac{\alpha |0\rangle + \beta e^{-\Gamma t/2} e^{i\phi} |1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, & \text{if not tunneled} \end{cases} \]

amplitude of state $|0\rangle$ grows without physical interaction

continuous null-result collapse

(better theory: Pryadko & A.K., 2007)

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)
Superconducting phase qubit at UCSB

Courtesy of Nadav Katz (UCSB)

Repeat 1000x
prob. 0,1
Experimental technique for partial collapse

Nadav Katz et al.
(John Martinis’ group)

Protocol:
1) State preparation by applying microwave pulse (via Rabi oscillations)
2) Partial measurement by lowering barrier for time $t$
3) State tomography (microwave + full measurement)

Measurement strength $p = 1 - \exp(-\Gamma t)$ is actually controlled by $\Gamma$, not by $t$

$p=0$: no measurement
$p=1$: orthodox collapse
Experimental tomography data

\[ \psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \]

\[ \theta_y \]

\[ \theta_x \]

\[ p = 0 \]

\[ p = 0.06 \]

\[ p = 0.14 \]

\[ p = 0.23 \]

\[ p = 0.32 \]

\[ p = 0.43 \]

\[ p = 0.56 \]

\[ p = 0.70 \]

\[ p = 0.83 \]

Nadav Katz et al. (UCSB)
Partial collapse: experimental results

N. Katz et al., Science-06

- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)

In (c) $T_1=110$ ns, $T_2=80$ ns (measured)

No fitting parameters in (a) and (b)
Uncollapsing of a phase qubit state

1) Start with an unknown state
2) Partial measurement of strength $\rho$
3) $\pi$-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the same strength $\rho$
5) $\pi$-pulse

If no tunneling for both measurements, then initial state is fully restored!

\[
\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} \rightarrow \frac{e^{i\phi} \alpha e^{-\Gamma t/2} |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} = e^{i\phi} (\alpha |0\rangle + \beta |1\rangle)
\]

phase is also restored (spin echo)
Experiment on wavefunction uncollapsing


Uncollapse protocol:
- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with X, Y, and no pulses

Background $P_B$ should be subtracted to find qubit density matrix

\[ \psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \]
Experimental results on Bloch sphere

<table>
<thead>
<tr>
<th>Initial state</th>
<th>1⟩</th>
<th>(0⟩+ 1⟩)/\sqrt{2}</th>
<th>(0⟩+ i 1⟩)/\sqrt{2}</th>
<th>0⟩</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial collapse</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncollapsed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Collapse strength: \(0.05 < p < 0.7\)

uncollapsing works well!
Same with polar angle dependence (another experimental run)

Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution
Quantum process tomography

Energy relaxation \( p_r = \frac{t}{T_1} = \frac{45\text{ns}}{450\text{ns}} = 0.1 \)

Selection affected when \( 1-p \sim p_r \)

Overall: uncollapsing is well-confirmed experimentally
Recent experiment on uncollapsing using single photons

Kim, Cho, Ra, Kim, arXiv:0903.3077

- very good fidelity of uncollapsing (>94%)
- measurement fidelity is probably not good (normalization by coincidence counts)
Conclusions

- Continuous quantum measurement is *not* equivalent to decoherence (environment) if detector output (information) is taken into account.

- It is easy to see what is “inside” collapse: simple Bayesian formalism works for many solid-state setups.

- Collapse can sometimes be undone (uncollapsing).

- A number of experimental predictions have been made.

- Three direct solid-state experiments have been realized; hopefully, more experiments are coming soon.