Non-projective measurement of solid-state qubits
(what is “inside” collapse)

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Outline:  
• Bayesian formalism for quantum measurement
• Persistent Rabi oscillations (+expt.)
• Wavefunction uncollapse (+expts.)
• New experimental proposals
  - decoherence suppression by uncollapsing
  - persistent Rabi oscillations revealed via noise

Ackn.:  
Theory: R. Ruskov, A. Jordan, K. Keane
Expt.: UCSB (J. Martinis, N. Katz et al.), Saclay (D. Esteve, P. Bertet et al.)

Funding:
Quantum mechanics =
Schrödinger equation + measurement postulate

1) Probability of measurement result \( r \):
   \[ p_r = \left| \langle \psi | \psi_r \rangle \right|^2 \]

where
   \[ \hat{A} | \psi_r \rangle = r | \psi_r \rangle \]

2) Wavefunction after measurement = \( | \psi_r \rangle \) (collapse)

Instantaneous collapse is surely an approximation (though often OK in optics, also the main point in Bell’s ineq.), especially obvious for solid-state systems

What is the evolution due to measurement?
(What is “inside” collapse?)
(controversial for last 80 years, many wrong answers, many correct answers)

Our limited scope:
(simplest system, experimental setups)
Superconducting “charge” qubit


\[ \hat{H} = \frac{(2e)^2}{2C} (\hat{n} - n_g)^2 - \frac{E_J}{2} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \]

Vion et al. (Saclay group); Science, 2002
Q-factor of coherent (Rabi) oscillations = 25,000
(“quantronium”)

Quantum coherent (Rabi) oscillations

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Charge qubits with SET readout

Cooper-pair box measured by single-electron transistor (rf-SET)

Setup can be used for continuous measurements

All results are averaged over many measurements (not “single-shot”)
Some other superconducting qubits

**Flux qubit**
Mooij et al. (Delft)

**Phase qubit**
J. Martinis et al. (UCSB and NIST)

**Charge qubit with circuit QED**
R. Schoelkopf et al. (Yale)
Some other superconducting qubits

**Flux qubit**

J. Clarke et al. (Berkeley)

![Flux qubit diagram](image)

![Scaled switching probability graph](image)

**“Quantronium” qubit**

I. Siddiqi, R. Schoelkopf, M. Devoret, et al. (Yale)

![Quantronium qubit circuit](image)

![Graph showing P_switch vs \( \tau \)](image)
Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003

Detector is not separated from qubit, also possible to use a separate detector

Rabi oscillations
Some other semiconductor qubits

**Spin qubit (QPC meas.)**
C. Marcus et al. (Harvard)

**Spin qubit**
L. Kouwenhoven et al. (Delft)

**Double-dot qubit**
Gorman, Hasko, Williams (Cambridge)
The system we consider: qubit + detector

**Qubit and Detector**

\[ H = H_{QB} + H_{DET} + H_{INT} \]

\[ H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1) \]

\[ \varepsilon \text{ – asymmetry, } H \text{ – tunneling} \]

\[ \Omega = (4H^2 + \varepsilon^2)^{1/2}/\hbar \text{ – frequency of quantum coherent (Rabi) oscillations} \]

Two levels of average detector current: \( I_1 \) for qubit state \(|1\rangle\), \( I_2 \) for \(|2\rangle\)

Response: \( \Delta I = I_1 - I_2 \)

**Detector Noise:** white, spectral density \( S_I \)

**DQD and QPC** (setup due to Gurvitz, 1997)

\[ H_{DET} = \sum_l E_la_l^+a_l + \sum_r E_ra_r^+a_r + \sum_{l,r} T(a_r^+a_l + a_l^+a_r) \]

\[ H_{INT} = \sum_{l,r} \Delta T (c_1^+c_1 - c_2^+c_2)(a_r^+a_l + a_l^+a_r) \]

\[ S_I = 2eI \]
What happens to a qubit state during measurement?

Start with density matrix evolution due to measurement only ($H = \epsilon = 0$)

"Orthodox" answer

\[
\begin{pmatrix}
1 & 1 \\
2 & 2 \\
1 & 1 \\
2 & 2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}
\]

"Decoherence" answer

\[
\begin{pmatrix}
1 & 1 \\
2 & 2 \\
1 & 1 \\
2 & 2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\frac{1}{2} & \exp(-\Gamma t) \\
\exp(-\Gamma t) & \frac{1}{2}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 \\
\frac{1}{2} & 0 \\
0 & \frac{1}{2}
\end{pmatrix}
\]

$|1\rangle$ or $|2\rangle$, depending on the result

no measurement result! (ensemble averaged)

Decoherence has nothing to do with collapse!

<table>
<thead>
<tr>
<th>applicable for:</th>
<th>single quant. system</th>
<th>continuous meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthodox</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Decoherence (ensemble)</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Bayesian, POVM, quant. traject., etc.</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Bayesian (POVM, quant. traj., etc.) formalism describes gradual collapse of a single quantum system, taking into account measurement result
Bayesian formalism for DQD-QPC system

Qubit evolution due to measurement (quantum back-action):

\[ |\psi(t)\rangle = \alpha(t) |1\rangle + \beta(t) |2\rangle \quad \text{or} \quad \rho_{ij}(t) \]

1) \(|\alpha(t)|^2\) and \(|\beta(t)|^2\) evolve as probabilities,
   i.e. according to the Bayes rule (same for \(\rho_{ii}\))

2) phases of \(\alpha(t)\) and \(\beta(t)\) do not change
   (no dephasing!), \(\rho_{ij}/(\rho_{ii}\rho_{jj})^{1/2} = \text{const} \)

Bayes rule (1763, Laplace-1812):

\[
P(A_i | \text{res}) = \frac{P(A_i) P(\text{res} | A_i)}{\sum_k P(A_k) P(\text{res} | A_k)}
\]

So simple because:
1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)
Bayesian formalism for a single qubit

- Time derivative of the quantum Bayes rule
- Add unitary evolution of the qubit
- Add decoherence (if any)

\[
\begin{align*}
\dot{\rho}_{11} &= -\dot{\rho}_{22} = -2\left(\frac{H}{\hbar}\right) \text{Im} \rho_{12} + \rho_{11}\rho_{22} (2\Delta I / S_I) [I(t) - I_0] \\
\dot{\rho}_{12} &= i(\varepsilon / \hbar)\rho_{12} + i(H / \hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I / S_I) [I(t) - I_0] - \gamma \rho_{12}
\end{align*}
\]

\[\hat{H}_{QB} = (\varepsilon / 2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)\]

|1⟩ → I₁,  |2⟩ → I₂,  \(\Delta I = I_1 - I_2\),  \(I_0 = (I_1 + I_2)/2\),  \(S_I\) – detector noise

\[
\gamma = \Gamma - (\Delta I)^2 / 4S_I, \quad \Gamma – \text{ensemble decoherence}
\]

Evolution of qubit wavefunction can be monitored if \(\gamma = 0\) (quantum-limited)

Averaging over result \(I(t)\) leads to conventional master equation:

\[
\begin{align*}
\frac{d\rho_{11}}{dt} &= -\frac{d\rho_{22}}{dt} = -2\left(\frac{H}{\hbar}\right) \text{Im} \rho_{12} \\
\frac{d\rho_{12}}{dt} &= i(\varepsilon / \hbar)\rho_{12} + i(H / \hbar)(\rho_{11} - \rho_{22}) - \Gamma \rho_{12}
\end{align*}
\]

Ensemble averaging includes averaging over measurement result!
Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved:
  \[ eV \gg \hbar\Omega, \quad eV \gg \hbar\Gamma, \quad \hbar/eV \ll (1/\Omega, 1/\Gamma) \]
  (no coherence in the detector, classical output, Markovian approximation)

- Simpler if weak response, \(|\Delta I| \ll I_0\), (coupling \(C \sim \Gamma/\Omega\) is arbitrary)

Derivations:

1) “logical”: via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) “microscopic”: Schr. eq. + collapse of the detector (A.K., 2000)

3) from “quantum trajectory” formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

5) from Keldysh formalism (Wei-Nazarov, 2007)
Why not just use Schrödinger equation for the whole system?

Impossible in principle!

Technical reason: Outgoing information (measurement result) makes it an open system

Philosophical reason: Random measurement result, but deterministic Schrödinger equation

Einstein: God does not play dice
Heisenberg: unavoidable quantum-classical boundary
Fundamental limit for ensemble decoherence

\[ \Gamma = (\Delta I)^2/4S_I + \gamma \]

ensemble decoherence rate

\[ \gamma \geq 0 \Rightarrow \Gamma \geq (\Delta I)^2/4S_I \]

single-qubit decoherence

\[ \sim \text{ information flow [bit/s]} \]

\[ \eta = 1 - \frac{\gamma}{\Gamma} = \frac{(\Delta I)^2 / 4S_I}{\Gamma} \]

detector ideality (quantum efficiency)

\[ \eta \leq 100\% \]

Transcribed into energy sensitivity: \((\varepsilon_O \varepsilon_{BA})^{1/2} \geq \hbar/2\)

where \(\varepsilon_O\) is output-noise-limited sensitivity [J/Hz]

and \(\varepsilon_{BA}\) is back-action-limited sensitivity [J/Hz]

Sensitivity limitation is known since 1980s (Caves, Clarke, Likharev, Zorin, Vorontsov, Khalili, etc.); also Zorin-1996, Averin-2000, Clerk et al.-2002, etc.

\[ (\varepsilon_O \varepsilon_{BA} - \varepsilon_{O,BA}^2)^{1/2} \geq \hbar/2 \iff \Gamma \geq (\Delta I)^2/4S_I + K^2S_I/4 \]

“measurement time” \((S/N=1)\)

\[ \tau_m = 2S_I / (\Delta I)^2 \]

(Shnirman & Schön, 1998)

\[ \Gamma \tau_m \geq \frac{1}{2} \]


S. Pilgram et al., 2002

A. Clerk et al., 2002

D. Averin, 2000,2003

\[ \eta \leq 100\% \]

Danilov, Likharev, Zorin, 1983

\[ \eta = \frac{\hbar^2 / 4}{\varepsilon_O \varepsilon_{BA}} = \eta_{opt} \]
POVM vs. Bayesian formalism

General quantum measurement (POVM formalism) (Nielsen-Chuang, p. 85,100):

Measurement (Kraus) operator $M_r$ (any linear operator in H.S.):

$$\psi \rightarrow \frac{M_r \psi}{||M_r \psi||} \quad \text{or} \quad \rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$$

Probability: $P_r = ||M_r \psi||^2$ or $P_r = \text{Tr}(M_r \rho M_r^\dagger)$

Completeness: $\sum_r M_r^\dagger M_r = 1$

- POVM is essentially a projective measurement in an extended Hilbert space
- Easy to derive: interaction with ancilla + projective measurement of ancilla
- For extra decoherence: incoherent sum over subsets of results

Relation between POVM and quantum Bayesian formalism:

So, mathematically, POVM and quantum Bayes are very close (Caves was possibly first to notice)

We emphasize not mathematical structures, but particular setups (goal: find a proper description) and experimental consequences
Experimental predictions and proposals from Bayesian formalism

- Direct experimental verification (1998)
- Bell-type correlation experiment (2000)
- Entanglement by measurement (2002)
- Violation of Leggett-Garg inequality (2005)
- Partial collapse of a phase qubit (2005)
- Undoing of a weak measurement (2006, 2008)
- Decoherence suppression by uncollapsing (2009)
Persistent Rabi oscillations

- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously
  (“reason”: attraction to two points on the Bloch sphere great circle)

Phase of Rabi oscillations fluctuates (dephasing)

Direct experiment is difficult (good quantum efficiency, bandwidth, control)

A.K., 1999
Indirect experiment: spectrum of persistent Rabi oscillations

\[ I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t) \]

(const + signal + noise)

amplifier noise ⇒ higher pedestal, poor quantum efficiency, but the peak is the same!!!

integral under the peak ⇔ variance \( \langle z^2 \rangle \)

How to distinguish experimentally persistent from non-persistent? Easy!

perfect Rabi oscillations: \( \langle z^2 \rangle = \langle \cos^2 \rangle = 1/2 \)

imperfect (non-persistent): \( \langle z^2 \rangle \ll 1/2 \)

quantum (Bayesian) result: \( \langle z^2 \rangle = 1 \) (!!!)

(demonstrated in Saclay expt.)

\[ S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2} \]

\[ \Omega = 2H \]

\[ C = (\Delta I)^2 / HS_I \]

peak-to-pedestal ratio = \( 4\eta \leq 4 \)
How to understand $\langle z^2 \rangle = 1$?

First way (mathematical)

We actually measure operator: $z \rightarrow \sigma_z$

$\ z^2 \rightarrow \sigma_z^2 = 1$

(What does it mean? Difficult to say…)

Second way (Bayesian)

$S_I(\omega) = S_{\xi \xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$

quantum back-action changes $z$ in accordance with the noise $\xi$

(what you see becomes reality)

Equal contributions (for weak coupling and $\eta=1$)

Can we explain it in a more reasonable way (without spooks/ghosts)?

No (under assumptions of macrorealism; Leggett-Garg, 1985)

$qubit$

$I(t)$

$+1$

$-1$

$z(t)$?

or some other $z(t)$?
Leggett-Garg-type inequalities for continuous measurement of a qubit

Assumptions of macrorealism (similar to Leggett-Garg'85):

\[ I(t) = I_0 + (\Delta I / 2) z(t) + \xi(t) \]

\[ |z(t)| \leq 1, \quad \langle \xi(t) z(t + \tau) \rangle = 0 \]

Then for correlation function

\[ K(\tau) = \langle I(t) I(t + \tau) \rangle \]

\[ K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \leq (\Delta I / 2)^2 \]

and for area under narrow spectral peak

\[ \int [S_I(f) - S_0] df \leq (8 / \pi^2) (\Delta I / 2)^2 \]

\[ \eta \text{ is not important!} \]

Experimentally measurable violation

(Saclay experiment)

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May be a physical (realistic) back-action?

\[ I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t) \]

OK, cannot explain without back-action

\[ \langle \xi(t) z(t + \tau) \rangle \neq 0 \]

But may be there is a simple classical back-action from the noise?

In principle, classical explanation cannot be ruled out (e.g. computer-generated \( I(t) \); no non-locality as in optics)

Try reasonable models: linear modulation of the qubit parameters \((H, \varepsilon)\) by noise \(\xi(t)\)

No, does not work!

Our (spooky) back-action is quite peculiar: \( \langle \xi(t) dz(t + 0) \rangle > 0 \)

“what you see is what you get”: observation becomes reality
Recent experiment (Saclay group, unpub.)

- superconducting charge qubit (transmon) in circuit QED setup (microwave reflection from cavity)
- driven Rabi oscillations
- perfect spectral peaks
- LGI violation (both $K$ and $S$)

A. Palacios-Laloy et al. (unpublished) courtesy of Patrice Bertet
**Next step: quantum feedback?**

Goal: persistent Rabi oscillations with zero linewidth (synchronized)

**Types of quantum feedback:**

- **Bayesian**
  - Best but very difficult
  - (monitor quantum state and control deviation)

- **Direct**
  - a la Wiseman-Milburn (1993)
  - (apply measurement signal to control with minimal processing)

- **“Simple”**
  - Imperfect but simple
  - (do as in usual classical feedback)

\[
\frac{\Delta H_{fb}}{H} = F \times \phi_m
\]

\[
\Delta H_{fb} / H = F \sin(\Omega t) \times \left( \frac{I(t) - I_0}{\Delta I / 2} - \cos(\Omega t) \right)
\]

**Graphs:**

- Ruskov & A.K., 2002
- A.K., 2005
Quantum feedback in optics


Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

J.M. Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.

First detailed theory:
Quantum feedback in optics


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PRL 94, 203002 (2005) also withdrawn

First detailed theory:

Recent experiment:
Cook, Martin, Geremia, Nature 446, 774 (2007) (coherent state discrimination)
Undoing a weak measurement of a qubit (“uncollapse”)  


It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement? (To restore a “precious” qubit accidentally measured)  

Yes! (but with a finite probability)

If undoing is successful, an unknown state is fully restored

\[ \Psi_0 \text{ (unknown)} \xrightarrow{\text{weak (partial) measurement}} \Psi_1 \text{ (partially collapsed)} \]

\[ \Psi_0 \text{ (still unknown)} \xrightarrow{\text{undoing (information erasure)}} \Psi_2 \]

\[ \Psi_1 \text{ (successful)} \]

\[ \Psi_1 \text{ (unsuccessful)} \]
Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons $\gamma_1$ and $\gamma_2$ produce interference pattern on screen. (b) Two-level atoms excited by laser pulse $l_1$, and emit $\gamma$ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse $l_1$ from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse $l_1$ from $c \rightarrow a$ followed by emission of $\gamma$ photons in $a \rightarrow b$ transition. Second pulse $l_2$ takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of $\phi$ photons.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Our idea of uncollapsing is quite different: we really extract quantum information and then erase it.
Uncollapse of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is non-unitary (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!

\[ |1\rangle \times |0\rangle = |0\rangle \]

need ideal (quantum-limited) detector

(similar to Koashi-Ueda, PRL-1999, also Nielsen-Caves-1997, Royer-1994, etc.)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)
Uncollapsing for DQD-QPC system

Simple strategy: continue measuring until result \( r(t) \) becomes zero! Then any unknown initial state is fully restored.

(same for an entangled qubit)

However, if \( r = 0 \) never happens, then undoing procedure is unsuccessful.

Probability of success:

\[
P_s = \frac{e^{-|r_0|}}{e^{ |r_0|} \rho_{11}(0) + e^{-|r_0|} \rho_{22}(0)}
\]
**General theory of uncollapsing**

POVM formalism (Nielsen-Chuang, p.100)

Measurement operator $M_r$: \[ \rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)} \]

Probability: $P_r = \text{Tr}(M_r \rho M_r^\dagger)$  
Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$  
(to satisfy completeness, eigenvalues cannot be $>1$)

$\max(C) = \min_i \sqrt{p_i}$, $p_i$ – eigenvalues of $M_r^\dagger M_r$

Probability of success: $P_S \leq \frac{\min P_r}{P_r(\rho_{\text{in}})}$  

$P_r(\rho_{\text{in}})$ – probability of result $r$ for initial state $\rho_{\text{in}}$,  
$\min P_r$ – probability of result $r$ minimized over all possible initial states

Averaged (over $r$) probability of success: $P_{av} \leq \sum_r \min P_r$  
(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)
Partial collapse of a Josephson phase qubit


How does a qubit state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit “ages” in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha \left| 0 \right> + \beta \left| 1 \right> \rightarrow \psi(t) = \begin{cases} \left| \text{out} \right>, \text{if tunneled} & \\ \alpha \left| 0 \right> + \beta e^{-\Gamma t/2} e^{i\varphi} \left| 1 \right>, \text{if not tunneled} & \end{cases}$$

$$\sqrt{\alpha^2 + \beta^2} e^{-\Gamma t}$$

(better theory: Pryadko & A.K., 2007)

amplitude of state $\left| 0 \right>$ grows \textbf{without physical interaction}

finite linewidth only after tunneling

\textbf{continuous null-result collapse}

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)
Superconducting phase qubit at UCSB

Courtesy of Nadav Katz (UCSB)

\[
|0\rangle \quad |1\rangle
\]

\[1 \Phi_0\]

\[\omega_{01}\]

\[I_{dc} + I_z\]

\[I_{dc} + I_{\mu W}\]

\[\text{Flux bias} \quad \text{Qubit} \quad \text{SQUID} \quad V_s\]

\[\text{Reset} \quad \text{Compute} \quad \text{Meas.} \quad \text{Readout}\]

\[\text{Repeat 1000x prob. 0,1}\]

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Experimental technique for partial collapse

Nadav Katz et al.
(John Martinis group)

Protocol:
1) State preparation by applying microwave pulse (via Rabi oscillations)
2) Partial measurement by lowering barrier for time $t$
3) State tomography (microwave + full measurement)

Measurement strength $p = 1 - \exp(-\Gamma t)$ is actually controlled by $\Gamma$, not by $t$

$p=0$: no measurement
$p=1$: orthodox collapse
Experimental tomography data

$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Nadav Katz et al. (UCSB, 2005)

$$\theta_x \theta_y \theta_\psi \pi \pi/2$$

$$p = 0, p = 0.06, p = 0.14, p = 0.23, p = 0.32, p = 0.43, p = 0.56, p = 0.70, p = 0.83$$

35/52

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Partial collapse: experimental results

- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for $T_1$ and $T_2$)

N. Katz et al., Science-06

$\eta_0 > 0.8$
Uncollapse of a phase qubit state

1) Start with an unknown state
2) Partial measurement of strength $p$
3) $\pi$-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the same strength $p$
5) $\pi$-pulse

If no tunneling for both measurements, then initial state is fully restored!

$$\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}}$$

$$\frac{e^{i\phi} \alpha e^{-\Gamma t/2} |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} = e^{i\phi} (\alpha |0\rangle + \beta |1\rangle)$$

phase is also restored (spin echo)
Experiment on wavefunction uncollapse


Uncollapse protocol:
- partial collapse
- $\pi$-pulse
- partial collapse (same strength)

State tomography with $X$, $Y$, and no pulses

Background $P_B$ should be subtracted to find qubit density matrix
Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution,
uncollapsing – undoing of a known, but non-unitary evolution
**Quantum process tomography**

Why getting worse at $p > 0.6$?

Energy relaxation $p_r = \frac{t}{T_1} = \frac{45\text{ns}}{450\text{ns}} = 0.1$

Selection affected when $1 - p \sim p_r$

**Overall:** uncollapsing is well-confirmed experimentally
Recent experiment on uncollapsing using single photons

Kim et al., Opt. Expr.-2009

- very good fidelity of uncollapsing (>94%)
- measurement fidelity is probably not good (normalization by coincidence counts)
Suppression of $T_1$-decoherence by uncollapsing

Ideal case ($T_1$ during storage only, $T=0$)

for initial state $|\psi_{\text{in}}\rangle=\alpha|0\rangle+\beta|1\rangle$

$|\psi_{\text{f}}\rangle=|\psi_{\text{in}}\rangle$ with probability $(1-p)e^{-t/T_1}$

$|\psi_{\text{f}}\rangle=|0\rangle$ with $(1-p)^2|\beta|^2e^{-t/T_1}(1-e^{-t/T_1})$

procedure preferentially selects events without energy decay

Trade-off: fidelity vs. selection probability

Unraveling of energy relaxation

\[
\begin{pmatrix}
|\beta|^2 e^{-t/T_1} & \alpha\beta^* e^{-t/2T_1} \\
\alpha^* \beta e^{-t/2T_1} & 1-|\beta|^2 e^{-t/T_1}
\end{pmatrix} = p_{\downarrow}|0\rangle\langle 0 | + (1 - p_{\downarrow})|\tilde{\psi}\rangle\langle \tilde{\psi} |
\]

where

$p_{\downarrow} = |\beta|^2(1-e^{-t/T_1})$

$|\tilde{\psi}\rangle = (\alpha|0\rangle + \beta e^{-t/2T_1}|1\rangle)$ / Norm

$\Rightarrow$ optimum: $1 - p_u = e^{-t/T_1}(1 - p)$
An issue with quantum process tomography (QPT)

QPT fidelity is usually $F_{\chi} = \text{Tr}(\chi_{\text{desired}} \chi)$ where $\chi$ is the QPT matrix.

However, QPT is developed for a linear quantum process, while uncollapsing (after renormalization) is non-linear.

A better way: average state fidelity

$$F_{av} = \text{Tr}(\rho_f U_0 |\psi_{in}\rangle \langle \psi_{in}|) d |\psi_{in}\rangle$$

Without selection

$$F_{\chi} = F_{av}^s = \frac{(d+1)F_{av} - 1}{d}, \quad d = 2$$

Another way: “naïve” QPT fidelity (via 4 standard initial states)

$$F_{\chi} = F_{av}^s = \frac{(d+1)F_{av} - 1}{d}$$

The two ways practically coincide (within line thickness)

Analytics for the ideal case

Average state fidelity

$$F_{av} = \frac{1}{2} + \frac{1}{C} + \frac{\ln(1+C)}{C^2}$$

“Naïve” QPT fidelity

$$F_{\chi} = -\frac{1}{4} + \frac{1}{4(1+C)} + \frac{4+C}{2(2+C)}$$

where

$$C = (1 - p)(1 - e^{-\Gamma t})$$

$$p_u = 1 - e^{-\Gamma t}(1 - p)$$

Ideal without uncollapsing

$p_u = 1 - e^{-t/T_1}(1-p)$

$p_u = p$ without uncollapsing

$e^{-t/T_1} = 0.3$
Realistic case ($T_1$ and $T_\phi$ at all stages)

- Easy to realize experimentally (similar to existing experiment)
- Increase of fidelity with $p$ can be observed experimentally
- Improved fidelity can be observed with just one partial measurement

Uncollapse seems to be the only way to protect against $T_1$-decoherence without encoding in a larger Hilbert space (QEC, DFS)


Trade-off: fidelity vs. selection probability
One more experimental proposal:

**Persistent Rabi oscillations revealed in low-frequency noise**

Hopefully, simple enough for semiconductor qubits

Goal: something easy for experiment, but still with a non-trivial measurement effect
Setup: one qubit & two detectors

\[ V_A(t) \quad \text{QPC } A \quad \text{qubit (DQD)} \quad \text{QPC } B \quad V_B(t) \]

\[ \Omega \]

For single-shot measurements partial collapse can be revealed via correlations of \( \int I_A \) and \( \int I_B \).

\[ (\text{Korotkov, PRB-2001}) \]

Same idea with another averaging \( \rightarrow \) weak values

\[ (\text{Romito et al., PRL-2008}) \]

Single-shot measurements are not yet available

\[ \Rightarrow \text{ use train (comb) of meas. pulses in QND regime} \]

One-detector stroboscopic QND measurement

\[ V(t) \quad \Delta t = 2\pi/\Omega \text{ (one pulse per Rabi period)} \]

Stroboscopic QND measurement synchronizes (!) phase of persistent Rabi oscillations (attracts to either 0 or \( \pi \))

\[ z(t) \]

Stroboscopic QND:

Braginsky, Vorontsov, Khalili, 1978
Jordan, Buttiker, 2005
Jordan, Korotkov, 2006
**Idea of experiment**

Perfect QND $\Rightarrow$ correlation/anticorr. between currents in two detectors

Imperfect QND $\Rightarrow$ random switching between two Rabi phases (0 and $\pi$) $\Rightarrow$ low-frequency telegraph noise

- **same combs on** $V_A$ and $V_B$
  - $V_A(t)$
  - $V_B(t)$
  - $z(t)$
  - anticorrelation between $I_A$ and $I_B$

- **$\pi$-shifted combs on** $V_A$ and $V_B$
  - $V_A(t)$
  - $V_B(t)$
  - $z(t)$
  - correlation (still QND!)

Correlation/anticorrelation between low-frequency (telegraph) noises indicates presence of persistent Rabi oscillations
Analytical results for current noise

\[ S_{IA}(\omega) \approx S_A \frac{\delta t_A}{T} \left( \frac{\delta I_A}{T} \right)^2 \left( \frac{1}{2\Gamma_S} \right) \]

\[ S_{IA,IB}(\omega) \approx \pm \frac{\delta t_A}{T} \frac{\delta t_B}{T} \frac{\Delta I_A}{T} \frac{\Delta I_B}{T} \left( \frac{1}{2\Gamma_S} \right) \]

(fully correlated/anticorrelated in first approx.)

\[ \Gamma_S \approx \frac{1}{4T_2} + \frac{\Omega}{4\pi} \left[ \phi^2 \frac{M AM B}{M^2 + M^2} + \frac{\delta t_A^2 M + \delta t_B^2 M}{12 T^2} \right] \]

\[ M_{A,B} = \delta t_{A,B}(\Delta I_{A,B})^2 / 4S_{A,B}, \quad S_{A,B} = 2eI_{A,B}(1 - T_{A,B}) \]

Assumed: \( \phi \ll 1, \, \delta t \ll T, \, \delta t \ll 4S/(\Delta I)^2, \, T_2 \gg T \)
Numerical results

Low-frequency telegraph noise (dashed) and cross-noise (solid)

Calculation based on numerical solution of the master equation

$1/T_2 = 0$
$T(\Delta I_{A,B})^2/4S_{A,B} = 1$

$\phi/2\pi$ (phase shift)
Estimates

Assume:

- QPC current \( I = 100 \) nA
- response \( \Delta I/I = 0.1 \)
- duty cycle \( \delta t/T = 0.2 \) (symmetric)
- Rabi frequency \( \sim 2 \) GHz

Then:

- “attraction” (collapse) time \( 1.5 \) ns (few Rabi periods)
- switching rate \( \Gamma_s \approx \frac{1}{4T_2} + \frac{1}{1 \mu s} + \frac{\varphi^2}{13 \text{ ns}} \) (many Rabi periods)
- need \( T_2 > 10 \) ns

\[
\frac{S_{\text{telegraph}}}{S_{\text{shot}}} \approx 600 \times \min\left(\frac{T_2}{250 \text{ ns}}, 1\right) \quad (\text{relatively large noise signal})
\]

seems to be reasonable and doable
Useful modification

(Zero average, easier for rf)

Any alternative explanation?

1) no oscillations – then no corr./anticorr.
2) unsynchronized Rabi oscillations – then different dependence on $\varphi$ ($\cos \varphi$ instead of $\varphi^{-2}$); also $\int S_{\text{telegr}}(f) \, df$ at least twice smaller
3) resonant frequency - driven Rabi? Then oscillations between $|g\rangle$ and $|e\rangle$ (both do not give a signal) with different frequency. Driven Rabi decreases corr./anticorr. (not an alternative explanation, but should be avoided)

Good news: both phases insensitive to driven Rabi

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Conclusions

- It is easy to see what is “inside” collapse: simple Bayesian formalism works for many solid-state setups
- Rabi oscillations are persistent if weakly measured
- Collapse can sometimes be undone (uncollapsing)
- Three direct solid-state experiments have been realized
- Many interesting experimental proposals are still waiting
  Two last proposals:
  - suppression of $T_1$-decoherence by uncollapsing
  - persistent Rabi oscillations revealed via noise correlation in two detectors