Quantum back-action during “fast” measurement of a phase qubit

How quantum state changes in time?

(what happens if measured for too short time?)

Main idea (for simplicity $\gamma=0$):

$$\psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi(t) = \begin{cases} |\text{out}\rangle, \text{if switched} \\ \alpha |0\rangle + \beta e^{-\Gamma t/2} |1\rangle / \text{Norm} \end{cases}$$

$$\text{Norm} = \sqrt{\alpha^2 + \beta^2 e^{-\Gamma t}}$$

continuous null-result collapse

(similar to optics, Dalibard et al., PRL-92)
Effect of remaining coherence after incomplete (too short) measurement

**Protocol:**
0) state preparation by rf pulse
1) incomplete measurement
2) additional rf pulse ($\theta$-pulse)
3) measurement again (complete)

$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$p = 1 - \exp(-\Gamma t)$ – probability of state $|1\rangle$ switching after incomplete measurement

$\varphi$ – extra phase (z-rotation)
**Formulas for ideal case**

**Step 1.** Rabi pulse $\theta_0$ prepares state $\cos(\theta_0/2)|0\rangle + \sin(\theta_0/2)|1\rangle$

**Step 2.** Incomplete measurement with strength $p = 1 - \exp(-\Gamma \tau)$ switches qubit with probability $P_1 = p \sin^2(\theta_0)$. With probability $1 - P_1$ the state becomes $\cos(\theta_m/2)|0\rangle + \sin(\theta_m/2)e^{-i\varphi_m}|1\rangle$, where $\varphi_m$ – accumulated phase shift in rotating frame (levels change) and $\theta_m = 2 \arctan(\sqrt{1 - p} \tan(\theta_0/2))$

**Step 3.** Z-rotation $\varphi$ and Rabi pulse $\theta$.

**Step 4.** Complete measurement, switching probability $P_2$.

Total switching probability $P_t = P_1 + P_2$

$$P_t = 1 - \frac{1}{2} [1 - p \sin^2(\frac{\theta_0}{2})][1 + \cos \theta_m \cos \theta - \sin \theta_m \sin \theta \cos(\varphi - \varphi_m)]$$

If $\varphi_m$ is compensated ($\varphi = \varphi_m$) then maximum oscillation amplitude:

$$P_t = 1 - \frac{1}{2} [1 - p \sin^2(\frac{\theta_0}{2})][1 + \cos(\theta_m + \theta)]$$
Dependence $P_t(\theta)$ for different $p$ and $\theta_0$ ($\phi_m$ is compensated)

- $\theta_0 = \pi/2$
- $\phi = \phi_m$

- $\theta_0 = \pi/3$
- $\phi = \phi_m$

- $\theta_0 = 2\pi/3$
- $\phi = \phi_m$