Measurement theory for phase qubits

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   (started in June 2005)
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Since last review

Published: 5 journal papers and 1 proceeding
Submitted (not published): 3 journal papers

Full-scale funding is starting in Year 2 (since June 2005)
(60% of that in Year 1)
Research accomplishments since last review

- Finished research supported by previous NSA/ARDA/ARO project (quantum feedback, etc.)
- Developed basic theoretical approach to quantum back-action during “fast” measurement of one phase qubit
- Developed improved semiclassical theory for measurement cross-talk for measurement of two phase qubits
- Derived Bell-like inequalities in time (similar to Leggett-Garg inequalities) for continuous measurement of a qubit
Quantum back-action during “fast” measurement of a phase qubit

How quantum state changes in time?

(what happens if measured for too short time?)

Main idea (for simplicity $\gamma=0$):

$$\psi(t) = \begin{cases} |\text{out}\rangle, \text{if switched} \\ \frac{\alpha |0\rangle + \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}}, \text{if not switched} \end{cases}$$

(similar to “quantum-jump” approach in optics)
Effect of remaining coherence after incomplete (too short) measurement

**Protocol:**
0) state preparation by rf pulse
1) incomplete measurement
2) additional rf pulse (θ-pulse)
3) measurement again (complete)

$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$p = 1 - \exp(-\Gamma t)$ – probability of state $|1\rangle$ switching after incomplete measurement

$\varphi$ – extra phase (z-rotation)
Measurement cross-talk for measurement of two phase qubits

Origin of the cross-talk:
Measurement of the first qubit and its tunneling into the deep well leads to damped oscillations, which produce microwave voltage perturbing the second qubit.

Detrimental effect of the cross-talk: For initial state $|10\rangle$ the cross-talk may excite second qubit resulting in a wrong measurement result $|11\rangle$.

Theoretical approaches for study of the cross-talk:
(a) Both qubits are modeled “classically”
(b) Second qubit is modeled quantum-mechanically, while first qubit evolution is still “classical” (reasonable since for the first qubit the quantum number is large, $n \sim 150$)
(c) Both qubits are modeled quantum-mechanically

So far we use and compare approaches (a) and (b)
Both qubits considered “classically”

Excitation of the II qubit after measurement of state $|10\rangle$:

Oscillator model, $T_1 = 25$ ns

(J. Martinis’ group, Science, 2005)

Real qubit potential, $N \equiv \frac{\Delta U}{\omega_p} = 5$:

$T_1 = 25$ ns. Qubit anharmonicity makes energy transfer less efficient (good news)

$T_1 = 500$ ns. Eventual classical escape from well for $T_1 > 400$ ns
Numerical solution of Schrödinger equation for the second qubit

Now second qubit is considered fully quantum-mechanically (still “classical” approach for the first qubit)

Energy levels (in units of the plasma frequency $\omega_p$) and wave functions

Level populations vs. time

Qubit potential barrier is $5 \times \tilde{n} \omega_p$ ($N=5$)
Measurement cross-talk in hybrid (classical-quantum) approach

Level populations at $t = 6$ ns

Mean energy $\bar{E}$

Populations of the bound states vs. $t$

Upper solid line – probability $P(t)$ to remain in the well. $n = 0$ - solid, 1 - dashed, 2 - dotted, 3 - red, 4 - green, 5 - blue.

Mean energy is less than the barrier height, but still finite escape probability (significant difference from classical consideration)
Bell-like inequalities in time for continuous measurement of a qubit
(R. Ruskov, A. Korotkov, A. Mizel, cond-mat/0505094)

Continuous monitoring of a qubit (charge, flux, or phase)

\[ I(t) = I_0 + z(t) \frac{\Delta I}{2} + \xi(t), \ \xi(t) - \text{noise}, \ \Delta I = I_1 - I_2 \]

Since |z| ≤ 1, and assuming non-invasive measurability in case of macro-realism (Leggett-Garg, 1985), we derive:

\[ K_I(\tau_1) + K_I(\tau_2) - K_I(\tau_1 + \tau_2) \leq (\Delta I / 2)^2 \]

for detector signal correlation function \( K_I(\tau) = \langle I(t)I(t + \tau) \rangle - \langle I \rangle^2 \)

Quantum calculation shows that \( K_I(\tau) + K_I(\tau) - K_I(2\tau) \)

can reach \( \frac{3}{2}(\Delta I / 2)^2 \) for weak continuous monitoring

Violation by factor up to 3/2
Consequences for measured detector signal spectral density
(Ruskov-Korotkov-Mizel, 2005)

Quantum case (earlier result, 1999)

\[ S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2} \]

Area under the spectral peak is \((\Delta I/2)^2\)
(independent of detector efficiency)

Macro-realistic bounds (this work)

If single spectral peak of the same (Lorentzian) shape, then

\[ \text{area} \leq \left( \frac{2}{3} \right) (\Delta I/2)^2 \]

For any peak at non-zero frequency a weaker bound
(still violated): \( \text{area} \leq \left( \frac{8}{\pi^2} \right) (\Delta I/2)^2 \)

Experimentally measurable violation of classical bound.
Research topics for the next year

• Quantum-rigorous theory of the classical measurement cross-talk of phase qubits
• Theoretical fidelity of one-shot measurements of phase qubits
• Quantum back-action for measurement of phase qubits
• Related problems of quantum measurement