Simple quantum feedback of a solid-state qubit

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Goal: keep coherent oscillations forever

Outline:
- Introduction (Bayesian formalism for continuous quantum measurement, Bayesian quantum feedback)
- Simple quantum feedback of a qubit

Support: cond-mat/0404696
The system we consider: qubit + detector

**qubit**

**detector**

\[ H = H_{QB} + H_{DET} + H_{INT} \]

\[ H_{QB} = \left( \frac{\varepsilon}{2} \right) (c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1) \]

\( \varepsilon \) – asymmetry, \( H \) – tunneling

\[ \Omega = \left( 4H^2 + \varepsilon^2 \right)^{1/2}/\bar{\sigma} \]

– frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: \( I_1 \) for qubit state \(|1\rangle\), \( I_2 \) for \(|2\rangle\)

Response: \( \Delta I = I_1 - I_2 \)

Detector noise: white, spectral density \( S_I \)

**DQD and QPC**

(setup due to Gurvitz, 1997)

\[ H_{DET} = \sum_l E_l a_l^+a_l + \sum_r E_r a_r^+a_r + \sum_{l,r} T(a_r^+a_l + a_l^+a_r) \]

\[ H_{INT} = \sum_{l,r} \Delta T (c_1^+c_1 - c_2^+c_2)(a_r^+a_l + a_l^+a_r) \]

\( S_I = 2eI \)
**Bayesian formalism for a single qubit**

\[ \hat{H}_{QB} = \frac{\epsilon}{2} (c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1) \]

\[ |1\rangle I_1, \quad |2\rangle I_2 \quad \Delta I = I_1 - I_2, \quad I_0 = (I_1 + I_2)/2, \quad S_I - \text{detector noise} \]

\[ \rho_{11} = -\rho_{22} = -2 (H / \hbar) \text{Im} \rho_{12} + \rho_{11} \rho_{22} (2\Delta I / S_I) [I(t) - I_0] \]

\[ \rho_{12} = i(\epsilon / \hbar) \rho_{12} + i(H / \hbar) (\rho_{11} - \rho_{22}) + \rho_{12} (\rho_{11} - \rho_{22}) (\Delta I / S_I) [I(t) - I_0] - \gamma \rho_{12} \]

\( \gamma = \Gamma - (\Delta I)^2 / 4S_I, \quad \Gamma - \text{ensemble decoherence} \)

\( \eta = 1 - \gamma / \Gamma = (\Delta I)^2 / 4S_I \Gamma \quad \text{– detector ideality (efficiency), } \eta \leq 100\% \)

Ideal detector (\( \eta = 1 \)) does not decohere a single qubit; then random evolution of qubit wavefunction can be monitored

For simulations: \( I(t) - I_0 = (\rho_{22} - \rho_{11}) \Delta I / 2 + \xi(t), \quad S_\xi = S_I \)

Averaging over \( \xi(t) \) μ conventional master equation

**Similar formalisms developed earlier.** Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

**Names:** Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, **Milburn, Wiseman**, Onofrio, Habib, Doherty, etc. (incomplete list)
Measured spectrum of qubit coherent oscillations

What is the spectral density $S_I(\omega)$ of detector current?

Assume classical output, $eV \gg \hbar \Omega$

$\varepsilon = 0, \quad \Gamma = \eta^{-1} (\Delta I)^2 / 4 S_0$

$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$

Spectral peak can be seen, but peak-to-pedestal ratio $\leq 4\eta \leq 4$

(result can be obtained using various methods, not only Bayesian method)

Weak coupling, $\alpha = C/8 \ll 1$

$S_I(\omega) = S_0 + \frac{\eta S_0 \varepsilon^2 / H^2}{1 + (\omega\hbar^2 \Omega^2 / 4 H^2 \Gamma)^2}$

$+ \frac{4\eta S_0 (1 + \varepsilon^2 / 2H^2)^{-1}}{1 + [\omega - \Omega \Gamma (1 - 2H^2 / \hbar^2 \Omega^2)]^2}$

Contrary:

Stace-Barrett, 2003
(PRL 2004)
Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit “fresh”)

Idea: monitor the Rabi phase $\phi$ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F \times \Delta \phi$

To monitor phase $\phi$ we plug detector output $I(t)$ into Bayesian equations

Ruskov-Korotkov, 2001
Performance of quantum feedback  
(no extra environment)

Qubit correlation function

\[ K_z(\tau) = \frac{\cos \Omega t}{2} \exp \left[ \frac{C}{16F} \left( e^{-2FH\tau/\hbar} - 1 \right) \right] \]

(for weak coupling and good fidelity)

Detector current correlation function

\[ K_I(\tau) = \frac{(\Delta I)^2}{4} \cos \Omega t \left( 1 + e^{-2FH\tau/\hbar} \right) \times \exp \left[ \frac{C}{16F} \left( e^{-2FH\tau/\hbar} - 1 \right) \right] + \frac{S_I}{2} \delta(\tau) \]

Fidelity (synchronization degree)

\[ D = 2 \langle \text{Tr} \rho \rho_{\text{desir}} \rangle - 1 \]

\[ C = \hbar (\Delta I)^2 / S_I H \quad \text{– coupling} \]
\[ \tau_a^{-1} \quad \text{– available bandwidth} \]
\[ F \quad \text{– feedback strength} \]

For ideal detector and wide bandwidth, fidelity can be arbitrarily close to 100%.

\[ D = \exp(-C/32F) \]

Ruskov & Korotkov, PRB 66, 041401(R) (2002)
Quantum feedback in presence of decoherence by environment

Big experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth (\(\gg \Omega\), GHz-range) of the line delivering noisy signal \(I(t)\) to the “processor”
Simple quantum feedback of a solid-state qubit

(A.K., cond-mat/0404696)

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations (a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau] \, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau] \, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: simplicity and relatively narrow bandwidth ($1/\tau \sim \Gamma_d << \Omega$)

Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures (surprisingly, situation is much better than anticipated!)

Goal: maintain coherent (Rabi) oscillations for arbitrary long time
Accuracy of phase monitoring via quadratures  
(no feedback yet)
Simple quantum feedback

\[ D, \langle X \rangle (4/\tau \Delta I) \]

How to verify feedback operation experimentally?
Simple: just check that in-phase quadrature \( \langle X \rangle \) of the detector current is positive

\[ D = \langle X \rangle (4/\tau \Delta I) \]

\( \langle X \rangle = 0 \) for any non-feedback Hamiltonian control of the qubit
Effect of nonidealities

- nonideal detectors (finite quantum efficiency $\eta$) and environment
- qubit energy asymmetry $\varepsilon$
- frequency mismatch $\Delta \Omega$

Quantum feedback still works quite well

Main features:

- Fidelity $F_Q$ up to $\sim 95\%$ achievable ($D \sim 90\%$)
- Natural, practically classical feedback setup
- Averaging $\tau \sim 1/\Gamma >> 1/\Omega$ (narrow bandwidth!)
- Detector efficiency (ideality) $\eta \sim 0.1$ still OK
- Robust to asymmetry $\varepsilon$ and frequency shift $\Delta \Omega$
- Simple verification: positive in-phase quadrature $\langle X \rangle$

Simple enough experiment?!
Quantum feedback in optics


Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia, John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.

Conclusion

- Very straightforward, practically classical feedback idea (monitoring the phase of oscillations via quadratures) works well for the qubit coherent oscillations

- Price for simplicity is a less-than-ideal operation (fidelity is limited by ~95%)

- Feedback operation is much better than expected

- Relatively simple experiment (simple setup, narrow bandwidth, inefficient detectors OK, simple verification)