Opportunistic UAV Navigation With Carrier Phase Measurements from Asynchronous Cellular Signals

Joe Khalife, Student Member, IEEE, and Zaher M. Kassas, Senior Member, IEEE

Abstract—A framework for opportunistic unmanned aerial vehicle (UAV) navigation is presented. The framework exploits carrier phase measurements from ambient cellular signals of opportunity, whose base transceiver stations (BTSs) are asynchronous. The framework employs an extended Kalman filter (EKF) to estimate the position and velocity of the UAV as well as the differences between the UAV-mounted receiver and each of the BTSs’ clock bias and clock drift. The observability of the estimation framework is analyzed. An initialization scheme for the EKF is developed and the boundedness of the EKF’s errors is studied. A lower bound for the EKF estimation error covariance is derived, and it is shown that the covariance remains bounded.

Two sets of experimental results are presented demonstrating a total position root mean-squared error of 2.94 m and 5.99 m for UAV trajectories of 2.6 km and 3.07 km, respectively.

Index Terms—Cellular signals, carrier phase, UAV, opportunistic navigation, observability, extended Kalman filter.

I. INTRODUCTION

Current unmanned aerial vehicle (UAV) navigation systems will not meet the stringent requirements on accuracy, resiliency, and robustness due to their heavy reliance on jammable and spoofable global navigation satellite system (GNSS) signals [1], [2]. When GNSS signals are unusable (e.g., in the presence of jamming or spoofing or in indoor and deep urban environments), cellular signals of opportunity (SOPs) could be used for navigation either in (1) a standalone fashion [3], [4] or (2) an integrated fashion, aiding the UAV’s inertial navigation system [5], [6]. These signals are attractive for navigation since they are abundant, received at a much higher power than GNSS signals, possess a favorable horizontal geometry, and are free to use. Moreover, cellular signals received by UAVs do not suffer from severe multipath by virtue of the favorable channel between base stations and UAVs. Several receiver designs have been published recently, producing time-of-arrival (TOA) and frequency-of-arrival (FOA) measurements from cellular code-division multiple access (CDMA) and long-term evolution (LTE) signals [7]–[10].

While TOA- and FOA-based navigation approaches are well-studied in the literature [11], [12], applying such approaches to cellular CDMA base transceiver stations (BTSs) or LTE eNodeBs requires perfect synchronization assumptions [13], [14]. However, cellular CDMA and LTE networks are not perfectly synchronized, and their protocols recommend synchronization of CDMA BTSs and LTE eNodeBs to within 3 microseconds from GPS time [15], [16]. This translates to ranging errors of about 900 meters. Several approaches in the literature have been proposed to account for the BTSs’ or eNodeBs’ clock biases and drifts, including using the round-trip time (RTT) instead of the TOA [17]. Although RTT-based methods could yield good results in asynchronous systems, two-way communication between the receiver and the BTSs or eNodeBs is needed. This limits the availability of RTT measurements to only paying subscribers to a particular cellular provider and compromises the privacy of the user. Some of the proposed navigation frameworks assume the BTSs’ or eNodeBs’ clock bias and drift to be constant [3], [18]. However, the clock bias and drift are dynamic and stochastic [19]; hence, must be continuously estimated.

To deal with this challenge, a framework employing a monitor receiver was put forth by [20]. Moreover, a mapper/navigator framework was proposed in [21], where the mapper, which was assumed to have complete knowledge of its states (e.g., by having access to GNSS signals), is estimating the clock states of BTSs in its environment, and is sharing these estimates with a navigating receiver that has no knowledge of its own states, but is making pseudorange measurements on the same BTSs in the environment. The mapper/navigator framework could yield centimeter-accurate UAV navigation when carrier phase observables extracted from cellular signals are exploited [22]. Having a mapper may be impractical in some environments or in the absence of a communication channel between the mapper and navigator. To alleviate the need of a monitor or a mapper in the case of code phase measurements from cellular signals, the navigator could estimate its states simultaneously with the states (position, clock bias, and clock drift) of the BTSs in the environment, i.e., perform radio simultaneous localization and mapping (radio SLAM) [5], [23], [24]. Alternatively, in the case where the navigating UAV is making carrier phase measurements from cellular signals, the relative frequency stability of cellular CDMA BTSs or LTE eNodeBs may be leveraged and measurement models that capture this stability may be employed to achieve centimeter-accurate navigation solutions without a mapper [22]. However, this method may fail if the frequency stability requirement is not met.

This paper considers UAV navigation with cellular carrier phase measurements without any assumptions on the synchronization between cellular BTSs or eNodeBs and makes three contributions. First, a UAV navigation framework with carrier phase measurements from cellular SOPs, which employs an extended Kalman filter (EKF) is presented. The precision of
carrier phase measurements is on the order of the carrier signal wavelength, making such measurements attractive for UAV navigation. Second, the EKF initialization is discussed and the observability of the proposed framework and the EKF error bounds are analyzed. Third, two sets of experimental results are presented demonstrating UAVs navigating with the proposed framework achieving a root mean-squared error (RMSE) of 2.94 m and 5.99 m for UAV trajectories of 2.6 km and 3.07 km, respectively.

The remainder of the paper is organized as follows. Section II describes the cellular SOP and receiver dynamics models and the cellular carrier phase observable. Section III describes the EKF-based navigation framework. Section IV gives the theoretical background on observability and boundedness of the EKF. Section V analyzes the observability and the EKF error boundedness of the proposed framework. Section VI provides experimental results demonstrating meter-level UAV navigation accuracy. Concluding remarks are given in Section VII.

II. MODEL DESCRIPTION

This section presents the dynamics model of the UAV-mounted receiver and cellular SOP as well as the cellular carrier phase measurement model. Note that an altimeter could be used to estimate the UAV’s altitude. Therefore, only the UAV’s two-dimensional (2–D) position is estimated in this paper. The subsequent analysis is readily extendable to 3–D; however, the vertical position estimate will suffer from large uncertainty due to the poor vertical diversity of cellular towers.

A. SOP Dynamics Model

The cellular SOPs emanate from spatially-stationary terrestrial BTSs or eNodeBs, and their states will consist of their known 2–D positions and unknown clock error states, namely the clock bias and clock drift. The position vector of the n-th SOP is given by \( r_n = [x_n, y_n]^T \). The state of the n-th SOP will only consist of its clock error state and is given by \( x_{clk,n} = [\delta t_{x,n}, \delta t_{y,n}, c]^T \), where \( \delta t_{x,n} \) and \( \delta t_{y,n} \) are the clock bias and clock drift, respectively, and \( c \) is the speed of light. The n-th cellular SOP’s dynamics can be described by the discretized state space model

\[
x_{clk,n}(k+1) = F_{clk} x_{clk,n}(k) + w_{clk,n}(k), \quad k = 1, 2, \ldots,
\]

where \( n = 1, \ldots, N \), with \( N \) being the total number of cellular SOPs, and \( w_{clk,n} \) is a zero-mean white noise sequence with covariance \( Q_{clk,n} \), with

\[
F_{clk} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad Q_{clk,n} = c^2 S_{\delta t_{x,n}}^2 T^2 + S_{\delta t_{y,n}}^2 T^2 + S_{\delta t_{x,n}} S_{\delta t_{y,n}} T^2.
\]

B. UAV-Mounted Receiver Dynamics Model

The UAV-mounted receiver state consists of its unknown position \( r_r = [x_r, y_r]^T \), velocity \( \dot{r}_r \), and clock error states \( x_{clk,r} = [\delta t_{r,x}, \delta t_{r,y}, c]^T \). Hence, the state vector of the receiver is given by \( x_r = [\dot{r}_r^T, r_r^T, x_{clk,r}]^T \). The receiver’s position \( r_r \) and velocity \( \dot{r}_r \) will be assumed to evolve according to continuous-time a velocity random walk model [27]. Therefore, the UAV-mounted receiver dynamics is modeled according to the discretized model

\[
x_r(k+1) = F_r x_r(k) + w_r(k), \quad k = 0, 1, 2, \ldots,
\]

where \( w_r = [w_{pv,r}^T, w_{clk,r}^T]^T \) is a discrete-time zero-mean white noise sequence with covariance \( Q_r = \text{diag}(Q_{pv,r}, Q_{clk,r}) \), with

\[
F_r = \begin{bmatrix} I_{2 \times 2} & T I_{2 \times 2} & 0_{2 \times 2} \\ 0_{2 \times 2} & I_{2 \times 2} & 0_{2 \times 2} \\ 0_{2 \times 2} & 0_{2 \times 2} & F_{clk} \end{bmatrix}, \quad F_{clk} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix},
\]

\[
Q_{clk,r} = \begin{bmatrix} S_{\delta t_{x,r}} T + S_{\delta t_{x,r}}^2 T^2 & S_{\delta t_{x,r}} T^2 \\ S_{\delta t_{y,r}}^2 T^2 & S_{\delta t_{y,r}} T^2 \end{bmatrix},
\]

\[
Q_{pv} = \begin{bmatrix} \tilde{q}_x T^3 & 0 & 0 \\ 0 & \tilde{q}_y T^3 & 0 \\ 0 & 0 & \tilde{q}_y T^3 \end{bmatrix},
\]

where \( \tilde{q}_x \) and \( \tilde{q}_y \) are the power spectral densities of the continuous-time \( x \) and \( y \) acceleration noise, respectively. The spectra \( S_{\delta t_{x,r}} \) and \( S_{\delta t_{y,r}} \) are modeled similarly to the SOP spectra, but with receiver-specific \( h_{0,r} \) and \( h_{-2,r} \).

C. Cellular Carrier Phase Measurement Model

A specialized navigation receivers (e.g., [3], [7], [9], [10]) could produce a carrier phase observable to the n-th cellular SOP given by

\[
\phi_n(t) = \phi_n(t_0) + \int_{t_0}^{t} f_{D,n}(\tau) d\tau, \quad n = 1, \ldots, N,
\]

where \( \phi_n(t_0) \) is the initial carrier phase and \( f_{D,n} \) is the Doppler frequency. The carrier phase observable in (1) could be parameterized in terms of the receiver and cellular SOP states to yield the discrete-time measurement model given by

\[
z_n(k) = \lambda \phi_n(t_0 + kT) + \| r_r(k) - r_n \|^2 + c [\delta t_r(k) - \delta t_{n,r}] + \lambda N_n + v_n(k),
\]

where \( \lambda \) is the wavelength of the carrier signal, \( N_n \) represents the carrier phase ambiguity corresponding to the n-th SOP.
(namely, the initial phase difference between the receiver and the \( n \)-th SOP), and \( v_n \) is the measurement noise, which is modeled as a discrete-time zero-mean white Gaussian sequence with variance \( \sigma^2_n(k) \).

III. NAVIGATION WITH CELLULAR SOP CARRIER PHASE MEASUREMENTS

This section formulates an EKF-based framework for standalone navigation with carrier phase measurements from asynchronous cellular SOPs.

A. Modified Clock Error States

Estimating the terms \( c\delta t_r \), \( c\delta t_s \), and \( \lambda N_n \) in (2) individually is unnecessary; hence, they will be lumped into one bias term defined as

\[
c\delta t_n(k) \triangleq c \left[ \delta t_r(k) - \delta t_s(k) + \frac{\lambda}{c} N_n(k) \right],
\]

with an associated drift state \( \dot{c}\delta t_n(k) \) given by

\[
\dot{c}\delta t_n(k) \triangleq c \left[ \dot{\delta t_r}(k) - \dot{\delta t_s}(k) \right].
\]

One may subsequently conclude that the dynamics of \( x_{clk,n} \) is expressed as

\[
x_{clk,n}(k+1) = F_{clk} x_{clk,n}(k) + w_{clk,n}(k), \quad n = 1, \ldots, N,
\]

where \( w_{clk,n} \) is a discrete-time zero-mean white noise sequence with covariance \( Q_{clk,n} = Q_{clk,r} + Q_{clk,s} \). Note that now \( w_{clk,n}(k) \) and \( w_{clk,m}(k) \) are correlated, with

\[
E \left[ w_{clk,n}(k) w_{clk,m}^T(k) \right] = \left\{ \begin{array}{ll} Q_{clk,n}, & \text{if } n = m, \\ Q_{clk,r}, & \text{otherwise.} \end{array} \right.
\]

B. EKF Model

the EKF estimates the UAV-mounted receiver’s position and velocity and the modified clock error states for all cellular SOPs, namely \( x \triangleq [r_r^T, c\delta t_1, \ldots, c\delta t_N, r_r^T, c\delta t_1, \ldots, c\delta t_N]^T \).

Note that \( x \) may be expressed as \( x = \Pi x', \) where \( x' \triangleq [r_r^T, r_r^T, x_{clk,1}^T, \ldots, x_{clk,N}^T]^T \) and \( \Pi \) is some permutation matrix that could be readily calculated. The EKF considers the system with the following dynamics and measurement model

\[
x(k+1) = F x(k) + w(k),
\]

\[
z(k) = h(x(k)) + v(k),
\]

with \( h(x(k)) \triangleq [h_1(x(k)), \ldots, h_N(x(k))]^T, h_n(x(k)) \triangleq \|r_r(k) - r_s(k)\| + c\delta t_n(k), z \triangleq [z_1, \ldots, z_N]^T, w \) is a discrete-time zero-mean white sequence with covariance \( \Sigma \triangleq \Pi Q' \Pi^T, \) where \( Q' \triangleq \text{diag}(Q_{p,v}, Q_{clk}), \)

\[
Q_{clk} \triangleq \begin{bmatrix} Q_{clk,1} & Q_{clk,r} & \cdots & Q_{clk,r} \\ Q_{clk,r} & Q_{clk,2} & \cdots & Q_{clk,r} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{clk,r} & Q_{clk,r} & \cdots & Q_{clk,N} \end{bmatrix}, \quad F \triangleq \begin{bmatrix} I_{p,wp} & T I_{p,wp} \\ 0_{p,wp} & I_{p,wp} \end{bmatrix},
\]

with \( p = N + 2, \) and \( v \triangleq [v_1, \ldots, v_n]^T \) is a discrete-time zero-mean white Gaussian sequence with covariance \( R(k) \triangleq \text{diag}([\sigma^2_1(k), \ldots, \sigma^2_N(k)]). \) Section VI discusses how the process and measurement noise covariance matrices \( Q \) and \( R, \) respectively, are selected in a practical environment.

The EKF is producing produces \( \hat{x}(k|j) = \mathbb{E} \left[ x(k) \mid z(1), \ldots, z(j) \right], j \leq k, \) with an associated estimation error covariance \( P(k|j) = \mathbb{E} \left[ (\hat{x}(k|j) - x(k))^T (\hat{x}(k|j) - x(k)) \right], \) where \( \hat{x}(k|j) \approx x(k) - \hat{x}(j) \) is the estimation error. The current state estimate \( \hat{x}(k|j) \) and its associated estimation error covariance \( P(k|j) \) are obtained using the standard EKF equations. The measurement Jacobian \( H \) used in the EKF estimation error covariance update is given by

\[
H(k) = \begin{bmatrix} G(k) & I_{N \times N} & 0_{(N+2) \times (N+2)} \end{bmatrix},
\]

\[
G(k) \triangleq \begin{bmatrix} r_r(k) - r_s \\ \|r_r(k) - r_s\| \end{bmatrix}^T,
\]

where \( G(k) \) is evaluated at \( \hat{x}(k+1|k). \)

C. EKF Initialization

It is assumed that the UAV has access to estimates of its position for two consecutive time-steps, denoted \( \hat{r}_r(-1) \) and \( \hat{r}_r(0), \) respectively, prior to navigating exclusively with cellular carrier phase measurements. It is also assumed that the estimation error covariance associated with these position estimates, denoted \( P_r(-1) \) and \( P_r(0), \) respectively, as well as their cross-covariance, denoted \( P_{rr}(-1, -1), \) are known. These estimates may be given for example by the UAV’s on-board GNSS receiver. An estimate of the receiver’s velocity could be formed according to

\[
\dot{\hat{r}}_r(0) \triangleq \frac{1}{T} [\hat{r}(0) - \hat{r}(-1)].
\]

Moreover, clock bias estimates could be produced according to

\[
c\delta t_n(j) \triangleq z_n(j) - \|\hat{r}_r(j) - r_{n}(j)\|,
\]

where \( j = -1, 0. \) Expanding (8) as a first-order Taylor series and substituting for \( z_n(j) \) yields

\[
c\delta t_n(j) \approx c\delta t_n(j) + h_{r,n}(j) [r_r(j) - \hat{r}_r(j)] + v_n(j),
\]

where \( h_{r,n}(j) \triangleq \frac{\dot{r}_r(j) - r_{n}(j)}{\|\hat{r}_r(j) - r_{n}(j)\|}. \) Next, a clock drift estimate is computed according to

\[
c\delta t_n(0) \approx \frac{1}{T} \left[ c\delta t_n(0) - c\delta t_n(-1) \right] \\
\approx c\delta t_n(0) + h_{r,n}(0) [r_r(0) - \hat{r}_r(0)] + v_n(0) \\
- \frac{h_{r,n}(1) [r_r(-1) - \hat{r}_r(-1)] + v_n(-1)}{T}.
\]

Next, an estimate of \( x(0) \) may be formed according to

\[
\dot{\hat{x}}(0) \triangleq \left[ \dot{\hat{r}}_r^T(0), \dot{c}\delta t_1(0), \ldots, \dot{c}\delta t_N(0), \right. \\
\left. \dot{r}_r(0), \dot{c}\delta t_1(0), \ldots, \dot{c}\delta t_N(0) \right]^T.
\]

Combining (7)–(10), the initial estimation error could be expressed as

\[
\hat{x}(0) = A_0 \hat{r}_r(0) + B_0 v_0,
\]
I

rank is full rank, i.e., matrix. The following theorem states a necessary and sufficient condition defined in (11)–(12) is usually determined by studying them. Consider the discrete-time linear time-varying system

\[ x(k + 1) = F(k) x(k) + \Gamma(k) u(k) + w(k), \]

(14)

\[ z(k) = H(k) x(k) + v(k), \]

(15)

where \( x \in \mathbb{R}^{n_x} \) is the system's state, \( u \in \mathbb{R}^{n_u} \) is the input, \( w \in \mathbb{R}^{n_w} \) is a zero-mean white sequence with covariance \( Q \), \( z \in \mathbb{R}^{s_z} \) is the measurement, and \( v \in \mathbb{R}^{s_v} \) is a zero-mean white sequence with covariance \( R \). Assume that \( w \) and \( v \) are uncorrelated. Let \( P(k+1|k) \) be a solution to the matrix Riccati difference equation in the Kalman filter estimating the state of system (14)–(15) given by

\[ P(k+1|k) = F(k) [ P(k|k-1) - P(k|k-1) H^T(k) \cdot 
\]

\[ H(k) P(k|k-1) H^T(k) + R(k) ]^{-1} \cdot 
\]

\[ H(k) P(k|k-1) \} F^T(k) + Q(k). \]

Let the following hold:

1) There are real numbers \( \underline{p}, \bar{p}, \underline{q}, \bar{q} > 0 \) such that \( Q(k) \) and \( R(k) \) are bounded by

\[ \underline{p} I \preceq Q(k) \preceq \bar{p} I, \quad \underline{q} I \preceq R(k) \preceq \bar{q} I. \]

2) The matrices \( F(k) \) and \( H(k) \) satisfy the uniform observability condition.

3) The initial condition \( P(0|0) \) of the matrix Riccati difference equation in the Kalman filter is positive definite.

Then, there are real numbers \( \underline{p}, \bar{p} > 0 \) such that \( P(k+1|k) \) is bounded via

\[ \underline{p} I \preceq P(k+1|k) \preceq \bar{p} I, \quad \forall k \geq 0. \]

Next, consider the discrete-time nonlinear stochastic system

\[ x(k + 1) = f [ x(k), u(k) ] + w(k), \]

(16)

\[ z(k) = h [ x(k) ] + v(k), \]

(17)

where \( x \in \mathbb{R}^{n_x} \) is the system's state, \( u \in \mathbb{R}^{n_u} \) is the input, \( w \in \mathbb{R}^{n_w} \) is a zero-mean white sequence with covariance \( Q(k) \), \( z \in \mathbb{R}^{s_z} \) is the measurement, and \( v \in \mathbb{R}^{s_v} \) is a zero-mean white sequence with covariance \( R(k) \).

An EKF is employed to estimate \( \hat{x} \). Define the EKF linearization errors

\[ \varphi(k) \triangleq f [ x(k), u(k) ] - f [ \hat{x}(k|k), u(k) ] \]

(18)

\[ \chi(k) \triangleq h [ x(k) ] - h [ \hat{x}(k+1|k) ] \]

(19)
where \( F \) and \( H \) are the dynamics and observation Jacobians, respectively, evaluated at \( \dot{x}(k|k) \) and \( \dot{x}(k+1|k) \), respectively.

**Definition IV.1.** [31] The stochastic sequence \( \dot{x}(k|k) \) is said to be exponentially bounded in mean square, if there are real numbers \( \eta, \nu > 0 \) and \( 0 < \delta < 1 \) such that

\[
\mathbb{E} \left[ \| \dot{x}(k|k) \|^2 \right] \leq \eta \| \dot{x}(0|0) \|^2 \delta^k + \nu
\]

holds for every \( k \geq 0 \).

**Definition IV.2.** [31] The stochastic sequence \( \dot{x}(k|k) \) is said to be exponentially bounded with probability one, if

\[
\sup_{k \geq 0} \| \dot{x}(k|k) \| < \infty
\]

holds with probability one.

**Theorem IV.2.** [31] Consider the system defined in (16)–(17) and consider an EKF estimating its state vector. Moreover, let the following assumptions hold

1) There are positive real numbers \( \bar{f}, \bar{h}, \bar{p}, \bar{q}, \bar{r} > 0 \) such that the following bounds hold for every \( k \geq 0 \)

\[
\| F(k) \| \leq \bar{f}
\]

\[
\| H(k) \| \leq \bar{h}
\]

\[
\bar{p} I \leq P(k+1|k) \leq \bar{p} I
\]

\[
\bar{q} I \leq Q(k)
\]

\[
\bar{r} I \leq R(k).
\]

2) The matrix \( F(k) \) is nonsingular for every \( k \geq 0 \).

3) There are positive real numbers \( \epsilon_\varphi, \epsilon_\chi, \kappa_\varphi, \kappa_\chi > 0 \) such that the nonlinear functions \( \varphi \) and \( \chi \) are bounded via

\[
\| \varphi(k) \| \leq \kappa_\varphi \| \dot{x}(k|k) \|^2
\]

\[
\| \chi(k) \| \leq \kappa_\chi \| \dot{x}(k|k) \|^2,
\]

with \( \| \dot{x}(k|k) \| \leq \epsilon_\varphi \) and \( \| \dot{x}(k|k) \| \leq \epsilon_\chi \).

Then, the estimation error \( \dot{x}(k|k) \) is exponentially bounded in mean square and bounded with probability one, provided that (i) the initial estimation error satisfies

\[
\| \dot{x}(0|0) \| \leq \epsilon
\]

and (ii) the covariance matrices of the noise terms are bounded via

\[
Q(k) \leq \delta I, \quad R(k) \leq \delta I,
\]

for some \( \epsilon, \delta > 0 \).

**V. Observability and EKF Estimation Error Bounds Analyses**

This section shows that the environment with the dynamics and observation model defined in (3)–(4) is observable for \( N \geq 2 \). Moreover, it shows that the EKF estimation error is exponentially bounded in the mean square sense and bounded with probability one. In the sequel, the following assumptions are made:

A1. The SOPs are not colocated.

A2. The UAV is not stationary nor is moving along a trajectory that is collinear with the vector connecting its receiver with any of the SOPs.

A3. The UAV is at a minimum distance \( d \) from each SOP at all time, i.e., \( \| r(k) - r_{sn} \| \geq d, \forall k \geq 0 \) and \( \forall n = 1, \ldots, N \).

**A. Observability Analysis**

The observability of an environment comprising multiple receivers making pseudorange measurements on multiple SOPs, assuming different \( a \ priori \) knowledge scenarios was analyzed in [29]. The observability analysis utilized the \( l \)-step observability matrix of the linearized system and considered the observability of the individual clock biases and drifts \( c_\delta t_n \), \( c_\delta t_{sn} \), \( \{ c_\delta t_{sn} \}_{n=1}^N \), and \( \{ c_\delta t_n \}_{n=1}^N \).

In contrast, the system in (3)–(4) considers a single receiver making carrier phase measurements on multiple SOPs, where the individual clock biases and carrier phase ambiguities are lumped into a single bias term \( \{ c_\delta t_n \}_{n=1}^N \) and the drifts are also lumped into a single drift term \( \{ c_\delta t_n \}_{n=1}^N \).

The observability results for the system defined in (3)–(4) is captured in the following theorem.

**Theorem V.1.** Under assumptions A1 and A2, the system defined in (3)–(4) is completely \( l \)-step observable for \( l \geq 4 \) and \( N \geq 2 \).

**Proof.** The linearization of the deterministic part of the system (3)–(4) into the form (11)–(12) yields

\[
F(k) \equiv \begin{bmatrix} I_{(N+2) \times (N+2)} & T I_{(N+2) \times (N+2)} \\ 0_{(N+2) \times (N+2)} & I_{(N+2) \times (N+2)} \end{bmatrix}, \quad \Gamma(k) \equiv 0,
\]

\[
H(k) \equiv \begin{bmatrix} H_{\xi}(k) \\ 0_{N \times (N+2)} \end{bmatrix}, \quad H_{\xi}(k) \equiv [G(k) \ I_{N \times N}].
\]

In the following, it will be proven by construction that the \( l \)-step observability matrix \( \mathcal{O}(k, k+l) \) of the linearized system is full rank, i.e.,

\[
\sum_{i=1}^{2L} \gamma_i \mathcal{O}(k, k+l) e_{2L,i} = 0
\]

is satisfied if and only if \( \gamma_i = 0 \), \( \forall i = 1, \ldots, 2L \), where \( L = N+2 \) and \( e_{L,i} \in \mathbb{R}^L \) is the standard basis vector consisting of a one at the \( i \)-th element and zeros elsewhere. Note that since \( N \geq 2 \), then \( l \geq 4 \). Let \( l = 4 \). Subsequently, \( \mathcal{O}(k, k+4) \) may be expressed as

\[
\mathcal{O}(k, k+4) = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} \\ \mathcal{O}_{21} & \mathcal{O}_{22} \end{bmatrix},
\]

\[
\mathcal{O}_{11} \equiv \begin{bmatrix} H_{d}(k) \\ H_{d}(k+1) \end{bmatrix}, \quad \mathcal{O}_{21} \equiv \begin{bmatrix} H_{d}(k+2) \\ H_{d}(k+3) \end{bmatrix},
\]

\[
\mathcal{O}_{12} \equiv \begin{bmatrix} 0 \\ T H_{d}(k+1) \end{bmatrix}, \quad \mathcal{O}_{22} \equiv \begin{bmatrix} 2 T H_{d}(k+2) \\ 3 T H_{d}(k+3) \end{bmatrix}.
\]

The matrix \( \mathcal{O}_{11} \) may also be expressed as

\[
\mathcal{O}_{11} = \begin{bmatrix} G(k) & I_{N \times N} \\ G(k+1) & I_{N \times N} \end{bmatrix}.
\]

Note that \( \mathcal{O}_{11} \in \mathbb{R}^{2N \times (N+2)} \). Moreover, the inequality \( 2N \geq N+2 \) holds, for \( N \geq 2 \). Therefore,

\[
\text{rank} \left[ \mathcal{O}_{11} \right] \leq N + 2.
\]
From (32), it can be seen that \( \text{rank}[\mathcal{O}_{11}] \geq N \). Moreover, for \( N \geq 2 \), and if A1 and A2 hold, then the \((N + 1)\)st and \((N + 2)\)nd rows of \( \mathcal{O}_{11} \) will be linearly independent from the first \( N \) rows and from each other, yielding

\[
\text{rank}[\mathcal{O}_{11}] \geq N + 2. \tag{34}
\]

Combining (33) and (34), it can be deduced that \( \text{rank}[\mathcal{O}_{11}] = N + 2 \). Similarly, it can be shown that \( \text{rank}[\mathcal{O}_{21}] = N + 2 \). Subsequently,

\[
\sum_{i=1}^{L} \alpha_i \mathcal{O}_{11} e_{L,i} = 0, \quad \sum_{i=1}^{L} \beta_i \mathcal{O}_{21} e_{L,i} = 0,
\]

are satisfied if and only if \( \alpha_i = \beta_i = 0, \forall i = 1, \ldots, L \). Therefore, the equality

\[
\sum_{i=1}^{L} \pi_i H_x (k + j) e_{L,i} = 0, \quad \forall j = 0, \ldots, 3 \tag{35}
\]

is satisfied if and only if \( \pi_i = 0, \forall i = 1, \ldots, L \). The left-hand side of (31) can be expressed as

\[
\sum_{i=1}^{2L} \gamma_i \mathcal{O}(k, k, 4) e_{2L,i} = [\rho_0^T, \ldots, \rho_3^T]^T, \tag{36}
\]

\[
\rho_j = \sum_{i=1}^{L} (\gamma_i + jT \gamma_{L+i}) H_x (k + j) e_{L,i}, \tag{37}
\]

where \( j = 0, \ldots, 3 \). It can be seen from (35) that \( \rho_j = 0 \) for all \( j = 0, \ldots, 3 \) if and only if

\[
\gamma_i + jT \gamma_{L+i} = 0, \quad \forall i = 1, \ldots, L; \quad \forall j = 0, \ldots, 3. \tag{38}
\]

Since (38) holds for all \( j = 0, \ldots, 3 \), then evaluating (38) at \( j = 0 \) yields

\[
\gamma_i = 0, \quad \forall i = 1, \ldots, L. \tag{39}
\]

Combining (38) and (39) for \( j > 0 \) yields

\[
\gamma_{L+i} = 0, \quad \forall i = 1, \ldots, L. \tag{40}
\]

Equations (38)–(40) imply (31); therefore, \( \mathcal{O}(k, k, 4) \) is full rank and the system is observable.

**Remark.** The result of Theorem V.1 is only valid locally and in a deterministic sense, i.e., with no process or measurement noise. However, this result can be extended to the stochastic system as well. Let the measurement Jacobian \( \mathbf{G} \) with respect to the position states (cf. (6)) be re-parameterized in terms of the bearing angles \( \{\theta_n\}_{n=1}^N \) between each SOP and the UAV according to

\[
\mathbf{G}(k) = \begin{bmatrix}
\cos[\theta_1(k)] & \ldots & \cos[\theta_N(k)] \\
\sin[\theta_1(k)] & \ldots & \sin[\theta_N(k)]
\end{bmatrix}^T.
\]

The presence of process noise will yield new bearing angle trajectories \( \theta_n'(k) = \theta_n(k) + \delta\theta_n(k) \). With assumptions A.1 and A.2, the new bearing angles will not change the rank properties of \( \mathcal{O}_{11} \) and \( \mathcal{O}_{21} \); which implies that the stochastic system is observable.

### B. Lower Bound on the EKF Estimation Error Covariance

The optimal geometric configuration of sensors (or navigation sources) around an emitter (or receiver) has been well studied in the literature [32], [33]. It was found that in the presence of independent and identically distributed measurement noise, the trace of the estimation error covariance in a nonlinear least-squares estimator is minimized when the end points of the unit line of sight vectors pointing from the receiver to each navigation source form a regular polygon around the receiver, i.e., \( \theta_n = \frac{2\pi(n-1)}{N}, n = 1, \ldots, N \geq 3 \) [34]. The aforementioned configuration will be referred to as the optimal configuration.

Although the system discussed in Subsection III-B is nonlinear, one may devise a scenario for \( N \geq 3 \) that will define a lower bound on the estimation error covariance in the EKF. To this end, it is assumed that the optimal SOP configuration around the receiver is maintained at all time, implying that assumption A1 is satisfied. Assumption A2 implies that the measurement Jacobian cannot be the same at all time. In order to satisfy A1 and A2 simultaneously, it is assumed that optimal configuration is maintained and that the SOPs rotate around the receiver on the unit circle by \( 2\pi/N \) at each time-step. Therefore, the optimal bearing angles at any given time-step \( k \) will be given by

\[
\theta^*_n(k) = \frac{2\pi \cdot \text{mod}(n-1+k,N)}{N}, n = 1, \ldots, N, k = 1, \ldots,
\]

where \( \text{mod}(\cdot, \cdot) \) is the modulo operator. Note that this parametrization is independent of the state. Therefore, the Riccati equation may be iterated off-line to produce a lower bound on the estimation error covariance in the EKF denoted \( \mathbf{P}_{\text{min}}(k+1|k) \), from which a real number \( \mathbf{p} > 0 \) such that

\[
\mathbf{p} \mathbf{I} \preceq \mathbf{P}_{\text{min}}(k+1|k)
\]

can be deduced. It is also important to note that while this scenario could never be physically realized, it is only used to define a lower bound on the estimation error covariance.

### C. EKF Estimation Error Bounds Analysis

From the system defined in (3)–(4), it can be seen that \( \mathbf{F}(k) = \mathbf{F} \) is nonsingular and

\[
\|\mathbf{F}\| = 1, \tag{41}
\]

for all \( k \geq 0 \). Moreover, from the definition of \( \mathbf{Q} \) and \( \mathbf{R} \) in Subsection III-B, it can be seen that \( \mathbf{Q}(k) = \mathbf{Q} \succ 0 \) and \( \mathbf{R}(k) \succ 0 \); hence, there exist real numbers \( \underline{\mathbf{Q}}, \underline{\mathbf{R}} > 0 \) such that

\[
\underline{\mathbf{Q}} \preceq \mathbf{Q}(k), \quad \mathbf{R}(k) \preceq \underline{\mathbf{R}}, \tag{42}
\]

for all \( k \geq 0 \). It was established in Theorem V.1 that the system is observable, hence there exist real numbers \( \underline{\mathbf{p}}, \bar{\mathbf{p}} > 0 \) such that

\[
\underline{\mathbf{p}} \mathbf{I} \preceq \mathbf{P}(k+1|k) \preceq \bar{\mathbf{p}} \mathbf{I}, \quad \forall \mathbf{k} \geq 0, \tag{43}
\]

An approach for obtaining \( \underline{\mathbf{p}} \) is given in Subsection V-B. Since the dynamics of the system in (3) are linear, then

\[
\|\varphi(k)\| = 0, \quad \forall \mathbf{k} \geq 0. \tag{44}
\]
The following two lemmas establish the rest of the conditions for Theorem IV.2 to hold.

**Lemma V.1.** The 2-norm of the measurement Jacobian defined in (5) is bounded by

$$
\|H(k)\| \leq \sqrt{N+1},
$$

(45)

for all $k \geq 0$.

**Proof.** Equation (45) follows from showing that

$$
H^T(k)H(k) \preceq (N+1)I.
$$

(46)

The matrix $\Delta \triangleq (N+1)I - H^T(k)H(k)$ is expressed as

$$
\Delta = \begin{bmatrix}
    M & 0 \\
    0 & (N+1)I
\end{bmatrix},
$$

$$
M \triangleq \begin{bmatrix}
    (N+1)I - G^T(k)G(k) & -G^T(k) \\
    -G(k) & NI
\end{bmatrix},
$$

which implies that (46) is satisfied when $M \succeq 0$. Since $NI \succ 0$, then $M$ is positive semi-definite if the Schur complement of its bottom-right block given by

$$
M_{\text{Schur}} \triangleq (N+1)I - G^T(k)G(k) - \frac{1}{N}G^T(k)G(k),
$$

is positive semi-definite. For any matrix $A$, the following holds

$$
A^TA \preceq \text{trace}(A^TA) I.
$$

It can be readily shown that $\text{trace}(G^T(k)G(k)) = N$ for all $k \geq 0$. Subsequently,

$$
M_{\text{Schur}} \succeq (N+1)I - NI = 0,
$$

which implies that $M \succeq 0$, yielding (46) and consequently (45).

**Lemma V.2.** Consider the system defined in (3)–(4). If $A^3$ holds, then

$$
\max_{1 \leq n \leq N} \sup_{x(k)} \|\text{Hess } h_n[x(k)]\| \leq \frac{1}{d},
$$

(47)

where Hess denotes the Hessian operator.

**Proof.** It can be readily shown that

$$
\text{Hess } h_n[x(k)] = \frac{1}{\|r_x(k) - r_{sn}\|} \text{diag}[U, 0_{(2N+2) \times (2N+2)}],
$$

where $U \triangleq I_{2 \times 2} - vv^T$ and $v \triangleq \frac{r_x(k) - r_{sn}}{\|r_x(k) - r_{sn}\|}$. It can be seen that the matrix $U$ is an annihilator matrix and therefore its eigenvalues consist of ones and zeros. Subsequently,

$$
\|\text{Hess } h_n[x(k)]\| = \frac{1}{\|r_x(k) - r_{sn}\|}.
$$

Since $A^3$ holds, i.e., $\|r_x(k) - r_{sn}\| \geq d$, then

$$
\|\text{Hess } h_n[x(k)]\| \leq \frac{1}{d},
$$

which in turn implies (47).

Using Taylor’s theorem and Lemma V.2, it can be deduced that

$$
\|\chi(k)\| \leq \kappa_x \|\hat{x}(k)\|^2,
$$

(48)

where $\kappa_x = \frac{1}{d}$ [31]. Now the main result for the EKF error bounds is stated.

**Theorem V.2.** Consider the system defined in (3)–(4) whose state is being estimated using an EKF as described in Subsection III-B. If $A1$–$A3$ hold, then the EKF error $\hat{x}(k|k)$ is exponentially bounded in the mean square and bounded with probability one assuming

$$
\|\hat{x}(0)\| \leq \epsilon, \ R(k) \preceq \delta I, \ Q \preceq \delta I,
$$

for some $\epsilon, \delta > 0$.

**Proof.** Combining (41)–(45), (48), and the fact that $F$ is nonsingular, one can see that all the conditions of Theorem IV.2 are satisfied, from which one concludes that $\hat{x}(k|k)$ is exponentially bounded and bounded with probability one.

**VI. EXPERIMENTAL RESULTS**

In this section, two experiments are conducted demonstrating UAV navigation via the framework developed in this paper. In the following experiments, the altitude of the UAVs was known from their on-board navigation system.

**A. Measurement Noise Statistics**

The CDMA and LTE receivers employed in the experiments use second-order coherent phase lock loops (PLLs), for which it can be shown that the measurement noise variance $\sigma_n^2$ is given by

$$
\sigma_n^2 = 2B_{\text{eq}} + \frac{B_{\text{PLL}}}{C/N_0},
$$

where $B_{\text{PLL}}$ is the receiver’s PLL noise equivalent bandwidth and $C/N_0$ is the $n$-th cellular SOP’s carrier-to-noise ratio measured by the receiver [35]. In the following experiments, $B_{\text{PLL}}$ was set to 3 Hz.

**B. Hardware and Filter Description**

An Autel Robotics X-Star Premium UAV was used for the first experiment and a DJI Matrice 600 was used for the second experiment. In each experiment, the UAVs were equipped with an Ettus E312 universal software radio peripheral (USRP), a consumer-grade 800/1900 MHz cellular antenna, and a small consumer-grade GPS antenna to discipline the on-board oscillator. The UAV-mounted receivers were tuned to a 882.75 MHz carrier frequency (i.e., $\lambda = 33.96$ cm), which is a cellular CDMA channel allocated for the U.S. cellular provider Verizon Wireless for both experiments. In the second experiment, the UAV was equipped with a second antenna and another E312 USRP, which was tuned to a 1955 MHz carrier frequency (i.e., $\lambda = 15.33$ cm), which is an LTE channel allocated for the U.S. cellular provider AT&T. Samples of the received signals were stored for offline post-processing. The cellular carrier phase measurements were taken at 0.267 Hz, i.e., $T = 0.0267$ ms. The ground-truth reference for each UAV trajectory was taken from its on-board navigation system, which uses GPS, an inertial measurement unit (IMU), and other sensors. The E312 USRPs are equipped with temperature-compensated crystal oscillators (TCXOs) with $h_{0, r} = 2 \times 10^{-19}$ and $h_{-2, r} = 2 \times 10^{-20}$. It was assumed that the cellular SOPs are equipped with oven-controlled crystal oscillators (OCXOs) with $h_{0, sn} = 8 \times 10^{-20}$ and $h_{-2, sn} = 4 \times 10^{-23}$. The $x$ and $y$ continuous-time acceleration noise spectra were set to $q_x = q_y = 0.03 m^2/s^3$ for both experiments. The EKF was initialized according to the
framework in Subsection III-C with initial position estimates obtained from the UAVs’ on-board navigation systems. The experimental setup and BTS and eNodeB layout is shown in Fig. 1. All BTS and eNodeB positions were mapped prior to the experiment according to the framework discussed in [33].

![Experimental setup and BTS and eNodeB layout. The environment consists of 9 cellular CDMA BTSs (cyan towers) and 2 LTE eNodeBs (magenta towers).](image)

C. Experiment 1: UAV Navigation Results

In the first experiment, the UAV’s total traversed trajectory was 2.6 km, which was completed in 4 minutes and 40 seconds. Over the course of the experiment, the UAV-mounted receiver was listening to 7 cellular CDMA BTSs as shown in Fig. 1 (denoted BTSs 1–7). Fig. 2 shows the true and estimated UAV trajectories. The total position RMSE was found to be 2.94 m with a final estimation error at the end of the UAV’s flight of 2.23 m. The EKF position error and the associated ±σ bounds as well as the position ±σ lower bound (LB) obtained according to Subsection V-B are shown in Fig. 3.

![Experiment 1: True UAV trajectory and estimated UAV trajectory via cellular carrier phase measurements with the proposed EKF framework. The true and estimated trajectories are shown in solid and dashed lines, respectively. Map data: Google Earth.](image)

D. Experiment 2: UAV Navigation Results

In the second experiment, the UAV’s total traversed trajectory was 3.07 km, which was completed in 5 minutes and 25 seconds. In this experiment, the receiver on-board the UAV was listening to 9 cellular CDMA BTSs and 2 LTE eNodeBs shown in Fig. 1. The true and estimated UAV trajectories are shown in Fig. 4. The total position RMSE was found to be 5.99 m with a final estimation error at the end of the UAV’s flight of 3.46 m. The EKF position error and the associated ±σ bounds as well as the position ±σ lower bound (LB) obtained according to Subsection V-B are shown in Fig. 5.

![Experiment 2: True UAV trajectory and estimated UAV trajectory via cellular carrier phase measurements with the proposed EKF framework. The true and estimated trajectories are shown in solid and dashed lines, respectively. Map data: Google Earth.](image)

**Remark.** The EKF employs statistical models to propagate the position and velocity of the UAV and the clock bias and drift differences. Such models will inherently mismatch the true dynamics of the UAV and clock states, possibly...
yielding large estimation errors. Using an IMU to propagate the position and velocity states of the UAV should yield better results [5]. Moreover, an adaptive filter may be employed to simultaneously estimate the clock states’ process noise covariance to reduce the clock model mismatch [36].

VII. Conclusion

This paper presented a framework for UAV navigation with asynchronous cellular signals. The framework employs precise cellular carrier phase measurements and an EKF to estimate the position and velocity of the UAV as well as the difference of the biases and drifts between the receiver’s and each cellular SOP’s clock. An EKF initialization scheme was also proposed. Moreover, it was shown that (1) this framework is observable and (2) the EKF error state is asymptotically stable in a mean square sense and bounded with probability one. Moreover, it was shown that the EKF’s estimation error covariance is lower and upper-bounded. Two sets of experimental results on two different UAVs showed that this framework can achieve 2.94 and 5.99 m position RMSE, over UAV trajectories of 2.6 km and 3.07 km, respectively.

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References


