

A Dual-Phase Power Allocation Scheme for Multicarrier Relay System With Direct Link

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Abstract—We present optimization algorithms for source and relay power allocations in a multicarrier relay system with direct link, where the source power is allowed to transmit in both phases in a two-phase relay scheme. We show that there is a significant benefit to the system capacity by allowing the source power to be distributed over both phases. Specifically, we consider the joint optimization of source and relay power to minimize a general cost function. The joint optimization problem is non-convex and the complexity of finding the optimal solution is extremely high. Using the alternating optimization (AO) method, the joint problem is decomposed into a convex source power allocation problem and a non-convex relay power allocation problem. By exploiting the specific structure of the problem, we present efficient algorithms that yield the exact optimal solutions for both source and (non-convex) relay power allocation problems. Then we show that the overall AO algorithm converges to a stationary point of the joint problem. Moreover, the proposed AO algorithm is asymptotically optimal for large relay transmit power or large source-relay channel gain. Finally, simulations show that the proposed AO algorithm achieves significant gain over various baselines.

Index Terms—Multicarrier relay system, multiuser relay system, OFDM relay system, optimal power allocations, three-node relay system.

I. INTRODUCTION

A three-node relay system as shown in Fig. 1 has attracted much attention in the literature. While the capacity of such a system remains an open problem, various attempts have been focused on specific relay schemes and their achievable regions. One of the most recent works is in [1] where the authors developed algorithms to maximize the relay system throughput by assuming that the relay node is non-regenerative and the powers at the source node and the relay node can be controlled over multiple subcarriers. Specifically, the relay communication in [1] is achieved in two phases. In the first phase, source node transmits signal to relay. In the second phase, relay amplifies and transmits the signal received from the first phase to the destination node. We call the first phase *source phase* and the second phase *relay phase*. In [1], there is no direct link between the source node and the destination node. As a result, the source

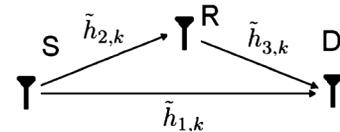


Fig. 1. A multicarrier three-node relay system, where $k = 0, 1, \dots, N - 1$ is the index of subcarriers, and the source transmits in both phases 1 and 2. S, R and D in the figures stands for source node, relay node and destination node, respectively.

node is always silent during the relay phase. With the presence of direct link between, the source node has an additional freedom to distribute its power among the two phases, instead of only transmitting in the source phase. This paper presents a power allocation algorithm to minimize a general cost function for the three-node relay systems with direct link. Moreover, we allow the source to transmit in both source and relay phase.

The effect of the direct link has also been considered in [2]–[8]. But in all these works, the source node and the relay node each transmit only in one of two separate time slots¹ for each carrier. In fact, except for [8], all other works mentioned above only allow the source to transmit in the source phase. Many other prior works such as [9]–[12] do not consider the direct link. More recent works on relays can be found in [13].

In this paper, we let the source repeat a transmission of the same information (transmitted in the source phase) to the destination in the relay phase. In other words, we split the total source power into two slots for transmitting the same information. This additional freedom makes the required power allocation algorithms differ from those published before. The achieved capacity is higher than the case where the source power is only limited to one of two time slots for each carrier, which is a special case of our relay scheme. However, this new scheme introduces several technical challenges as elaborated below.

- **More Complicated Non-convex Optimization Objective:** Since source node can transmit in both phases, we need to consider the joint optimization of source power allocation at both phases and relay power allocation at the relay phase. This not only increases the number of optimization variables but also makes the optimization objective a more complicated non-convex function.
- **More Complex Coupling between Source and Relay Power Allocation:** The source and relay power optimization is coupled in a more complex way as the destination node gets signal from both source and relay nodes during the relay phase.

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¹We use “phase” and “slot” interchangeably.

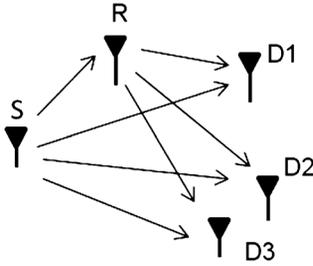


Fig. 2. A multiuser application of the relay system shown in Fig. 1 where the destination nodes (users) can select different subcarriers embedded in OFDM frames in each phase.

From an application point of view, the three-node multicarrier relay system shown in Fig. 1 can be a simple abstraction of a multiuser multicarrier downlink relay system shown in Fig. 2 where the destination nodes can select different subcarriers in each phase. In the multiuser case, the channel gains of a link among different subcarriers are generally diverse, and hence the power allocations at both the source and relay nodes over all subcarriers are particularly important. The proposed algorithm can also be used to solve the power allocation problems in the multiuser multicarrier relay systems for given subcarrier allocation.

Several of the previous works such as [2]–[7] considered MIMO relay systems. The work in this paper is limited to single-antenna nodes. While our contributions do not entirely generalize the previous works on MIMO relay systems, the insight shown in this paper could help achieve that goal in the future.

In the next section, we present in detail the system model and the problem formulation. The problem formulated is a joint optimization of source power allocation over two phases and relay power allocation in the second phase. Two types of cost functions are considered. Using the alternating optimization (AO) method [11], the joint optimization problem is decomposed into two subproblems, namely the optimization of relay power allocation for given source power allocation, and the optimization of source power allocation for given relay power allocation. These two subproblems are respectively addressed in Section III and Section IV. Then the overall AO algorithm and the optimality analysis is given in Section V. Although the exact solution to the joint optimization problem is not guaranteed, the AO algorithm converges to a stationary point which is asymptotically optimal for large relay transmit power or large source-relay channel gain. Simulation examples are given in Section VI to illustrate the superior performance of the proposed AO algorithm.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The relay architecture is shown in Fig. 1. We consider an OFDM based multicarrier system. Each packet of information is encoded into N independent complex symbols, \tilde{x}_k , $k = 0, 1, \dots, N - 1$, of zero mean and unit variance. To transmit a packet of information from the source to the destination, the relay scheme has two phases.

In source phase, the source transmits $\tilde{x}_{1,k} = \sqrt{p_{1,k}}\tilde{x}_k$, $k = 0, 1, \dots, N - 1$, over N subcarriers towards the destination and the relay, the relay does not transmit but receives

$\tilde{y}_{r,k}$, $k = 0, 1, \dots, N - 1$, and the destination receives $\tilde{y}_{d1,k}$, $k = 0, 1, \dots, N - 1$. The power of $\tilde{x}_{1,k}$ is $p_{1,k}$.

In relay phase, the source transmits again but it transmits $\tilde{x}_{2,k} = \sqrt{p_{2,k}}\tilde{x}_k$, $k = 0, 1, \dots, N - 1$, the relay does not receive but, concurrently with the source, transmits $\tilde{x}_{r,k} = \tilde{g}_k\tilde{y}_{r,k}$, $k = 0, 1, \dots, N - 1$, and the destination receives $\tilde{y}_{d2,k}$, $k = 0, 1, \dots, N - 1$. The relay factor \tilde{g}_k has the amplitude g_k and the phase β_k . The power of $\tilde{x}_{2,k}$ is $p_{2,k}$.

The channel coefficient of the k -th subcarrier from the source to the destination is denoted by $\tilde{h}_{1,k}$, that from the source to the relay is $\tilde{h}_{2,k}$, and that from the relay to the destination is $\tilde{h}_{3,k}$. We use $h_{l,k}$ as the amplitude of $\tilde{h}_{l,k}$, and $\alpha_{l,k}$ the phase of $\tilde{h}_{l,k}$. For clarity, we assume that all the channel coefficients are non-zero. The algorithms developed in this paper can be easily extended to solve the cases when some of the channel coefficients are zero.

The N subcarriers used in source phase for $\tilde{x}_{1,k}$ are indexed by $k = 0, 1, \dots, N - 1$. But the N subcarriers used in relay phase for $\tilde{x}_{2,k}$ (from source to destination) and $\tilde{x}_{r,k}$ (from relay to destination) are indexed by $\bar{k} = \bar{0}, \bar{1}, \dots, \bar{N} - \bar{1}$, which can be a permutation of those used in source phase. Namely, \bar{k} is a function of k via a given permutation. Permutation can be used to model the subcarrier allocation in multi-user relay systems, where a user may be allocated with a different subcarrier in the relay phase.

Now, we can write that in source phase, the destination and the relay receive, respectively,

$$\tilde{y}_{d1,k} = \tilde{h}_{1,k}\tilde{x}_{1,k} + \tilde{n}_{d1,k} \quad (1)$$

$$\tilde{y}_{r,k} = \tilde{h}_{2,k}\tilde{x}_{1,k} + \tilde{n}_{r,k} \quad (2)$$

and in relay phase, the destination receives

$$\tilde{y}_{d2,k} = \tilde{h}_{3,\bar{k}}\tilde{x}_{r,k} + \tilde{h}_{1,\bar{k}}\tilde{x}_{2,k} + \tilde{n}_{d2,\bar{k}} \quad (3)$$

where \bar{k} in $\tilde{h}_{3,\bar{k}}$ and $\tilde{h}_{1,\bar{k}}$ of (3) is a permuted version of k in $\tilde{h}_{1,k}$ and $\tilde{h}_{2,k}$ of (1) and (2). All the noise terms $\tilde{n}_{d1,k}$, $\tilde{n}_{r,k}$ and $\tilde{n}_{d2,k}$ are assumed to be independent of each other and across subcarriers, and have zero mean and unit variance. Note that there is no loss of generality to assume the unit variance. If the variance of $\tilde{n}_{d1,k}$ and $\tilde{n}_{d2,k}$ is γ_d , we can divide (1) and (3) by $\sqrt{\gamma_d}$ and adjust the notations. And if the variance of $\tilde{n}_{r,k}$ is γ_r , we can divide (2) by $\sqrt{\gamma_r}$ and adjust the notations.

Combining all the previous equations, we can write the received signals in both phases as a vector:

$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{h}}_k\tilde{\mathbf{x}}_k + \tilde{\mathbf{n}}_k \quad (4)$$

with $\tilde{\mathbf{y}}_k = [\tilde{y}_{d1,k}, \tilde{y}_{d2,k}]^T$ and

$$\tilde{\mathbf{h}}_k = \begin{bmatrix} \sqrt{p_{1,k}}\tilde{h}_{1,k} & \sqrt{p_{1,k}}\tilde{g}_k\tilde{h}_{2,k}\tilde{h}_{3,\bar{k}} + \sqrt{p_{2,k}}\tilde{h}_{1,\bar{k}} \end{bmatrix}^T \quad (5)$$

$$\tilde{\mathbf{n}}_k = \begin{bmatrix} \tilde{n}_{d1,k} & \tilde{g}_k\tilde{h}_{3,\bar{k}}\tilde{n}_{r,k} + \tilde{n}_{d2,\bar{k}} \end{bmatrix}^T \quad (6)$$

Since the covariance matrix of the noise vector $\tilde{\mathbf{n}}_k$ is $\mathbf{C}_k = \text{diag}(1, g_k^2 h_{3,\bar{k}}^2 + 1)$, a sufficient statistics for $\tilde{\mathbf{x}}_k$ is

$$\hat{\tilde{\mathbf{x}}}_k = \tilde{\mathbf{h}}_k^H \mathbf{C}_k^{-1} \tilde{\mathbf{y}}_k \quad (7)$$

One can then verify that the SNR of \hat{x}_k is

$$\text{SNR}_k = p_{1,k} h_{1,k}^2 + \frac{|\sqrt{p_{1,k}} \tilde{g}_k \tilde{h}_{2,k} \tilde{h}_{3,\bar{k}} + \sqrt{p_{2,k}} \tilde{h}_{1,\bar{k}}|^2}{g_k^2 h_{3,\bar{k}}^2 + 1}. \quad (8)$$

It follows from (8) that $\beta_k = \alpha_{1,\bar{k}} - \alpha_{2,k} - \alpha_{3,\bar{k}}$ is the phase of \tilde{g}_k that maximizes SNR_k . With the optimal phase β_k , (8) becomes

$$\text{SNR}_k = p_{1,k} h_{1,k}^2 + \frac{(\sqrt{p_{1,k}} g_k h_{2,k} h_{3,\bar{k}} + \sqrt{p_{2,k}} h_{1,\bar{k}})^2}{g_k^2 h_{3,\bar{k}}^2 + 1}. \quad (9)$$

Let source node transmit in relay phase will result in a better SNR especially when relay power is limited or direct channel is strong. The simulation results also show that the optimized $p_{2,k}$'s are typically larger than zero. A more rigorous discussion will be given at the end of Section IV.

We will consider two types of cost functions. The first is

$$J_1 = -\min_k \text{SNR}_k. \quad (10)$$

This cost could be particularly useful if each of the N subcarriers is occupied by a distinct downlink user (see Fig. 2) and we want to ensure a fair quality for all users. The second cost is

$$J_2 = \sum_{k=0}^{N-1} f_k(\text{SNR}_k), \quad (11)$$

where $f_k(\text{SNR}_k)$ is any decreasing convex function of $\text{SNR}_k > 0$, i.e., $f'_k \doteq (\partial f_k(x)/\partial x)|_{x=\text{SNR}_k} < 0$ and $f''_k \doteq (\partial^2 f_k(x)/\partial^2 x)|_{x=\text{SNR}_k} > 0$. A special form of J_2 is

$$J_2 = -\frac{1}{2N} \sum_{k=0}^{N-1} \log_2(1 + \text{SNR}_k), \quad (12)$$

which is the negative of the capacity of the relay system under the previously defined relay scheme. Clearly, there are many more variations of the cost functions. But we hope that our results for J_1 and J_2 can help solve other related problems.

The joint source and relay power allocation problem is formulated as:

$$\min_{p_{1,k}, p_{2,k}, g_k, \forall k} J. \quad (13)$$

subject to

$$\sum_{k=0}^{N-1} (p_{1,k} + p_{2,k}) \leq P_s, \quad (14)$$

$$\sum_{k=0}^{N-1} (g_k^2 h_{2,k}^2 p_{1,k} + g_k^2) \leq P_r. \quad (15)$$

along with $g_k \geq 0$, $p_{1,k} \geq 0$ and $p_{2,k} \geq 0$. Here, J is either J_1 or J_2 , P_r is the power constraint at the relay, and P_s is the sum power constraint at the source.

The above problem differs from those in [3]–[7] in a fundamental way. As mentioned earlier, all those references assume

that the source node is silent during the relay phase. As shown later in this paper, by allowing the source power to be distributed in both phases, a substantial performance gain can be achieved.

The exact solution to (13) is difficult to find because both the cost function and the relay power constraint in (15) are non-convex. In this paper, we provide a local optimal solution by adopting an alternating optimization approach where we optimize $g_k \forall k$ with previously given $p_{1,k}$ and $p_{2,k} \forall k$, then optimize $p_{1,k}$ and $p_{2,k} \forall k$ with previously given $g_k \forall k$, and then repeat the process until convergence. For fixed source power allocation $p_{1,k}, p_{2,k} \forall k$, the relay power allocation problem (i.e., the optimization of $g_k \forall k$) is still non-convex. However, by exploiting the specific structure of the SNR expression in (9), we propose efficient algorithms to find the unique optimal solution for this non-convex problem in Section III. For fixed relay amplifying factor g_k , the dual-phase source power allocation problem (i.e., the optimization of $p_{1,k}, p_{2,k} \forall k$) is convex and we propose efficient algorithms to find the optimal solution in Section IV. The overall algorithm to solve Problem (13) is given in Section V.

III. OPTIMAL RELAY POWER ALLOCATION GIVEN SOURCE POWER ALLOCATION

For fixed $p_{1,k}$ and $p_{2,k}$, Problem (13) becomes

$$\min_{g_k \geq 0, \forall k} J, \quad (16)$$

subject to

$$\sum_{k=0}^{N-1} c_k g_k^2 \leq P_r, \quad (17)$$

where $c_k = h_{2,k}^2 p_{1,k} + 1$.

Neither J_1 nor J_2 is a convex functions of $g_k \forall k$. In particular, the SNR_k in (9) is not a concave function of g_k . However, it is easy to verify that

$$\frac{d\text{SNR}_k}{dg_k} = 2(\sqrt{p_{1,k}} g_k h_{2,k} h_{3,\bar{k}} + \sqrt{p_{2,k}} h_{1,\bar{k}}) \times \frac{(\sqrt{p_{1,k}} h_{2,k} h_{3,\bar{k}} - \sqrt{p_{2,k}} g_k h_{1,\bar{k}} h_{3,\bar{k}})}{(g_k^2 h_{3,\bar{k}}^2 + 1)^2}. \quad (18)$$

It follows that SNR_k is a quasi-concave function of g_k , which has a unique maximum at²

$$g_k^* = \frac{h_{2,k} \sqrt{p_{1,k}}}{h_{1,\bar{k}} h_{3,\bar{k}} \sqrt{p_{2,k}}}. \quad (19)$$

Namely, SNR_k is an increasing function of g_k for $g_k \in [0, g_k^*]$ and a decreasing function of g_k for $g_k \in [g_k^*, \infty)$. Because of the power constraint (17), we only need to search the optimal g_k within $[0, g_k^*]$. A typical plot of SNR_k is shown in Fig. 3.

Note that if the source does not transmit in relay phase, then $p_{2,k} = 0$ and hence SNR_k is simply an increasing concave function of g_k (and J_1 and J_2 are convex functions of $g_k \forall k$).

²Note that if $p_{2,k} = 0$, we let $g_k^* = \infty$.

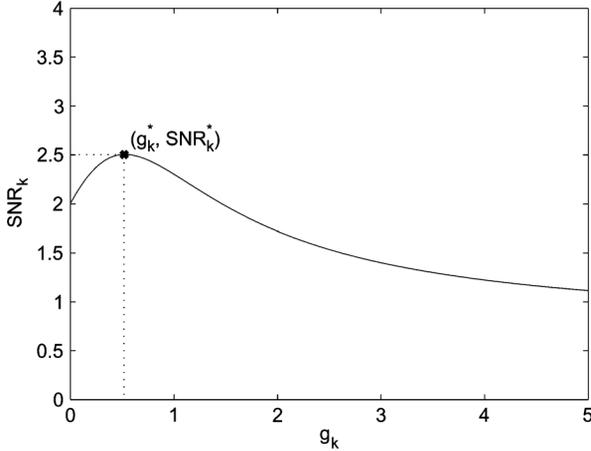


Fig. 3. A typical plot of SNR_k vs. g_k where g_k^* is the position of the peak value SNR_k^* .

A. Using the Cost $J = J_1$

Recall $J_1 = -\min_k \text{SNR}_k$ and $\min_{\{g_k \forall k\}} J_1 = \max_{\{g_l \forall l\}} \min_k \text{SNR}_k$. Denote $\text{SNR}_k^* = \text{SNR}_k|_{g_k=g_k^*}$. Also denote the solution to (16) by \hat{g}_k and the corresponding optimal SNR by $\widehat{\text{SNR}}_k \triangleq \text{SNR}_k|_{g_k=\hat{g}_k}$. From (9), if we let $g_k = 0$, we have the corresponding $\text{SNR}_k = p_{1,k}h_{1,k}^2 + p_{2,k}h_{1,\bar{k}}^2$. Since SNR_k is a monotonic increasing function of g_k over $[0, g_k^*]$, we must have:

$$p_{1,k}h_{1,k}^2 + p_{2,k}h_{1,\bar{k}}^2 \leq \widehat{\text{SNR}}_k \leq \text{SNR}_k^*, \forall k. \quad (20)$$

Also, because SNR_k is an increasing function of g_k for $g_k \in [0, g_k^*]$, it follows that

$$\widehat{\text{SNR}}_k = \min_l \widehat{\text{SNR}}_l, \forall k \in \{0 \leq k' \leq N-1 : \hat{g}_{k'} > 0\}.$$

It is easy to verify from (9) that for each $\mu > 0$, the equation $\text{SNR}_k = \mu$ can be reduced to a quadratic equation of g_k , which has two (closed-form) solutions $g_k^{(1)}$ and $g_k^{(2)}$. Furthermore, if $0 < \mu \leq \text{SNR}_k^*$, one (and only one value) of $g_k^{(1)}$ and $g_k^{(2)}$ is less than or equal to g_k^* , which is denoted by $g_k(\mu)$.

With the above discussions, one can verify that the following algorithm yields the exact solution to (16) with $J = J_1$:

Algorithm 1

- 1) Compute g_k^* , SNR_k^* and $\mu^* = \min_k \text{SNR}_k^*$.
 - 2) For each k , solve the equation $\text{SNR}_k = \mu^*$ to obtain $g_k(\mu^*)$. If (17) is satisfied with $g_k = \max(0, g_k(\mu^*))$, $\forall k$, then set $\hat{g}_k = \max(0, g_k(\mu^*))$, $\forall k$ and stop. Otherwise, go to the next step.
 - 3) Use the bisection search method³ [14] to find $\hat{\mu}$ within $(0, \mu^*)$ such that (17) is satisfied with equality as $\sum_{k=0}^{N-1} c_k g_k^2 = P_r$ and $g_k = \max(0, g_k(\hat{\mu}))$, $\forall k$. Then, set $\hat{g}_k = \max(0, g_k(\hat{\mu}))$, $\forall k$, and stop.
-

B. Using the Cost $J = J_2$

Recall $J_2 = \sum_k f_k(\text{SNR}_k)$ where $f_k(x)$ is a decreasing convex function of $x \geq 0$. To find the solution to (16), the KKT

conditions [14] are the necessary conditions (but not sufficient conditions due to non-convexity of J_2 with respect to $g_k \forall k$). But we can still distill the exact solution from the KKT conditions as follows. Among the KKT conditions,⁴ we have

$$f'_k \frac{d\text{SNR}_k}{dg_k} + 2\mu g_k c_k^2 - \gamma_k = 0, \forall k, \quad (21)$$

where $f'_k = \frac{df_k(\text{SNR}_k)}{d\text{SNR}_k} < 0$ and $\frac{d\text{SNR}_k}{dg_k}$ is given in (18), μ is the Lagrange multiplier associated with the relay power constraint in (17), and γ_k is the Lagrange multiplier for the constraint $g_k \geq 0$. Also, $\mu > 0$ if the equality in (17) holds, and $\gamma_k = 0$ if $g_k > 0$.

The following proposition characterizes the properties of the solution of the KKT conditions.

Proposition 1: The KKT conditions of the relay power allocation problem in (16) with $J = J_2$ has a unique solution denoted by $\hat{\mu}, \{\hat{\gamma}_k, \hat{g}_k : \forall k\}$. Moreover, $\hat{\mu}, \{\hat{\gamma}_k, \hat{g}_k : \forall k\}$ has the following properties:

- 1) $g_k = 0, \gamma_k = 0, \forall k \in \mathcal{K}_0 \triangleq \{k' : p_{1,k'} = 0\}$ and $g_k > 0, \gamma_k = 0, \forall k \in \bar{\mathcal{K}}_0 \triangleq \{k' : p_{1,k'} > 0\}$.
- 2) If $\sum_{k=0}^{N-1} c_k g_k^{*2} \leq P_r$, we have $\hat{g}_k = g_k^*, \forall k$; otherwise $\hat{g}_k = 0, \forall k \in \mathcal{K}_0$ and $\hat{\mu}, \{\hat{g}_k : \forall k \in \bar{\mathcal{K}}_0\}$ is the unique solution of the following equations:

$$-f'_k \frac{d\text{SNR}_k}{2g_k dg_k} = \mu c_k, \quad \forall k \in \bar{\mathcal{K}}_0, \quad (22)$$

$$\sum_{k \in \bar{\mathcal{K}}_0} c_k g_k^2 = P_r. \quad (23)$$

- 3) In (22), $-f'_k \frac{d\text{SNR}_k}{2g_k dg_k}$ is positive and decreasing with $g_k \in [0, g_k^*]$.
- 4) Let $g_k(\mu), \forall k \in \bar{\mathcal{K}}_0$ denote the solution of (22) with fixed μ . Then we have $g_k(\mu) \in [0, g_k^*]$ and $\sum_{k \in \bar{\mathcal{K}}_0} c_k g_k^2(\mu)$ is decreasing with μ .

Please refer to Appendix A for the proof.

By Result 3) in Proposition 1, for a given feasible μ , the solution $g_k(\mu), \forall k \in \bar{\mathcal{K}}_0$ of (22) can be easily found by a bisection search. On the other hand, if $\sum_{k=0}^{N-1} c_k g_k^{*2} > P_r$ the optimal Lagrange multiplier $\hat{\mu}$ is given by the solution of

$$\sum_{k \in \bar{\mathcal{K}}_0} c_k g_k^2(\mu) = P_r, \quad (24)$$

which can also be found by a bisection search according to the Result 4) in Proposition 1.

Based on Proposition 1, one can verify that the following algorithm yields the exact solution to Problem (16) with $J = J_2$:

Algorithm 2

- 1) Compute $g_k^* \forall k$. If (17) holds with $g_k = g_k^*$, set $\hat{g}_k = g_k^* \forall k$ and stop. Otherwise, go to the next step.
 - 2) Use a bisection search to find $\hat{\mu}$ within $[0, \infty]$ such that (24) is satisfied, where for a given μ , the $g_k(\mu), \forall k \in \bar{\mathcal{K}}_0$ in (24) is found as the solution to (22) via an inner-loop bisection search. Then set $\hat{g}_k = g_k(\hat{\mu})$ for all k .
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³It converges exponentially fast.

⁴For brevity, we do not list all KKT conditions. We assume that the reader understands the basic components of the KKT conditions.

IV. OPTIMAL SOURCE POWER ALLOCATION GIVEN RELAY POWER ALLOCATION

Given a relay power allocation (i.e., given a set of $g_k \forall k$), we now consider the optimization of the source power allocation by solving the following problem:

$$\min_{p_{1,k} \geq 0, p_{2,k} \geq 0, \forall k} J, \quad (25)$$

subject to (14) and (15), which is later proved convex.

 A. Using the Cost $J = J_1$

Since $J_1 = -\min_k \text{SNR}_k$, Problem (25) can be reformulated as

$$\min_{\theta, p_{1,k} \geq 0, p_{2,k} \geq 0, \forall k} -\theta, \quad (26)$$

subject to (14),(15) and

$$\text{SNR}_k \geq \theta, \quad \forall k. \quad (27)$$

It follows from (9) that

$$\text{SNR}_k = r_k p_{1,k} + s_k \sqrt{p_{1,k} p_{2,k}} + t_k p_{2,k}, \quad (28)$$

where $r_k = h_{1,k}^2 + \frac{g_k^2 h_{2,k}^2 h_{3,k}^2}{g_k^2 h_{3,k}^2 + 1}$, $s_k = \frac{g_k h_{1,k} h_{2,k} h_{3,k}}{g_k^2 h_{3,k}^2 + 1}$ and $t_k = \frac{h_{1,k}^2}{g_k^2 h_{3,k}^2 + 1}$. It is obvious that SNR_k is increasing with $p_{1,k}$ and $p_{2,k}$.

It is also obvious that the optimal $p_{1,k}$ and $p_{2,k}$ must be such that the equality in (14) is satisfied. If $p_{1,k}$ and $p_{2,k}$ are such that the strict inequality in (14) holds, we can increase some or all of $p_{2,k}$ such that the equality in (14) is achieved and θ is increased while (15) is not affected.

Furthermore, the optimal $p_{1,k}$ and $p_{2,k}$ must be such that $\text{SNR}_k = \theta, \forall k$. If there is SNR_l satisfying $\text{SNR}_l > \min_k \text{SNR}_k$, we can reduce $p_{1,l}$ to make $\text{SNR}_l = \min_k \text{SNR}_k$ without affecting θ and then we can increase some or all of $p_{2,k}$ such that the equality in (14) is achieved and θ is increased while (15) is not affected.

Applying the KKT conditions [14] to the problem (26), we have

$$-\lambda_k \left(r_k + \frac{s_k \sqrt{p_{2,k}}}{2\sqrt{p_{1,k}}} \right) + \mu_2 g_k^2 h_{2,k}^2 + \mu_1 = 0, \quad \forall k, \quad (29)$$

$$-\lambda_k \left(t_k + \frac{s_k \sqrt{p_{1,k}}}{2\sqrt{p_{2,k}}} \right) + \mu_1 = 0, \quad \forall k, \quad (30)$$

where λ_k is the k -th multiplier due to (27), μ_1 is the multiplier due to (14), and μ_2 is the multiplier due to (15). The discussions shown previously imply that $\mu_1 > 0$ and $\lambda_k > 0, \forall k$.

The following proposition gives the solution to the above KKT conditions for fixed μ_1, μ_2 .

Proposition 2: For fixed μ_1, μ_2 , the solution to the KKT conditions of Problem (26) with $J = J_1$ is given by

$$p_{1,k} = \begin{cases} 0, & t_k \gamma - r_k > 0, s_k = 0 \\ \frac{\theta}{r_k}, & t_k \gamma - r_k < 0, s_k = 0 \\ \frac{\theta}{r_k + \gamma t_k}, & t_k \gamma - r_k = 0, s_k = 0 \\ \frac{\theta}{r_k + \eta_k s_k + \eta_k^2 t_k}, & s_k > 0, \end{cases} \quad (31)$$

$$p_{2,k} = \begin{cases} \frac{\theta}{t_k}, & t_k \gamma - r_k > 0, s_k = 0 \\ 0, & t_k \gamma - r_k < 0, s_k = 0 \\ \frac{\theta}{\gamma r_k + t_k}, & t_k \gamma - r_k = 0, s_k = 0 \\ \eta_k^2 p_{1,k}, & s_k > 0, \end{cases} \quad (32)$$

where $\gamma = g_k^2 h_{2,k}^2 \frac{\mu_2}{\mu_1} + 1$; $\eta_k, \forall k \in \{k' : s_k > 0\}$ is given by

$$\eta_k = \sqrt{\frac{p_{2,k}}{p_{1,k}}} = \frac{1}{s_k} \left(t_k \gamma - r_k + \sqrt{(t_k \gamma - r_k)^2 + s_k^2 \gamma} \right), \quad (33)$$

and θ is chosen such that (14) is satisfied with equality.

Please refer to Appendix B for the proof.

By Proposition 2, for fixed μ_1, μ_2 , the solution of the KKT conditions only depends on the intermediate variable $\gamma = g_k^2 h_{2,k}^2 \frac{\mu_2}{\mu_1} + 1$. Hence, if the solution in Proposition 2 with $\gamma = 1$ (i.e., $\mu_2 = 0$) satisfies the relay power constraint (15), it is also the optimal solution for Problem (26) with $J = J_1$. Otherwise, we can use bisection search to find the γ such that the solution in Proposition 2 satisfies (15) with equality and this solution is optimal for Problem (26). Note that the probability for $s_k = r_k = t_k = 0$ is virtually zero. So, the last expressions in (31) and (32) should be stable numerically.

From the above analysis, we obtain the following algorithm which finds the exact solution to (26) with $J = J_1$:

Algorithm 3

- 1) Initialize $\gamma = 1$ and $\theta > 0$.
 - 2) Determine η_k from (33) for all k where $s_k > 0$.
 - 3) Determine $p_{1,k}$ and $p_{2,k}$ by (31) and (32).
 - 4) One step bisection:⁵ If the left side of (14) is smaller than its right side, increase⁶ θ . If the opposite is true, decrease θ . Go to step 3) until convergence.
 - 5) If $\gamma = 1$ and (15) is satisfied, stop. Otherwise, go to the next step.
 - 6) One step bisection:⁷ If the left side of (15) is larger than its right side, increase γ . If the opposite is true, decrease γ . Go to step 2) until convergence.
-

⁵The lower bound of θ is obviously zero. An upper bound of θ for each γ can be found by doubling its previous (nonzero) value each time initially as θ is increased consecutively in the bisection search.

⁶In bisection, "increase a variable" means "increase the variable halfway towards its nearest larger estimate obtained previously", and "decrease a variable" means "decrease the variable halfway towards its nearest smaller estimate obtained previously".

⁷An upper bound of γ can be found by doubling its previous (nonzero) value each time initially as γ is increased consecutively in the bisection search.

In our simulations, “convergence” means that the difference between the coefficients obtained in two consecutive iterations is less than a pre-set threshold.

B. Using the cost $J = J_2$

The cost function J_2 can be expressed as

$$J_2 = \sum_{k=0}^{N-1} f_k(r_k p_{1,k} + s_k \sqrt{p_{1,k} p_{2,k}} + t_k p_{2,k}). \quad (34)$$

Let $\mathbf{p} = [p_{1,0}, \dots, p_{1,N-1}, p_{2,0}, \dots, p_{2,N-1}]^T$ and let \mathbf{H} be the Hessian matrix of J_2 with respect to \mathbf{p} . Then, one can verify that for any real vector $\mathbf{v} \in \mathbb{R}^{2N}$,

$$\begin{aligned} \mathbf{v}^T \mathbf{H} \mathbf{v} = \sum_{k=0}^{N-1} \left\{ f_k'' \cdot \left(a_k \left[r_k + s_k \frac{\sqrt{p_{2,k}}}{2\sqrt{p_{1,k}}} \right] \right. \right. \\ \left. \left. + b_k \left[s_k \frac{\sqrt{p_{1,k}}}{2\sqrt{p_{2,k}}} + t_k \right] \right)^2 - f_k' \right. \\ \left. \cdot a_k^2 s_k \frac{\sqrt{p_{2,k}}}{4\sqrt{p_{1,k}^3}} - f_k' \cdot b_k^2 s_k \frac{\sqrt{p_{1,k}}}{4\sqrt{p_{2,k}^3}} \right\} \geq 0, \end{aligned} \quad (35)$$

where we have applied $f_k' < 0$ and $f_k'' > 0$. Hence, J_2 is convex with respect to \mathbf{p} . The KKT conditions of (25) with $J = J_2$ include

$$f_k' \cdot \frac{d\text{SNR}_k}{dp_{1,k}} + \mu_2 (g_k^2 h_{2,k}^2) + \mu_1 = 0, \quad \forall k, \quad (36)$$

$$f_k' \cdot \frac{d\text{SNR}_k}{dp_{2,k}} + \mu_1 = 0, \quad \forall k, \quad (37)$$

where, as discussed previously, $\mu_1 > 0$ and $\mu_2 \geq 0$. The following proposition gives the solution to the above KKT conditions for fixed μ_1, μ_2 .

Proposition 3: For fixed μ_1, μ_2 , the solution to the KKT conditions of Problem (26) with $J = J_2$ is given by

$$p_{1,k} = \begin{cases} 0, & t_k \frac{a_2}{a_1} - r_k > 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\frac{a_2}{a_1} \mu_1}{\frac{a_2}{a_1} t_k + \left(\frac{a_2}{a_1} - r_k \right)^2} \right) \right)^+}{\frac{r_k}{r_k + \frac{a_2}{a_1} t_k}}, & t_k \frac{a_2}{a_1} - r_k < 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k} \right) \right)^+}{\frac{a_2}{a_1} r_k + t_k}, & t_k \frac{a_2}{a_1} - r_k = 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k + \frac{s_k}{2\eta_k}} \right) \right)^+}{r_k + \eta_k s_k + \eta_k^2 t_k}, & s_k > 0 \end{cases} \quad (38)$$

$$p_{2,k} = \begin{cases} \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k} \right) \right)^+}{t_k}, & t_k \frac{a_2}{a_1} - r_k > 0 \\ 0, & t_k \frac{a_2}{a_1} - r_k < 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k} \right) \right)^+}{\frac{a_2}{a_1} r_k + t_k}, & t_k \frac{a_2}{a_1} - r_k = 0 \\ \eta_k^2 p_{1,k}, & s_k > 0 \end{cases} \quad (39)$$

where $\gamma = g_k^2 h_{2,k}^2 \frac{\mu_2}{\mu_1} + 1$, and $\eta_k, \forall k \in \{k' : s_k > 0\}$ is given in (33).

Please refer to Appendix C for the proof.

Summarizing the above analysis, we have the following algorithm for finding the exact solution to (25) with $J = J_2$:

Algorithm 4

- 1) Initialize $\gamma = 1$ and $\mu_1 > 0$.
- 2) Compute η_k from (33) for all k where $s_k > 0$. Compute $p_{1,k}$ and $p_{2,k}$ by (38) and (39) for all k .
- 3) One step bisection: if the left side of (14) is smaller than its right side, decrease μ_1 . If the opposite is true, increase μ_1 . Go to step 2) until convergence.
- 4) If $\gamma = 1$ and (15) is satisfied, stop. Otherwise, go to next step.
- 5) One step bisection: If the left side of (15) is larger than its right side, increase γ . If the opposite is true, decrease γ . Go to step 2) until convergence.

C. Discussion of the Optimal Source Power Ratio $\frac{p_{2,k}}{p_{1,k}}$

Some properties of the optimal source power ratio $\frac{p_{2,k}}{p_{1,k}}$ are shown below.

- **Conditions for non-zero source transmit power in the relay phase:** For any given $g_k \forall k$, the optimal $\frac{p_{2,k}}{p_{1,k}}$ is given in (33), which is larger than zero if $s_k > 0$. It follows from the definition of s_k , shown below (28), that $s_k > 0$ if and only if $g_k > 0$, $h_{1,\bar{k}} > 0$, $h_{2,k} > 0$ and $h_{3,\bar{k}} > 0$. If $s_k = 0$, it follows from (28) (i.e., $\max_{p_{1,k}, p_{2,k}} \text{SNR}_k$ subject to $p_{1,k} + p_{2,k} \leq p_k$ where p_k is any amount of power for the k th subcarrier) that $p_{1,k} = p_k$ and $p_{2,k} = 0$ if $r_k > t_k$, or otherwise $p_{1,k} = 0$ and $p_{2,k} = p_k$ if $r_k < t_k$. It is obvious from the definitions of r_k and t_k that $r_k > t_k$ if $k = \bar{k}$. But if $k \neq \bar{k}$, r_k may not always be larger than t_k .
- **Optimal $\frac{p_{2,k}}{p_{1,k}}$ under weak direct link channel gain $h_{1,k}$:** If the direct link has an infinite attenuation (or weak in the extreme), i.e., $h_{1,k} \approx 0 \forall k$, then $s_k = 0$, $r_k \geq t_k = 0$ and hence $\frac{p_{2,k}}{p_{1,k}} = 0$. In general, as the direct link becomes weaker, $\frac{p_{2,k}}{p_{1,k}}$ should become smaller.
- **Optimal $\frac{p_{2,k}}{p_{1,k}}$ under strong direct link channel gain $h_{1,k}$:** if the direct link ($h_{1,k} \forall k$) becomes relatively strong compared to the relay links ($h_{2,k}$ and $h_{3,k} \forall k$), then r_k and t_k become more dominant than s_k . In this case, $\frac{p_{2,k}}{p_{1,k}}$ becomes either very large or very small depending on which of r_k and t_k is larger than the other.

V. OVERALL ALGORITHM AND OPTIMALITY ANALYSIS

The overall algorithm (named Algorithm AO) for solving Problem (13) is summarized below.

Algorithm AO

- 1) Use algorithm 1 (or algorithm 2) with a pre-select source power to find a new set of relay power, then go to Step 2);
- 2) Use algorithm 3 (or algorithm 4) with newly updated relay power to find a new set of source power, then go to Step 3);
- 3) Use algorithm 1 (or algorithm 2) with newly updated source power to find a new set of relay power, then go to Step 2) until converge, where $|\Delta J| \leq \epsilon_I$.

In general, the alternating optimization method may fail to locate the stationary points, not to mention the global convergence to the optimal solution. However, since the constraint functions in ((14)–(15)) are convex either for fixed $\{p_{1,k}, p_{2,k}\}$ or for fixed $\{g_k\}$, and Algorithm AO belongs to the nonlinear Gauss–Seidel (GS) method [15] where the optimization vector is only partitioned into two component vectors, it follows from [15] that Algorithm AO converges to a stationary point of Problem (13) as stated in the following theorem.

Theorem 1 (Local Convergence of Alg. AO): Any limit point $\{\tilde{p}_{1,k}, \tilde{p}_{2,k}, \tilde{g}_k : \forall k\}$ generated by Algorithm AO is a stationary point of Problem (13).

In the following, we derive the asymptotically optimal solutions respectively for large relay transmit power P_r and large source-relay channel gain $h_2 \triangleq \min_k h_{2,k}$. Based on this, we show that the stationary point found by Algorithm AO is asymptotically optimal for large h_2 or P_r . Throughout this section, we will explicitly express the cost $J(\{p_{1,k}, p_{2,k}, g_k\})$ as a function of $\{p_{1,k}, p_{2,k}, g_k\}$ for clearness.

A. Asymptotically Optimal Solution for Large Relay Transmit Power

First, we give an upper bound for the SNR and show that the SNR upper bound can be approached as P_r grows large.

Lemma 1: For fixed $p_{1,k} + p_{2,k} = P_k$, the SNR on the k -th subcarrier is upper bounded by

$$\text{SNR}_k \leq \text{SNR}_k^{\text{U1}} \triangleq P_k \max \left(h_{2,k}^2 + h_{1,k}^2, h_{1,\bar{k}}^2 \right).$$

Moreover, if we let

$$p_{1,k} = P_k \mathbf{I} \left(h_{2,k}^2 + h_{1,k}^2 > h_{1,\bar{k}}^2 \right), \quad (40)$$

$$p_{2,k} = P_k \mathbf{I} \left(h_{2,k}^2 + h_{1,k}^2 \leq h_{1,\bar{k}}^2 \right), \quad (41)$$

$$g_k = \begin{cases} 0, & \text{if } h_{2,k}^2 + h_{1,k}^2 \leq h_{1,\bar{k}}^2, \\ \sqrt{\frac{P_r h_{2,k}^2}{N(1+h_{2,k}^2 p_{1,k})}}, & \text{otherwise,} \end{cases} \quad (42)$$

then the corresponding SNR_k satisfies $\text{SNR}_k^{\text{U1}} - \text{SNR}_k = \mathcal{O}\left(\frac{1}{P_r}\right)$ for sufficiently large P_r , where $\mathbf{I}(\cdot)$ denotes the indication function such that $\mathbf{I}(E) = 1$ if the event E is true and $\mathbf{I}(E) = 0$ otherwise.

Please refer to Appendix D for the proof.

Using Lemma 1, we can obtain an asymptotically optimal solution of Problem (13) for large P_r by solving a convex optimization problem as stated in the following theorem.

Theorem 2 (Asymptotically Optimal Solution For Large P_r): Let $\{P_k^\circ : \forall k\}$ denote the optimal solution of the following convex optimization problem

$$\min_{\{P_k \geq 0 : \forall k\}} \check{J}, \quad \text{s.t.} \quad \sum_{k=0}^{N-1} P_k \leq P_s, \quad (43)$$

where either $\check{J} = -\min_k \text{SNR}_k^{\text{U1}}$ or $\check{J} = \sum_{k=0}^{N-1} f_k(\text{SNR}_k^{\text{U1}})$. Obtain $\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ : \forall k\}$ using ((40)–(42)) with P_k replaced by P_k° . Then for sufficiently large P_r , $\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ : \forall k\}$

is an asymptotically optimal solution of Problem (13), i.e., $\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ : \forall k\}$ satisfies the power constraints in ((14)–(15)) and

$$J(\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ\}) - J^* = \mathcal{O}\left(\frac{1}{P_r}\right), \quad (44)$$

where J^* is the optimal value of Problem (13).

Please refer to Appendix E for the proof.

Based on Theorem 2, we establish the asymptotic optimality of Algorithm AO for large P_r .

Theorem 3 (Asymptotic Optimality of Alg. AO for Large P_r): Suppose that in Algorithm AO, the initial point is chosen such that $p_{1,k} = \mathcal{O}(1)$, $p_{2,k} = \mathcal{O}\left(\frac{1}{P_r}\right)$ if $h_{2,k}^2 + h_{1,k}^2 > h_{1,\bar{k}}^2$ and $p_{1,k} = \mathcal{O}\left(\frac{1}{P_r}\right)$, $p_{2,k} = \mathcal{O}(1)$ otherwise. Then for sufficiently large P_r , any limit point $\{\tilde{p}_{1,k}, \tilde{p}_{2,k}, \tilde{g}_k : \forall k\}$ generated by Algorithm AO is an asymptotically optimal solution of Problem (13), i.e.,

$$J(\{\tilde{p}_{1,k}, \tilde{p}_{2,k}, \tilde{g}_k\}) - J^* = \mathcal{O}\left(\frac{1}{P_r}\right), \quad (45)$$

and $\{\tilde{p}_{1,k}, \tilde{p}_{2,k}, \tilde{g}_k : \forall k\}$ satisfies the power constraints in ((14), (15)).

Please refer to Appendix F for the proof.

B. Asymptotically Optimal Solution for Large Source-Relay Channel Gain

We first give an upper bound for the SNR and show that the SNR upper bound can be approached as h_2 grows large.

Lemma 2: For given non-negative $p_{1,k}, p_{2,k}, g_k$, the SNR on the k -th subcarrier is upper bounded by $\text{SNR}_k(p_{1,k}, p_{2,k}, g_k) \leq \text{SNR}_k^{\text{U2}}(p_{1,k}, p_{2,k}, \rho_k)$, where

$$\text{SNR}_k^{\text{U2}}(p_{1,k}, p_{2,k}, \rho_k) \triangleq h_{1,k}^2 p_{1,k} + (h_{3,\bar{k}} \sqrt{\rho_k} + h_{1,\bar{k}} \sqrt{p_{2,k}})^2,$$

and $\rho_k = g_k^2 h_{2,k}^2 p_{1,k}$. Moreover, for sufficiently large h_2 and given non-negative $p'_{1,k}, g'_k$, if $g'_k = \mathcal{O}\left(\frac{1}{\sqrt{h_2}}\right)$, $p'_{1,k} - p_{1,k} = \mathcal{O}\left(\frac{1}{h_2}\right)$ and $g_k'^2 h_{2,k}^2 p'_{1,k} = \rho_k$, then

$$\text{SNR}_k^{\text{U2}}(p_{1,k}, p_{2,k}, \rho_k) - \text{SNR}_k(p'_{1,k}, p_{2,k}, g'_k) = \mathcal{O}\left(\frac{1}{h_2}\right).$$

Lemma 2 follows straightforward and the proof is omitted for conciseness.

Using Lemma 2, we can obtain an asymptotically optimal solution of Problem (13) for large h_2 by solving a convex optimization problem as stated in the following theorem.

Theorem 4 (Asymptotically Optimal Solution for large h_2): Let $\{p_{1,k}^\circ, p_{2,k}^\circ, \rho_k^\circ : \forall k\}$ be an optimal solution of the following convex optimization problem

$$\begin{aligned} & \min_{\{p_{1,k} \geq 0, p_{2,k} \geq 0, \rho_k \geq 0 : \forall k\}} \hat{J}, \\ & \text{s.t.} \quad \sum_{k=0}^{N-1} \rho_k \leq P_r, \quad \sum_{k=0}^{N-1} (p_{1,k} + p_{2,k}) \leq P_s, \end{aligned} \quad (46)$$

where either $\hat{J} = -\min_k \text{SNR}_k^{U2}$ or $\hat{J} = \sum_{k=0}^{N-1} f_k(\text{SNR}_k^{U2})$. Define

$$\hat{g}_k^\circ = \begin{cases} \frac{1}{h_{2,k}} \sqrt{\frac{\rho_k^\circ}{p_{1,k}^\circ}}, & \text{if } p_{1,k}^\circ > 0, \\ \frac{(\rho_k^\circ)^{\frac{1}{4}}}{\sqrt{h_2}}, & \text{otherwise.} \end{cases}$$

$$\hat{p}_{1,k}^\circ = \begin{cases} p_{1,k}^\circ, & \text{if } p_{1,k}^\circ > 0, \\ \frac{\sqrt{\rho_k^\circ}}{h_2}, & \text{otherwise.} \end{cases}$$

Then for sufficiently large h_2 , $\{\hat{p}_{1,k}^\circ, \hat{p}_{2,k}^\circ, \hat{g}_k^\circ : \forall k\}$ is an asymptotically optimal solution of Problem (13), i.e.,

$$J(\{\hat{p}_{1,k}^\circ, \hat{p}_{2,k}^\circ, \hat{g}_k^\circ\}) - J^* = \mathcal{O}\left(\frac{1}{h_2}\right), \quad (47)$$

and $\sum_{k=0}^{N-1} (\hat{p}_{1,k}^\circ + p_{2,k}^\circ) - P_s \leq \mathcal{O}\left(\frac{1}{h_2}\right)$, $\sum_{k=0}^{N-1} (h_{2,k}^2 \hat{p}_{1,k}^\circ + 1) (\hat{g}_k^\circ)^2 - P_r \leq \mathcal{O}\left(\frac{1}{h_2}\right)$.

Please refer to Appendix G for the proof.

Obviously, if we use the $\{\hat{p}_{1,k}^\circ, \hat{p}_{2,k}^\circ, \hat{g}_k^\circ\}$ in Theorem 4 as initial point, the solution $\{\tilde{p}_{1,k}, \tilde{p}_{2,k}, \tilde{g}_k : \forall k\}$ found by Algorithm AO will be an asymptotically optimal solution of Problem (13), i.e.,

$$J(\{\tilde{p}_{1,k}, \tilde{p}_{2,k}, \tilde{g}_k\}) - J^* = \mathcal{O}\left(\frac{1}{h_2}\right). \quad (48)$$

VI. IMPLEMENTATION ISSUE AND SIMULATION RESULT

A. Implementation Issue

In a real system, the AO algorithm could be executed at the source node. The global channel state information (CSI) must be supplied to this node. In relay systems, the source-relay/source-destination and relay-destination channels can be estimated directly at the destination node by utilizing pilot signals [16]. The destination must then feedback the CSI to the source node.

- **Complexity Analysis:** Here, each of addition, subtraction, multiplication and division is counted as one FLOP. The operations for comparisons are neglected.
 - **Complexity of Algorithm 1:** Step 1 requires $6N$ FLOPs (floating point operation), while Step 2 and 3 together require $21N$ FLOPs, where N is the number of subcarrier. Then the complexity order of the overall bisection search process is $\mathcal{O}\left(21N \log_2\left(\frac{\mu^*}{\epsilon}\right) + 6N\right)$, where μ^* is the length of the search range in Algorithm 1 and ϵ is the tolerable error.
 - **Complexity of Algorithm 2:** Step 1 requires $8N$, while step 2 requires $(21 + M_1)N$, where the M_1 additional FLOPs per subcarrier is introduced by convex function f . A different function f may require a different number of FLOPs in calculating f' , f'^{-1} and f'' . Given $\mu \leq \mu^*$, the complexity order of the overall bisection search process is upper-bounded by $\mathcal{O}\left((21 + M_1)N \log_2\left(\frac{\mu^*}{\epsilon}\right) \log_2\left(\frac{\mu^*}{\epsilon}\right) + 8N\right)$.
 - **Complexity of Algorithm 3:** Steps 1 through 3 require $46N$ FLOPs. The search ranges for

θ and γ are denoted by R_θ and R_γ , respectively. The complexity order is upper-bounded by $\mathcal{O}\left((37N + 9N \log_2\left(\frac{R_\theta}{\epsilon}\right)) \log_2\left(\frac{R_\gamma}{\epsilon}\right) + 46N\right)$. The complexity order increases logarithmically with R_θ and R_γ .

- **Complexity of Algorithm 4:** Step 1 through 2 require $40N + M_2$, where the additional M_2 FLOPs are introduced by the convex function f . Then, the complexity order is upper-bounded by

$$\mathcal{O}\left(\left(26N + M_2N + 14N \log_2\left(\frac{R_\theta}{\epsilon}\right)\right) \times \log_2\left(\frac{R_\gamma}{\epsilon}\right) + 40N + M_2N\right) \quad (49)$$

- **Overall Complexity of Algorithm AO:** With the above analysis, one can obtain the total number of FLOPs per iteration of the AO algorithm. Basically, the complexity grows linearly with the number of carrier and logarithmically with precision requirement. The number of iterations of the AO algorithm is difficult to quantify, which depends on the stop criterion. In our simulation, the required number of iterations to achieve $|\Delta J| \leq \epsilon_I$ with $\epsilon_I = 0.005$ is about 2–4, where ΔJ is the difference of J between two consecutive iterations.

B. Simulation Results

We have tested Algorithms 1–4 individually. They all converged rapidly to the exact optimal solutions.

For problem (13) with $J = J_1$, we apply Algorithms 1 and 3 alternately until convergence. For problem (13) with $J = J_2$, we apply Algorithms 2 and 4 alternately until convergence. We determine the convergence when both μ_1 and μ_2 satisfy $|\mu_1^r - \mu_1^{(r-1)}| < 10^{-5}$ and $|\mu_2^r - \mu_2^{(r-1)}| < 10^{-5}$ where r is the current index of alternation. For all tested cases, the convergence was reached within three to six alternations. The initialization of the source power allocation was chosen to be uniform, i.e., $p_{1,k} = p_{2,k} = \frac{P_s}{2N}$.

The optimization of the permutation function \bar{k} is computationally costly. In our simulation, we have compared three choices of \bar{k} : 1) $\bar{k} = k$, 2) \bar{k} is such that $h_{3,\bar{k}}$ is aligned with $h_{1,k} \forall k$, and 3) \bar{k} is such that $h_{3,\bar{k}}$ is aligned with $h_{2,k} \forall k$. Two sequences S_1 and S_2 are said to be aligned if the largest element in S_1 is located at the same position as the largest element in S_2 , the second largest element in S_1 is located at the same position as the second largest element in S_2 , and so on. We have found that the third choice of \bar{k} always yields slightly better result than the first two. Also note that with the third choice of \bar{k} , the overall relay channel gains defined by $h_{2,k} h_{3,\bar{k}} \forall k$ have a larger dynamics than $h_{2,k} h_{3,k}$, which intuitively makes the relay power scheduling more effective. In the following, we use the third choice of \bar{k} .

We generated the channel parameters $\tilde{h}_{1,k}$, $\tilde{h}_{2,k}$ and $\tilde{h}_{3,k} \forall k$ as independent circular complex Gaussian random variables of zero means. We choose $\tilde{h}_{2,k}$ and $\tilde{h}_{3,k} \forall k$ to have the variance two, and $\tilde{h}_{1,k} \forall k$ to have the variance $2 \times 10^{\frac{\alpha}{10}}$. When $\alpha = 0$, the direct link has the same strength as the relay links. As α decreases from zero, the direct link weakens. The value of α

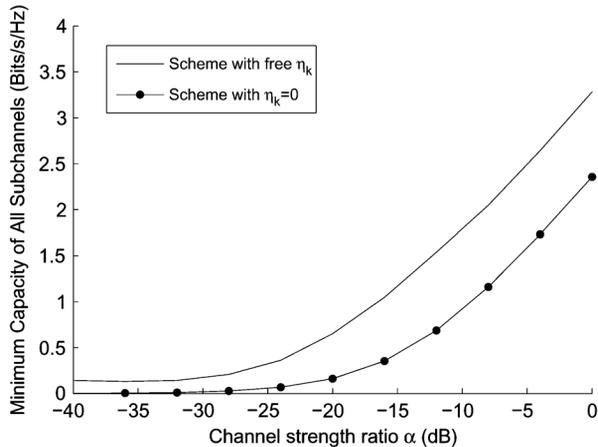


Fig. 4. Minimum subchannel capacity determined by Algorithms 1 and 3 (for J_1) with $N = 300$ and $P_s = P_r = 50$. The scheme with free η_k is based on Algorithms 1 and 3 with no constraint on η_k . The scheme with $\eta_k = 0$ is based on Algorithms 1 and 3 with $\eta_k = 0$.

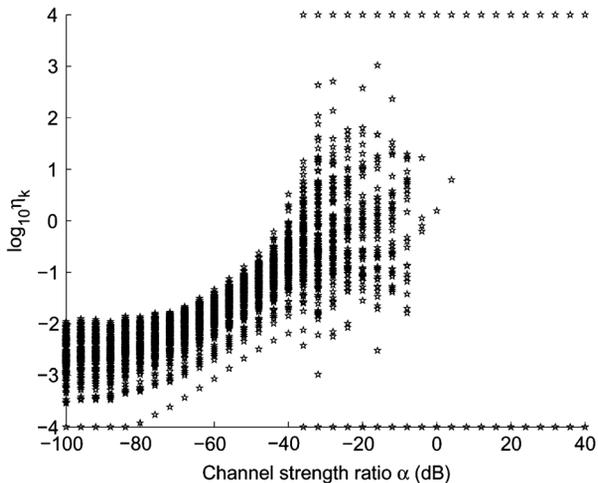


Fig. 5. Distribution of the optimal $\eta_k \forall k$ determined by Algorithms 1 and 3 (for J_1) with $N = 300$ and $P_s = P_r = 50$.

measures a relative (averaged) strength in dB of the direct link over the relay links.

In Fig. 4, we show the minimum capacity among all subchannels versus α . This capacity was averaged over ten independent channel realizations. For each channel realization (of $\tilde{h}_{1,k}$, $\tilde{h}_{2,k}$ and $\tilde{h}_{3,k} \forall k$), Algorithms 1 and 3 were used to maximize $\min_k \text{SNR}_k$ (or minimize $-\min_k \text{SNR}_k$). The resulting $\min_k \text{SNR}_k$ is used to determine the minimum capacity (in bits/s/Hz) by $\frac{1}{2} \log_2(1 + \min_k \text{SNR}_k)$. We see a significant gap of capacity between the case of free (optimal) η_k and the case of $\eta_k = 0$ when the direct link is not much weaker than the relay links. When the direct link becomes much weaker than the relay links, the gap diminishes as expected.

The distribution of the optimal $\log_{10} \eta_k \forall k$ determined by Algorithms 1 and 3 for a typical channel realization is shown in Fig. 5. As predicted by the analysis shown in Section IV-C, when the direct link becomes weaker (α smaller), η_k approaches to zero. And when the direct link becomes stronger (α larger), η_k is either very large or very small. For the plot, any value of

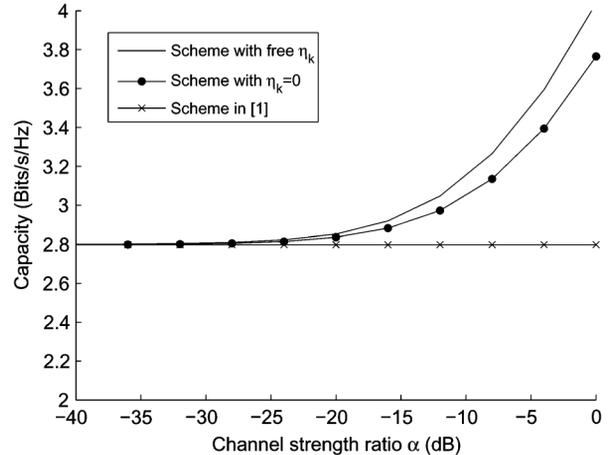


Fig. 6. Average subchannel capacity determined by Algorithms 2 and 4 (for J_2) with $N = 300$ and $P_s = P_r = 50$. The scheme with free η_k is that by Algorithms 2 and 4 with no constraint on η_k , the scheme with $\eta_k = 0$ is that by Algorithms 2 and 4 with $\eta_k = 0$, and the scheme in [1] is the WF-MCAF method in [1] which ignores the direct link completely.

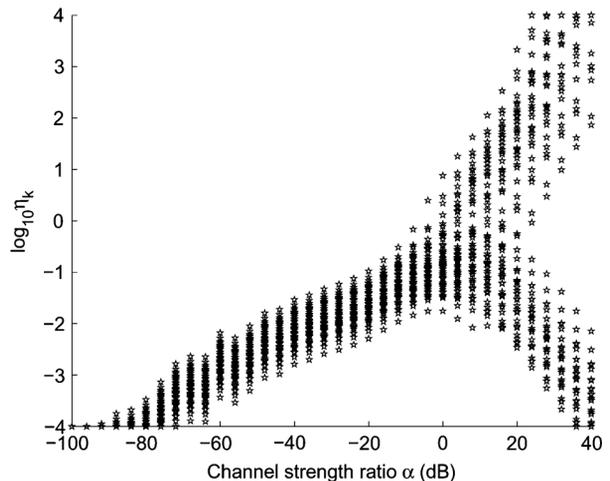


Fig. 7. Distribution of optimal $\eta_k \forall k$ determined by Algorithms 2 and 4 (for J_2) with $N = 300$ and $P_s = P_r = 50$.

$\log_{10} \eta_k$ larger than 4 is set to 4, and any value of $\log_{10} \eta_k$ less than -4 is set to -4 .

Fig. 6 shows the averaged subchannel capacity determined by Algorithms 2 and 4 versus α . For each channel realization, Algorithms 2 and 4 were used to minimize J_2 shown in (12). The resulting $-J_2$ was further averaged over ten independent channel realizations and then plotted into this figure. Once again, we see an important gap between the case of free (optimal) η_k and the case of $\eta_k = 0$ when the direct link is not very weak.

The distribution of the optimal $\eta_k \forall k$ determined by Algorithms 2 and 4 for a typical channel realization is shown in Fig. 7. The general trends of the values of η_k as α increases or decreases are consistent with the analysis shown in Section IV-C.

Comparing the two costs J_1 and J_2 , or equivalently, comparing Figs. 4 and 5 versus Figs. 6 and 7, we see that allowing η_k to be nonzero is more important for J_1 than for J_2 , i.e., the capacity gap for J_1 is larger than that for J_2 . It takes a much

weaker direct link, for J_1 than for J_2 , that η_k approaches to zero. We also see that the distribution of η_k becomes more binary for J_1 than for J_2 as the direct link becomes stronger.

VII. CONCLUSION

We have studied a dual-phase power allocation problem for a multicarrier relay system with direct link. We have developed four efficient algorithms that yield the exact solutions to the four corresponding optimization problems, including two that are not convex. Unlike the previous works, we allow the source power to be distributed in both phases for each carrier. Our analysis and simulation show that this additional freedom is important when the direct link is not too weak compared to the relay links.

APPENDIX

A. Proof of Proposition 1

- 1) It follows from $\frac{d\text{SNR}_k}{dg_k^2} = \frac{d\text{SNR}_k}{2g_k dg_k}$ and (18) that $\frac{d\text{SNR}_k}{dg_k^2} = \infty$ when $g_k = 0$, and $\frac{d\text{SNR}_k}{dg_k^2} < \infty$ when $g_k > 0$. Also note that the left-side function of the power constraint (17) is a (finitely) weighted sum of g_k^2 . As a result, if there is a $g_k = 0$, we can always increase g_k and decrease a positive $g_{k'}$ without affecting the power constraint but decreasing the cost J_2 . Therefore, the optimal g_k for the cost J_2 must be positive for all k , and hence $\gamma_k = 0, \forall k \in \bar{K}_0$.
- 2) Since the optimal g_k (also denoted by \hat{g}_k) to Problem (16) must fall in $[0, g_k^*]$ where J_2 is decreasing with g_k , the optimal $g_k \forall k$ must be such that either (17) holds with $\hat{g}_k = g_k^* \forall k$ or the equality in (17) holds with $\hat{g}_k < g_k^*$ for at least one k .
- 3) It follows from (18) that

$$\frac{d\text{SNR}_k}{2g_k dg_k} = \frac{(h_{3,\bar{k}} h_{2,k} g_k + h_{1,\bar{k}} \eta_k)}{g_k} \times \frac{(h_{3,\bar{k}} h_{2,k} - \eta_k h_{3,\bar{k}}^2 h_{1,\bar{k}} g_k)}{(h_{3,\bar{k}}^2 g_k^2 + 1)^2} p_{1,k}, \quad (50)$$

where both factors in the right-hand side are positive and decreasing with $g_k \in [0, g_k^*]$. Since $f_k(x)$ is a decreasing convex function, $-f'_k$ is also positive and decreasing with $g_k \in [0, g_k^*]$. For a given feasible μ , the corresponding g_k for each k can be easily found by a bisection search based on (22), the solution of which is denoted by $g_k(\mu)$.

B. Proof of Proposition 2

The constraint (15) may or may not be active (i.e., the equality may or may not hold at the optimal solution). If the solution to all the KKT conditions, except with $\mu_2 = 0$, satisfies (15) with equality, then $\mu_2 = 0$ is optimal. Otherwise, we need to have $\mu_2 > 0$.

Let $a_1 = \mu_1$ and $a_2 = \mu_2 g_k^2 h_{2,k}^2 + \mu_1$. Then, for each pair of μ_1 and μ_2 , (29) and (30) become

$$r_k + s_k \frac{\eta_k}{2} = \frac{a_2}{\lambda_k}, \quad (51)$$

$$t_k + s_k \frac{1}{2\eta_k} = \frac{a_1}{\lambda_k}. \quad (52)$$

Taking the ratio of the above two equations leads to a quadratic equation in terms of η_k , from which we have one unique (positive) solution in (33).

One can verify from (33) that η_k is a monotonically increasing function of $\frac{\mu_2}{\mu_1}$, i.e., $\frac{\partial \eta_k}{\partial \left(\frac{\mu_2}{\mu_1}\right)} > 0$.

For any given η_k in $\sqrt{p_{2,k}} = \eta_k \sqrt{p_{1,k}}$, we have from (28) that $\text{SNR}_k = (r_k + \eta_k s_k + \eta_k^2 t_k) p_{1,k}$ and hence for each given θ we can choose

$$p_{1,k} = \frac{\theta}{r_k + \eta_k s_k + \eta_k^2 t_k} \quad (53)$$

to achieve $\text{SNR}_k = \theta \forall k$.

The search for θ should be such that the equality in (14) is satisfied. Obviously, the left side of (14) is also a monotonically increasing function of θ subject to given $\eta_k \forall k$.

If the inequality in (15) is satisfied by the optimal solution to (26), $\frac{\mu_2}{\mu_1} = 0$ is optimal. Otherwise, we must have $\frac{\mu_2}{\mu_1} > 0$. So, we should start the search for $\frac{\mu_2}{\mu_1}$ from $\frac{\mu_2}{\mu_1} = 0$. The left side of (15) is a decreasing function of $\frac{\mu_2}{\mu_1}$ subject to (33) and the equality in (14).

The expression of η_k in (33) is numerically unstable when s_k is small (especially since g_k , and hence s_k , for some k can be exactly zero). To solve the numerical problem, we need to consider the case when s_k is arbitrarily small. As $s_k \rightarrow 0$, we can use the Taylor series expansion of (33) and write that

$$\eta_k = \begin{cases} \frac{2}{s_k} \left(t_k \frac{a_2}{a_1} - r_k \right) \rightarrow \infty, & t_k \frac{a_2}{a_1} - r_k > 0 \\ \frac{s_k \frac{a_2}{a_1}}{2 \left(t_k \frac{a_2}{a_1} - r_k \right)^2} \rightarrow 0, & t_k \frac{a_2}{a_1} - r_k < 0 \\ \sqrt{\frac{a_2}{a_1}}, & t_k \frac{a_2}{a_1} - r_k = 0 \end{cases} \quad (54)$$

and using this in (53) leads to (31) and (32).

C. Proof of Proposition 3

The (36) and (37) are equivalent to

$$r_k + s_k \frac{\eta_k}{2} = -\frac{a_2}{f'_k}, \quad (55)$$

$$t_k + s_k \frac{1}{2\eta_k} = -\frac{a_1}{f'_k}, \quad (56)$$

which are virtually the same as (51) and (52).

Taking the ratio of (55) and (56) and solving the resulting quadratic equation in terms of η_k , we have the same solution as shown in (33). Hence, for any given $\frac{\mu_2}{\mu_1}$, we can find $\eta_k \forall k$ from (33).

Since f'_k is monotonically increasing with SNR_k , it is also monotonically increasing with $p_{1,k}$ subject to $\sqrt{p_{2,k}} = \eta_k \sqrt{p_{1,k}}$. Hence, if we are also given $\mu_1 = a_1$ (in addition to γ), we can find from (56) a unique $p_{1,k}$, and then the corresponding $p_{2,k}, \forall k$. Specifically, we recall $\text{SNR}_k = (r_k + \eta_k s_k + \eta_k^2 t_k) p_{1,k}$, and hence have from (56) that $f'_k \doteq f'_k(\text{SNR}_k) = f'_k((r_k + \eta_k s_k + \eta_k^2 t_k) p_{1,k}) = \frac{-\mu_1/s_k}{t_k + 2\eta_k}$. Therefore,

$$p_{1,k} = \frac{\left(f'_k{}^{-1} \left(\frac{-\mu_1}{t_k + 2\eta_k} \right) \right)^+}{r_k + \eta_k s_k + \eta_k^2 t_k}, \quad (57)$$

where $f_k'^{-1}(x)$ is the inverse function of $f_k'(x)$, and $(x)^+ = \max(x, 0)$.

It is clear from (57) that $p_{1,k}, p_{2,k}, \forall k$ increase as μ_1 decrease, subject to any given γ . So, we can use the bisection search to determine μ_1 such that the equality in (14) holds.

The equality in (15) may or may not be satisfied by the optimal solution subject to $\gamma = 1$ (or equivalently $\mu_2 = 0$). If it is satisfied, $\gamma = 1$ is optimal and no further search is needed. Otherwise, we need to increase γ . Since $\sqrt{\frac{p_{2,k}}{p_{1,k}}}$ increases with γ , the left side of (15) is monotonically decreasing with γ subject to the equality in (14). Hence, we can use the bisection search to find γ such that the equality in (15) is satisfied.

Similar to the discussions for (31) and (32), as $s_k \rightarrow 0$, we should apply (54) to (57), which yields

$$p_{1,k} = \begin{cases} 0, & t_k \frac{a_2}{a_1} - r_k > 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\frac{a_2}{a_1} \mu_1}{\frac{a_2}{a_1} t_k + \left(t_k \frac{a_2}{a_1} - r_k \right)^2} \right) \right)^+}{r_k}, & t_k \frac{a_2}{a_1} - r_k < 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k} \right) \right)^+}{r_k + \frac{a_2}{a_1} t_k}, & t_k \frac{a_2}{a_1} - r_k = 0 \end{cases}, \quad (58)$$

and

$$p_{2,k} = \begin{cases} \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k} \right) \right)^+}{t_k}, & t_k \frac{a_2}{a_1} - r_k > 0 \\ 0, & t_k \frac{a_2}{a_1} - r_k < 0 \\ \frac{\left(f_k'^{-1} \left(\frac{-\mu_1}{t_k} \right) \right)^+}{\frac{a_2}{a_1} r_k + t_k}, & t_k \frac{a_2}{a_1} - r_k = 0 \end{cases} \quad (59)$$

(57), (58) and (59) together conclude the proof.

D. Proof of Lemma 1

For fixed $p_{1,k}, p_{2,k}$, we have $\text{SNR}_k \leq \text{SNR}_k^* = p_{1,k}(h_{2,k}^2 + h_{1,k}^2) + p_{2,k}h_{1,k}^2$. Then for fixed $p_{1,k} + p_{2,k} = P_k$, the SNR upper bounded can be obtained by solving

$$\max_{p_{1,k}, p_{2,k}} \text{SNR}_k^*, \text{ s.t. } p_{1,k} + p_{2,k} = P_k. \quad (60)$$

It is easy to see that the optimal solution of (60) is given by ((40)–(41)) and the optimal value is SNR_k^{U1} . Substituting ((40)–(42)) into the SNR expression in (9), it can be verified that $\text{SNR}_k^{\text{U1}} - \text{SNR}_k = \mathcal{O}\left(\frac{1}{P_r}\right)$.

E. Proof of Theorem 2

By Lemma 1, we have

$$\check{J}(P_k^\circ) \leq J^*. \quad (61)$$

$$\text{SNR}_k^{\text{U1}}(P_k^\circ) - \text{SNR}_k(p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ) = \mathcal{O}\left(\frac{1}{P_r}\right). \quad (62)$$

From (62), we have

$$\check{J}(P_k^\circ) - J(\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ\}) = \mathcal{O}\left(\frac{1}{P_r}\right). \quad (63)$$

Then (44) follows immediately from (61) and (63). Finally, using the definition of $\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ : \forall k\}$, it can be verified that $\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^\circ : \forall k\}$ also satisfies the power constraints in ((14), (15)).

F. Proof of Theorem 3

Using the KKT condition in (22), it can be shown that for any initial point $\{p_{1,k}, p_{2,k}\}$ that satisfies the condition in Theorem 3, the optimal $g_k^{(1)}$ obtained in step 1 of Algorithm AO satisfies $g_k^{(1)} = \mathcal{O}\left(\frac{1}{\sqrt{P_r}}\right)$ if $h_{2,k}^2 + h_{1,k}^2 \leq h_{1,\bar{k}}^2$, and $g_k^{(1)} = \mathcal{O}(\sqrt{P_r})$ otherwise. Following similar analysis as in the proof of Lemma 1, it can be shown that

$$\check{J}(\{P_k^\circ\}) - J(\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^{(1)}\}) = \mathcal{O}\left(\frac{1}{P_r}\right). \quad (64)$$

By Lemma 1, we have

$$\check{J}(\{P_k^\circ\}) \leq J^*. \quad (65)$$

For fixed $g_k^{(1)}$, the optimal $\{p_{1,k}^{(1)}, p_{2,k}^{(1)}\}$ obtained in step 2 of Algorithm AO satisfies

$$J(\{p_{1,k}^{(1)}, p_{2,k}^{(1)}, g_k^{(1)}\}) \leq J(\{p_{1,k}^\circ, p_{2,k}^\circ, g_k^{(1)}\}). \quad (66)$$

From ((64)–(66)), we have $J(\{p_{1,k}^{(1)}, p_{2,k}^{(1)}, g_k^{(1)}\}) - J^* = \mathcal{O}\left(\frac{1}{P_r}\right)$, i.e., after the first iteration of Algorithm AO, the solution is already asymptotically optimal for large P_r . The rest iterations will only decrease the cost function. This completes the proof.

G. Proof of Theorem 4

Using Lemma 2 and the fact that the constraint $\sum_{k=0}^{N-1} \rho_k \leq P_r$ in (46) is a more relaxed relay power constraint compared to the original one in (15), we have

$$\hat{J}(p_{1,k}^\circ, p_{2,k}^\circ, \rho_k^\circ) \leq J^*. \quad (67)$$

Note that $(\hat{g}_k^\circ)^2 h_{2,k}^2 \hat{p}_{1,k}^\circ = \rho_k^\circ$. Then it follows from Lemma 2, $\hat{g}_k^\circ = \mathcal{O}\left(\frac{1}{\sqrt{h_2}}\right)$ and $\hat{p}_{1,k}^\circ - p_{1,k}^\circ = \mathcal{O}\left(\frac{1}{h_2}\right)$ that

$$\text{SNR}_k^{\text{U2}}(p_{1,k}^\circ, p_{2,k}^\circ, \rho_k^\circ) - \text{SNR}_k(\hat{p}_{1,k}^\circ, p_{2,k}^\circ, \hat{g}_k^\circ) = \mathcal{O}\left(\frac{1}{h_2}\right). \quad (68)$$

From (68), we have

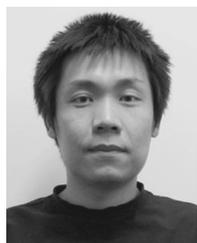
$$\hat{J}(p_{1,k}^\circ, p_{2,k}^\circ, \rho_k^\circ) - J(\{\hat{p}_{1,k}^\circ, p_{2,k}^\circ, \hat{g}_k^\circ\}) = \mathcal{O}\left(\frac{1}{h_2}\right). \quad (69)$$

Then (47) follows immediately from (67) and (69). Finally, using $\hat{g}_k^\circ = \mathcal{O}\left(\frac{1}{\sqrt{h_2}}\right)$, $\hat{p}_{1,k}^\circ - p_{1,k}^\circ = \mathcal{O}\left(\frac{1}{h_2}\right)$, and the definition of $\{p_{1,k}^\circ, p_{2,k}^\circ, \rho_k^\circ : \forall k\}$, it can be verified that $\{\hat{p}_{1,k}^\circ, p_{2,k}^\circ, \hat{g}_k^\circ : \forall k\}$ satisfies $\sum_{k=0}^{N-1} (\hat{p}_{1,k}^\circ + p_{2,k}^\circ) - P_s \leq \mathcal{O}\left(\frac{1}{h_2}\right)$, $\sum_{k=0}^{N-1} (h_{2,k}^2 \hat{p}_{1,k}^\circ + 1)(\hat{g}_k^\circ)^2 - P_r \leq \mathcal{O}\left(\frac{1}{h_2}\right)$.

REFERENCES

- [1] W. Zhang, U. Mitra, and M. Chiang, "Optimization of amplify-and-forward multicarrier two-hop transmission," *IEEE Trans. Commun.*, vol. 59, no. 5, pp. 1434–1445, May 2011.
- [2] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1398–1407, Apr. 2007.
- [3] Y. Rong, "MMSE-based non-regenerative multicarrier MIMO wireless relay communications with direct source-destination link," in *Proc. IEEE Int. Conf. Commun. (ICC '09)*, Jun. 2009, pp. 1–5.

- [4] Y. Rong and F. Gao, "Optimal beamforming for non-regenerative MIMO relays with direct link," *IEEE Commun. Lett.*, vol. 13, no. 12, pp. 926–928, Dec. 2009.
- [5] Y. Rong, "Optimal joint source and relay beamforming for MIMO relays with direct link," *IEEE Commun. Lett.*, vol. 14, no. 5, pp. 390–392, May 2010.
- [6] H. Wan and W. Chen, "Joint source and relay design for multiuser MIMO nonregenerative relay networks with direct links," *IEEE Trans. Veh. Technol.*, vol. 61, no. 6, pp. 2871–2876, Jul. 2012.
- [7] H. Wan, W. Chen, and J. Ji, "Efficient linear transmission strategy for MIMO relaying broadcast channels with direct links," *IEEE Wireless Commun. Lett.*, vol. 1, no. 1, pp. 14–17, February 2012.
- [8] X. Liu, L. Chen, X. Zhang, and D. Yang, "Resource allocation for OFDM-based multi-destination relay networks with direct links," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2011)*, Dec. 2011, pp. 1–6.
- [9] C.-B. Chae, T. Tang, R. W. Heath, and S. Cho, "MIMO relaying with linear processing for multiuser transmission in fixed relay networks," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 727–738, Feb. 2008.
- [10] B. K. Chalise and L. Vandendorpe, "MIMO relay design for multi-point-to-multipoint communications with imperfect channel state information," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2785–2796, Jul. 2009.
- [11] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear nonregenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4837–4851, Dec. 2009.
- [12] Y. Yu and Y. Hua, "Power allocation for a MIMO relay system with multiple-antenna users," *IEEE Trans. Signal Process.*, vol. 58, no. 5, pp. 2823–2835, May 2010.
- [13] Y. Hua, D. Bliss, S. Gazor, Y. Rong, and Y. Sung, Eds., *IEEE J. Sel. Areas Commun. (Special Issue on Theories and Methods for Advanced Wireless Relays)*, vol. 30, no. 8, Sep. 2012.
- [14] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [15] L. Grippo and M. Sciandrone, "On the convergence of the block non-linear Gauss-Seidel method under convex constraints," *IEEE Trans. Inf. Theory*, vol. 26, no. 3, pp. 127–136, 2000.
- [16] T. Kong and Y. Hua, "Optimal design of source and relay pilots for MIMO relay channel estimation," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4438–4446, 2011.



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