INSTANTON EFFECTS IN TWO-DIMENSIONAL SCALAR ELECTRODYNAMICS

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An effective Lagrangian of two-dimensional scalar electrodynamics that takes into account the effects of the complicated vacuum structure in the model is found.

1. Introduction

It is traditionally assumed that spontaneous symmetry breaking plays a decisive role in the acquisition of mass by the vector bosons. However, the idea of unbroken gauge symmetry also has its adherence, and their number has been increasing recently (see [1-4]). It should be noted that there are many different approaches to this problem.

In the present paper, for the example of scalar electrodynamics it will be shown that allowance for nonperturbative effects associated with the existence of instantons makes it possible to go over to an effective Lagrangian containing only neutral fields. It is shown that in the physical space there are particles present which correspond to just such fields. Thus, a definite mechanism leading to uncharged physical states is demonstrated. However, if tunneling is weak, the complicated vacuum structure has hardly any influence on the mass spectrum of the particles, which is the same as in the Higgs phase.

The paper is arranged as follows. In Sec. 2, we construct many-instanton configurations and calculate their interaction energy. In Sec. 3, we construct the approximation of a rarefied instanton gas [5,6] and on its basis find an effective low-energy Lagrangian which contains neutral scalar and pseudoscalar fields. In Sec. 4, the effective Lagrangian is used to calculate the Green's functions for the gauge-invariant operators that interpolate the scalar and pseudoscalar fields.

2. The Model

We consider the Lagrangian

$$L=\frac{1}{4}F_{\mu\nu}^2+|D_\mu\varphi|^2+\frac{\lambda}{2}\left(\varphi^+\varphi-\frac{c^2}{2}\right)^2 \tag{1}$$

of scalar electrodynamics in two-dimensional Euclidean space $x = (x_1, x_2)$. Here, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, $D_\mu=\partial_\mu-ieA_\mu$, A_μ is a real vector field, φ is a complex scalar field.

The Lagrangian (1) has gauge invariance. If it is assumed that spontaneous symmetry breaking has concentrated the field φ near the value $c/\sqrt{2}$ and perturbation theory is used, then the vector field A_μ acquires the mass $m_V = ec$; the mass of the scalar field is $m_S = \sqrt{\lambda c^2}$ (the constants e, c, and λ are positive).

However, the existence in the model of instantons, which was proved in [7], leads to nonperturbative effects and shows that it is incorrect to use only perturbation theory.

The basic configuration of instanton type (instanton) has the form

$$A_\mu(x)=\frac{2\pi}{e}\varepsilon_{\mu\nu}\partial_\nu\Phi_I(x),\quad \varphi(x)=\frac{c}{\sqrt{2}}e^{-i\theta(x)}(1-\Psi_I(x)), \tag{2}$$

where $\varepsilon_{12}=-\varepsilon_{21}=1$, $\varepsilon_{00}=\varepsilon_{11}=0$, $\mathrm{tg}\,\theta(x)=x_1/x_2$, $x=(x_1, x_2)$. Note that $\partial_\nu\Phi_I(0) = 0$, $\Psi_I(0) = 1$. The explicit form of the functions $\Phi_I(x)$ and $\Psi_I(x)$ is unknown, but the asymptotic behaviors follow from the equations of motion:

$$\Phi_I(x)\underset{x\to\infty}{\longrightarrow}D(x)-f_v\Delta^v(x),\quad \Psi_I(x)\underset{x\to\infty}{\longrightarrow}f_s\Delta^s(x). \tag{3}$$

Here, f_V and f_S are certain real constants, $D(x)=-(\ln x^2)/(4\pi)$, $\Delta^{v,s}(x)=\frac{1}{2\pi}K_0(m_{v,s}|x|)$, where K_0

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is the Macdonald function of zeroth order.

It is clear from the asymptotic behaviors that in the model the sizes of the instanton are fixed (in contrast to QCD) and are measured by $1/m_V$ for the field A_μ and $1/m_s$ for the field φ.

We note that the functions Φ_I, Ψ_I, Δ^v, Δ^s, D have axial symmetry, i.e., depend only on $|x|$. We also note the relations

$$-\partial^2 D(x)=\delta(x), \quad (-\partial^2+m_{v,s}^2)\Delta^{v,s}(x)=\delta(x). \tag{4}$$

The translational invariance of the theory makes it possible to place the center of the instanton (2) at an arbitrary point z, for which it is sufficient in (2) to replace $\Phi_I(x)$, $\Psi_I(x)$, $\theta(x)$ by $\Phi_I(x-z)$, $\Psi_I(x-z)$, $\theta(x-z)$, respectively.

The discrete symmetry of the Lagrangian (1) with respect to the operation $\varphi\to\varphi^+$, $A_\mu\to -A_\mu$ makes it possible to introduce the anti-instanton, which is obtained from (2) by the given transformations.

It is obvious that the instanton action (S_{inst}) is equal to the anti-instanton action. In what follows, we shall need the assumption $\exp(-S_{inst}) \ll 1$ (we can achieve this, since the parameters e, λ, and c are arbitrary).

The topological charge of the configuration, which becomes a pure gauge as $x \to \infty$, is determined by the equation

$$q=\frac{e}{2\pi}\oint A_\mu\, d\xi_\mu,$$

where the integral is around a circle of infinite radius, $d\xi$ is the element of arc, and the integral is taken counterclockwise. Note that q is also the number of "turns" of the phase of φ on the passage around the infinite contour.

Transforming the contour integral to a double integral, we obtain

$$q=\int q(x)\,d^2x, \quad q(x)=\frac{e}{4\pi}\varepsilon_{\mu\nu}F_{\mu\nu}(x). \tag{5}$$

The pseudoscalar quantity q(x) is called the density of the topological charge.

It is easy to show that for the instanton (2) q = 1 holds, for the anti-instanton q = −1.

We now consider a set of N equally spaced instantons and anti-instantons (the distances between the centers are much greater than $1/m_{V,S}$):

$$\begin{gathered}A_\mu(x)=\frac{2\pi}{e}\varepsilon_{\mu\nu}\partial_\nu\Phi(x), \quad \Phi(x)=\sum_{i=1}^{N} q_i\Phi_I(x-z_i),\\ \varphi(x)=\frac{c}{\sqrt{2}}\exp\Big(-i\sum_{i=1}^{N} q_i\theta(x-z_i)\Big)\prod_{i=1}^{N}(1-\Psi_I(x-z_i)).\end{gathered} \tag{6}$$

Here, $q_i = \pm 1$ are the topological charges, which distinguish the instanton and anti-instanton, and z_i is the position of the center of the i-th instanton or anti-instanton.

The configuration (6) is only an approximate solution of the Euclidean equations of motion. This corresponds to the fact that the instantons interact.

We find the interaction of two widely separated instantons (having the same or different signs of the topological charge). We use the singular gauge for which the configurations (6) correspond to

$$A_\mu(x)=\frac{2\pi}{e}\varepsilon_{\mu\nu}\partial_\nu\Phi(x)+\frac{1}{e}\sum_{i=1}^{N} q_i\partial_\mu\theta(x-z_i), \quad \varphi(x)=\frac{c}{\sqrt{2}}\prod_{i=1}^{N}(1-\Psi_I(x-z_i)). \tag{7}$$

Here N = 2. The interaction energy is the excess of the action for (7) over twice the instanton action. In calculating the corresponding double integral, we divide the (x_1, x_2) into three parts (see Fig. 1). We call the disk of radius R, where $1/m_{v,s}\ll R\ll|z_2-z_1|$, concentric with the first instanton region I. Region II is a similar disk concentric with the

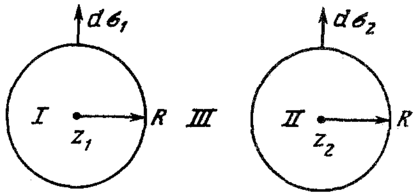

Fig. 1

second instanton, and region III is the remaining part of the plane.

We denote by $\Delta S_{\rm I}$, $\Delta S_{\rm II}$, $\Delta S_{\rm III}$ the parts of the interaction energy accumulated in the regions I, II, III, respectively. We calculate $\Delta S_{\rm I}$. In region I, the influence of the second instanton reduces to small corrections to the fields A_μ, φ produced by the first instanton. For the integration of the corresponding increment of the Lagrangian (1), we can use integration by parts. Then there remain only integrals over the boundary of region I, since the double integrals are equal to each other by virtue of the fact that the fields of the first instanton satisfy the equations of motion. Therefore, we write

$$\Delta S_{\rm I}=\oint_{\sigma_1}\frac{\partial L}{\partial\varphi_{;k}}\delta\varphi\,d\sigma^k+\oint_{\sigma_1}\frac{\partial L}{\partial A_{\mu;k}}\delta A_\mu\,d\sigma^k.$$

Here, $d\sigma$ corresponds to the outer normal of the boundary of region I. The increment of the fields (the influence of the second instanton) can be readily found from (7) with allowance for (3):

$$\delta A_\mu=q_2\frac{2\pi}{e}\varepsilon_{\mu\tau}\partial_\tau(-f_v\Delta^v(x-z_2)),\quad \delta\varphi=-\frac{c}{\sqrt{2}}f_s\Delta^s(x-z_2).$$

Here, we have noted that $\Psi_{\rm I}(x-z_1)\ll 1$ on the boundary of region I. Using (3) and (4) for the fields of the first instanton (since $R\gg 1/m_{V,S}$), we have

$$\frac{\partial L}{\partial\varphi_{;k}}=2\partial_k\varphi=-\sqrt{2}\,cf_s\partial_k\Delta^s(x-z_1),\quad \frac{\partial L}{\partial A_{\mu;k}}=F_{k\mu}=-q_1\frac{2\pi}{e}\partial^2\Phi=q_1\frac{2\pi}{e}f_vm_v^2\Delta^v(x-z_1).$$

We can therefore write

$$\Delta S_{\rm I}=c^2f_s^2I_1^s+\left(\frac{2\pi}{e}\right)^2q_1q_2f_v^2m_v^2I_2^v,$$

where we have used the notation

$$I_1^{v,s}=\oint_{\sigma_1}(\partial_i\Delta^{v,s}(x-z_1))\Delta^{v,s}(x-z_2)\,d\sigma^i,\quad I_2^{v,s}=\oint_{\sigma_1}(\partial_i\Delta^{v,s}(x-z_2))\Delta^{v,s}(x-z_1)\,d\sigma^i. \tag{8}$$

From symmetry considerations, $\Delta S_{\rm II}=\Delta S_{\rm I}$.

In calculating $\Delta S_{\rm III}$, we take into account only the principal crossed terms, since the fields of both the first and the second instanton are here small. Using the asymptotic behaviors (3), we obtain

$$\Delta L=q_1q_2\left(\frac{2\pi}{e}\right)^2f_v^2m_v^4\Delta_1^v\Delta_2^v+c^2f_s^2(\partial_\mu\Delta_1^s)(\partial_\mu\Delta_2^s)+e^2c^2\left(\frac{2\pi}{e}\right)^2f_v^2q_1q_2(\partial_\mu\Delta_1^v)(\partial_\mu\Delta_2^v)+\lambda c^4f_s^2\Delta_1^s\Delta_2^s.$$

For brevity, we here use the notation $\Delta_i^{v,s}=\Delta^{v,s}(x-z_i)$. It is readily seen that when ΔL is integrated by parts the double integrals are equal to zero by virtue of (4), and there therefore remain only the integrals over the boundaries of regions I and II. The integral over the boundary of region II can be readily reduced to one over the boundary of region I by symmetry considerations,

$$\Delta S_{\rm III}=\left(\frac{2\pi}{e}\right)^2q_1q_2f_v^2m_v^2(-I_1^v-I_2^v)+c^2f_s^2(-I_1^s-I_2^s).$$

The minus sign takes into account the direction of the outer normal,

$$\Delta S_{\rm I}+\Delta S_{\rm II}+\Delta S_{\rm III}=c^2f_s^2(I_1^s-I_2^s)+\left(\frac{2\pi}{e}\right)^2f_v^2m_v^2q_1q_2(I_2^v-I_1^v).$$

To calculate $I_1^{v,s}-I_2^{v,s}$, we consider the following integral over region I:

$$\int\limits_{I} \{(\partial_\mu \Delta^{v,s}(x-z_1))(\partial_\mu \Delta^{v,s}(x-z_2)) + m_{v,s}^2 \Delta^{v,s}(x-z_1)\Delta^{v,s}(x-z_2)\} d^2x.$$

We shall integrate by parts in two ways, transferring the derivatives differently. By means of (4), we readily obtain

$$I_1^{v,s} - I_2^{v,s} = -\int\limits_{I} \delta(x-z_1)\Delta^{v,s}(x-z_2)\, d^2x = -\Delta^{v,s}(z_1-z_2).$$

Thus, we have calculated the interaction energy of two widely spaced instantons with centers at the points z_1 and z_2 and topological charges q_1 and q_2:

$$U_{q_1q_2}(z_1, z_2) = q_1 q_2 \left(\frac{2\pi}{e}\right)^2 m_v^2 f_v^2 \Delta^v(z_1-z_2) - c^2 f_s^2 \Delta^s(z_1-z_2). \tag{9}$$

3. Effective Lagrangian

In this part, we find an effective low-energy Lagrangian of the model (1) that takes into account nonperturbative effects and contains fields that are not transformed by the gauge group.

For this, we calculate the amplitude of a vacuum-vacuum transition during infinite time. Using the path integral technique, we write

$$Z = \lim_{\tau \to \infty} \langle 0 | e^{-H_{\text{Eucl.}}\tau} | 0 \rangle = \mathcal{N} \int \mathcal{D}A_\mu \mathcal{D}\varphi \mathcal{D}\varphi^+ \exp\left(-\int\limits_{\infty} L(A_\mu, \varphi)\, d^2x\right). \tag{10}$$

Here, $\mathcal{N}$ is the normalization, the integration is over configurations that become a pure gauge as $x_2 \to \pm\infty$, and the double integral under the exponential sign is taken over the complete two-dimensional Euclidean space.

Because of the gauge freedom, the integral over $\mathcal{D}A_\mu \mathcal{D}\varphi \mathcal{D}\varphi^+$ contains classes of equivalent configurations, from which it is necessary in each case to select one representative. We shall not consider this question, but assume that it has already been done.

Using the method of steepest descent, we shall take into account the contribution to (10) from only the neighborhoods of the configuration $\varphi = c/\sqrt{2}$, $A_\mu = 0$ and configurations of the type (6).

We shall denote by Z_{PT} the contribution to (10) from the neighborhood of the configuration $\varphi = c/\sqrt{2}$, $A_\mu = 0$. Let the contribution to (10) from the neighborhood of the one-instanton configuration be $Z_{PT} \cdot a$, where $a = c\exp(-S_{inst})$, the constant c taking into account the ratios of the corresponding determinants and the presence of the zero mode (here, there is no integration with respect to the degree of freedom associated with the zero mode; we do that integration separately). The contribution from the neighborhood of the N-instanton configuration (6) (without integration with respect to the degrees of freedom associated with displacements of the centers of the instantons) is $Z_{PT} a^N \exp(-U_N)$, where U_N is the interaction energy of the instantons, this, when allowance is made for only binary interactions (see (9)), being

$$U_N = \sum_{i<j}^{N} U_{q_i q_j}(z_i, z_j). \tag{11}$$

In what follows, we shall use the letter x instead of z in the notation for the centers of the instantons.

Summing (integrating) the contributions from all possible instanton configurations and the configurations $A_\mu = 0$, $\varphi = c/\sqrt{2}$, we obtain

$$Z = Z_{PT} \sum_{N^+=0}^{\infty} \sum_{N^-=0}^{\infty} \int d^2x_1^+ \dots d^2x_{N^+}^+ d^2x_1^- \dots d^2x_{N^-}^- \frac{a^{N^+ + N^-}}{N^+!\, N^-!} e^{-U_{N^+ + N^-}}. \tag{12}$$

Here, the summation is separately over the number of instantons (N^+) and anti-instantons N^-), x_i^+ is the center of instanton i, and x_j^- is the center of anti-instanton j. The combinatorial factor $1/(N^+! N^-!)$ eliminates the additional summation of equivalent configurations obtained by permutation of the instantons.

In what follows, we require the generating functional, which we determine by the formula

$$Z[\mu^+(x),\mu^-(x)] = Z_{\text{PT}} \sum_{N^+=0}^{\infty}\sum_{N^-=0}^{\infty}\int d^2x_1^+\dots d^2x_{N^+}^+ \times$$
$$d^2x_1^-\dots d^2x_{N^-}^- \frac{a^{N^++N^-}}{N^+!\,N^-!} e^{-U_{N^++N^-}} \prod_{i=1}^{N^+} e^{\mu^+(x_i^+)} \prod_{j=1}^{N^-} e^{\mu^-(x_j^-)}. \tag{13}$$

Here, the arbitrary functions $\mu^+(x)$ and $\mu^-(x)$ (chemical potentials) play the role of the current in the generating functional of ordinary field theory.

The validity of taking into account in (12) and (13) configurations with closely spaced instantons may be questioned. However, we may note, first, that for all fixed N^+ and N^- the relative volume of such configurations in the $(N^+ + N^-)$-dimensional space is infinitesimally small and, second, that in Sec. 4 it will be shown that the mean density of the instantons is of order $\exp(-S_{\text{inst}}) \ll 1$, i.e., they rarely approach close to each other, and, third, that we are interested in only the properties of the theory at large distances.

Below, we write down a Lagrangian for which the amplitude of the vacuum–vacuum transition during infinite time is equal to (12) apart from a constant, so that we are justified in regarding it as an effective Lagrangian of the model (1).

We consider in two-dimensional Euclidean space the Lagrangian

$$L_{\text{eff}} = \frac{(\nabla\Sigma)^2+m_v^2\Sigma^2}{2} + \frac{(\nabla\sigma)^2+m_s^2\sigma^2}{2} - 2a\left(\cos\frac{\Sigma}{F_v}\right)\exp\frac{\sigma}{F_s}, \tag{14}$$

where $F_v=(2\pi c f_v)^{-1}$, $F_s=(f_s c)^{-1}$.

Subsequently, we shall see that the field σ is a true scalar and the field Σ a pseudoscalar. There is some difficulty associated with the pseudoscalar nature of Σ. The point is that on the transition to Minkowski space we must multiply the pseudoscalar fields by the imaginary unit, as a result of which the metric of the field Σ becomes negative. But if we wish to have positivity of the metric in the Minkowski space, then we will have a negative metric in the Euclidean space. We proceed as follows. In the Euclidean space, we will work with the Lagrangian (14) but bear in mind that the physical field is the field $\Sigma' = i\Sigma$. In Minkowski space the Lagrangian (14) will correspond to

$$L_{\text{eff.Mink}} = \frac{1}{2}\left[(\nabla\Sigma')^2 - m_v^2(\Sigma')^2\right] + \frac{1}{2}\left[(\nabla\sigma)^2 - m_s^2\sigma^2\right] + 2a\left(\cos\frac{\Sigma'}{F_v}\right)\exp\frac{\sigma}{F_s}.$$

We calculate the amplitude of vacuum–vacuum transition in the Euclidean space during infinite time for (14):

$$Z_{\text{eff}} = \mathcal{N}_{\text{eff}} \int \mathcal{D}\sigma\mathcal{D}\Sigma \exp\left(-\int L_{\text{eff}}\, d^2x\right).$$

Denoting by $S_0(\Sigma)$ and $S_0(\sigma)$ the actions corresponding to the free fields Σ and σ, and expanding in a series the exponential of the integrated final term of (14), we obtain

$$Z_{\text{eff}} = \mathcal{N}_{\text{eff}} \sum_{N=0}^{\infty} \frac{(2a)^N}{N!} \int d^2x_1\dots d^2x_N \left[\int e^{-S_0(\sigma)} \prod_{i=1}^{N} \exp\left(\frac{\sigma(x_i)}{F_s}\right)\mathcal{D}\sigma\right]\left[\int e^{-S_0(\Sigma)} \prod_{i=1}^{N} \cos\frac{\Sigma(x_i)}{F_v}\,\mathcal{D}\Sigma\right]. \tag{15}$$

We denote the expressions in the square brackets by $I(\sigma)$ and $I(\Sigma)$, respectively. We can regard $I(\sigma)$ as the vacuum expectation value of $\exp\left(\int J_\sigma(x)\sigma(x)\,d^2x\right)$, where $J_\sigma(x) = \left[\sum_{i=1}^{N}\delta(x-x_i)\right]\Big/F_s$. Then the standard technique gives

$$I(\sigma) = \exp\left(\frac{1}{2F_s^2}\sum_i^N\sum_j^N \Delta^s(x_i-x_j)\right)\int \mathcal{D}\sigma e^{-S_0(\sigma)}.$$

Here, the function Δ^s is identical to the one used in Sec. 2. It can be seen that the $\Delta^s(0)$ will be encountered in the sum. These unphysical infinities disappear if we insert in the Lagrangian (14) the symbol of normal ordering. Therefore, in the double sum we can set i ≠ j.

The calculation of $I(\Sigma)$ is a little bit more difficult. Expanding the cosine as a sum of exponentials, we find that the product of the cosines decomposes into a sum of N + 1 classes in accordance with the number of minuses under the exponential symbol. In each class, the terms differ only by a permutation of the order of 1, 2, ..., N, which is unimportant for the subsequent integration over dx_i. Therefore, we replace the product of the cosines by

$$2^{-N}\sum_{N^-=0}^{N}\frac{N!}{N^-!(N-N^-)!}\exp\Big[\frac{i}{F_v}\left(-\Sigma(x_1)-\ldots-\Sigma(x_{N^-})+\Sigma(x_{N^-+1})+\ldots+\Sigma(x_N)\right)\Big].$$

Here, N^- characterizes the class, and the combinatorial factor is equal to the number of terms in the class. Denoting $N^+ = N - N^-$ and making the obvious transformations for x_i, we write

$$\tilde{I}(\Sigma)=\int e^{-S_0(\Sigma)}\frac{1}{2^N}\sum_{N^-=0}^{N}\frac{N!}{N^+!N^-!}\exp\Big[\frac{-i}{F_v}\sum_i^{N^-}\Sigma(x_i^-)+\frac{i}{F_v}\sum_j^{N^+}\Sigma(x_j^+)\Big]\mathscr{D}\Sigma.$$

The tilde above I recalls the permutation made of the points x_i, but in (15) we can use $\tilde{I}(\Sigma)$ instead of $I(\Sigma)$.

Repeating the calculations with the field σ, we find

$$\tilde{I}(\Sigma)=\frac{1}{2^N}\sum_{N^-}^{N}\frac{N!}{N^-!N^+!}\exp\Big[\frac{1}{2}\int d^2x\,d^2y\,J_\Sigma(x)J_\Sigma(y)\Delta^v(x-y)\Big]\int\mathscr{D}\Sigma e^{-S_0(\Sigma)},$$

where

$$J_\Sigma(x)=\frac{1}{F_v}\Big[\sum_{i=1}^{N^+}\delta(x-x_i^+)-\sum_{j=1}^{N^-}\delta(x-x_j^-)\Big].$$

When the integral over d^2xd^2y is replaced by a sum, it is again necessary to eliminate the unphysical $\Delta^v(0)$.

Substituting $I(\sigma)$ and $\tilde{I}(\Sigma)$ in (15) and replacing the double summation over N and N^- by summation over N^+ and N^-, we obtain

$$Z_{\text{eff}}=Z'_{\text{PT}}\sum_{N^+=0}^{\infty}\sum_{N^-=0}^{\infty}\int d^2x_1^+\ldots d^2x_{N^+}^+\,d^2x_1^-\ldots d^2x_{N^-}^-\,\frac{a^{N^++N^-}}{N^+!\,N^-!}e^{-U_{N^++N^-}},$$

where

$$Z'_{\text{PT}}=\mathscr{N}_{\text{eff}}\int\mathscr{D}\Sigma\mathscr{D}\sigma\exp(-S_0(\Sigma)-S_0(\sigma)),$$

and the function $U_{N^++N^-}$ is equal to the one introduced in (11). Comparing the expression for Z_{eff} and (12), we obtain the important equation

$$Z/Z_{\text{PT}}=Z_{\text{eff}}/Z'_{\text{PT}},$$

which gives us grounds for regarding L_{eff} as the effective Lagrangian of the model (1).

We now determine the generating functional

$$Z_{\text{eff}}[\mu^+(x),\mu^-(x)]=\mathscr{N}_{\text{eff}}\int\mathscr{D}\sigma\mathscr{D}\Sigma\Big\{\exp[-S_0(\Sigma)-S_0(\sigma)]\times$$
$$\exp\Big[2a\int d^2x\cos\Big(\frac{\Sigma(x)}{F_v}-i\frac{\mu^+(x)-\mu^-(x)}{2}\Big)\exp\Big(\frac{\sigma(x)}{F_s}+\frac{\mu^+(x)+\mu^-(x)}{2}\Big)\Big]\Big\}. \quad (16)$$

Proceeding as in the calculation of Z_{eff}, we can readily show that

$$Z[\mu^+(x),\mu^-(x)]/Z_{\text{PT}}=Z_{\text{eff}}[\mu^+(x),\mu^-(x)]/Z'_{\text{PT}}. \quad (17)$$

Note that the imaginary unit in the argument of the cosine in (16) is due to the pseudo-scalar nature of the field Σ.

Thus, we have obtained the effective low-energy Lagrangian (14) for the model (1). In the following part we shall show how the Lagrangian (14) can be used to calculate different vacuum expectation values in the theory (1).

We note that Σ and σ are real fields and that they are not transformed by the gauge group (they are neutral with respect to it). This suggests that in the theory with the Lagrangian (1) spontaneous symmetry breaking does not occur.

We consider more closely the effective Lagrangian. It can be seen that $\langle\Sigma\rangle = 0$ but $\langle\sigma\rangle = \sigma_0 \neq 0$. Bearing in mind that $a \sim \exp(-S_{inst}) \ll 1$, we obtain in the first approximation $\sigma_0 = 2a/(F_s m_s^2)$. The masses of the particles Σ and σ satisfy the relations

$$m_\sigma^2 = m_s^2 - \frac{2a}{F_s^2} e^{\sigma_0/F_s}, \quad m_\Sigma^2 = m_v^2 + \frac{2a}{F_v^2} e^{\sigma_0/F_s}. \tag{18}$$

We see that in the theory there have appeared particle masses shifted somewhat compared with m_s and m_v. In the following part, we shall show that in the physical subspace particles with precisely these masses are present.

4. Green's Functions

In this part, we develop a technique for calculating Green's functions of gauge-invariant operators in two-dimensional scalar electrodynamics, calculate explicitly the Fourier transforms of $\langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle, \langle T\varphi^2(x)\varphi^2(y)\rangle$, and find approximating operators for the fields σ and Σ.

We calculate $\langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle$. Representing this quantity as a functional integral and taking into account only the two leading terms, we write

$$\langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle = \langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle^{\mathrm{PT}} + \langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle^{\mathrm{inst}}.$$

Here, the first term is the result of calculations in accordance with perturbation theory (without allowance for the instantons), and the second term gives the mean value over the instanton configurations in the approximation of a rarefied instanton gas.

We determine the transition to the momentum space for arbitrary operators B and C by the formula

$$\langle B(p)C(k)\rangle = \frac{1}{(2\pi)^2}\int d^2x\, d^2y\, e^{-ipx-iky}\langle TB(x)C(y)\rangle. \tag{19}$$

The standard technique for the free vector field A_μ with mass m_v leads to the formula

$$\langle F_{\mu\nu}(k)F_{\tau\rho}(p)\rangle^{\mathrm{PT}} = \varepsilon_{\mu\nu}\varepsilon_{\tau\rho}\delta(p+k)\frac{k^2}{k^2+m_v^2}. \tag{20}$$

To calculate the mean values over the instanton configurations, we introduce statistical operators that have the meaning of instanton densities:

$$\hat{n}^+(x) = \sum_{i=1}^{N^+}\delta(x-x_i^+), \quad \hat{n}^-(x) = \sum_{i=1}^{N^-}\delta(x-x_i^-), \quad \hat{n}(x) = \hat{n}^+(x) + \hat{n}^-(x).$$

Here, x_i^+ and x_i^- are certain fixed points (the centers of the instantons and anti-instantons).

The mean values of the statistical operators can be calculated by means of the generating functional (13), using (16) and (17):

$$\langle\hat{n}^\pm(x)\rangle^{\mathrm{inst}} = \frac{1}{Z}\frac{\delta Z[\mu^+,\mu^-]}{\delta\mu^\pm(x)}\Bigg|_{\substack{\mu^+\equiv 0\\ \mu^-\equiv 0}} = \frac{1}{Z_{\mathrm{eff}}}\frac{\delta Z_{\mathrm{eff}}[\mu^+,\mu^-]}{\delta\mu^\pm(x)}\Bigg|_{\substack{\mu^+\equiv 0\\ \mu^-\equiv 0}} =$$

$$a\left\langle\left(\mathrm{c}\quad\frac{\Sigma(x)}{F_v} \mp i\sin\frac{\Sigma(x)}{F_v}\right)\exp\frac{\sigma(x)}{F_s}\right\rangle^{\Sigma,\sigma}.$$

Here, $\langle\ \rangle^{\Sigma,\sigma}$ denotes the vacuum expectation value for the theory with the Lagrangian (14). Taking into account the first terms in the expansion of the cosine and the exponential, we find

$$\langle\hat{n}^\pm(x)\rangle^{\mathrm{inst}} = a\exp\frac{\sigma_0}{F_s}, \text{ where } \sigma_0 = \frac{2a}{F_s m_s^2}, \quad n = \langle\hat{n}(x)\rangle^{\mathrm{inst}} = 2a\exp\frac{\sigma_0}{F_s}.$$

The mean density n of the instantons (with allowance for the anti-instantons) is a small quantity, since $a \sim \exp(-S_{\text{inst}}) \ll 1$, and therefore the instanton gas can indeed be assumed to be rarefied.

To calculate $\langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle^{\text{inst}}$, we write in accordance with (6)

$$F_{\mu\nu}(x)=\frac{2\pi}{e}\varepsilon_{\mu\nu}\partial^2\left[\int\Phi_I(x-z)\,(\hat{n}^+(z)-\hat{n}^-(z))\,d^2z\right].$$

The mean value of the product of the statistical operators can be calculated with allowance for (13), (16), and (17):

$$\langle T\,(\hat{n}^+(z)-\hat{n}^-(z))\,(\hat{n}^+(u)-\hat{n}^-(u))\rangle^{\text{inst}}=\frac{1}{Z_{\text{eff}}}\left(\frac{\delta Z_{\text{eff}}[\mu^+,\mu^-]}{\delta\mu^+(z)\,\delta\mu^+(u)}+\right.$$

$$\left.\frac{\delta Z_{\text{eff}}[\mu^+,\mu^-]}{\delta\mu^-(z)\,\delta\mu^-(u)}-\frac{\delta Z_{\text{eff}}[\mu^+,\mu^-]}{\delta\mu^+(z)\,\delta\mu^-(u)}-\frac{\delta Z_{\text{eff}}[\mu^+,\mu^-]}{\delta\mu^-(z)\,\delta\mu^+(u)}\right)\Bigg|_{\substack{\mu^+\equiv 0\\ \mu^-\equiv 0}}=$$

$$n\delta(z-u)-4a^2\left\langle T\,\exp\frac{\sigma(z)}{F_s}\exp\frac{\sigma(u)}{F_s}\sin\frac{\Sigma(z)}{F_v}\sin\frac{\Sigma(u)}{F_v}\right\rangle^{\Sigma,\sigma}.$$

Developing perturbation theory near the vacuum $\langle\Sigma\rangle=0$, $\langle\sigma\rangle=\sigma_0$, we obtain

$$\langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle^{\text{inst}}=\left(\frac{2\pi}{e}\right)^2\varepsilon_{\mu\nu}\varepsilon_{\tau\rho}\int\partial^2\Phi_I(x-z)\,\partial^2\Phi_I(y-u)\times$$

$$\left[n\delta(z-u)-\frac{4a^2}{F^2}\exp\left(\frac{2\sigma_0}{F_s}\right)\Delta^\Sigma(z-u)\right]d^2z\,d^2u.$$

Here, the function Δ^Σ satisfies Eq. (4) with mass m_Σ. Going over to the momentum space and taking into account (19), (4), and (18), we obtain

$$\langle F_{\mu\nu}(p)F_{\tau\rho}(k)\rangle^{\text{inst}}=\varepsilon_{\mu\nu}\varepsilon_{\tau\rho}\left(\frac{2\pi}{e}\right)^2(2\pi)^2\delta(p+k)\,n\,|(\partial^2\Phi_I)(p)|^2\frac{p^2+m_v^2}{p^2+m_\Sigma^2}.$$

Here, $(\partial^2\Phi_I)(p)$ is the Fourier transform of the function $\partial^2\Phi_I(x)$, and in accordance with the asymptotic behavior (3) it has at small p the form $m_v^4f_v^2[2\pi(p^2+m_v^2)]^{-1}$.

Adding the obtained result to the result (20) of perturbation theory, we obtain

$$\langle F_{\mu\nu}(p)F_{\tau\rho}(k)\rangle=\varepsilon_{\mu\nu}\varepsilon_{\tau\rho}\delta(p+k)\,\frac{p^2+m_\Sigma^2-m_v^2}{p^2+m_\Sigma^2}. \tag{21}$$

Note that for the density of the topological charge (see (5)) we have

$$\langle q(p)q(k)\rangle=\left(\frac{e}{2\pi}\right)^2\delta(p+k)\,\frac{p^2+m_\Sigma^2-m_v^2}{p^2+m_\Sigma^2}. \tag{22}$$

Note that in (21) and (22) the perturbation-theory pole $1/(p^2+m_V^2)$ is indeed replaced by the pole $1/(p^2+m_\Sigma^2)$.

Postponing for a moment the discussion of the obtained relations, we give the results of the calculation of $\langle\varphi^2(p)\varphi^2(k)\rangle$, which is done like the above calculation.

For this Green's function, the principle of correlation weakening holds:

$$\lim_{|x-y|\to\infty}\langle T\varphi^2(x)\varphi^2(y)\rangle=(\langle\varphi^2(x)\rangle)^2,$$

where $\langle\varphi^2\rangle$ is found to be

$$\langle\varphi^2(x)\rangle=\frac{c^2}{2}\left[1-2n\int\Psi_I(x)\,d^2x+n\int\Psi_I^2(x)\,d^2x+\right.$$

$$\left.4a^2\left[\int\Psi_I(x)\,d^2x\right]^2+\frac{4a^2}{F_s^2}\int\Psi_I(x)\Psi_I(y)\Delta^\sigma(x-y)\,d^2x\,d^2y\right].$$

The function Δ^σ satisfies (4) with mass m_σ. For small momenta ($p \ll 1/m_{V,s}$)

$$\langle\varphi^2(k)\varphi^2(p)\rangle^{\text{inst}}=(2\pi)^2\delta(p+k)\delta(k)\,(\langle\varphi^2(x)\rangle)^2+\frac{c^4}{4}(2\pi)^2\delta(p+k)\,n\left[\frac{4f_s^2}{(2\pi)^2}\frac{1}{(k^2+m_s^2)(k^2+m_\sigma^2)}-\right.$$

$$\left.\frac{4f_s}{2\pi}\frac{\Psi_I^2(k)}{k^2+m_\sigma^2}+\Psi_I^2(k)\Psi_I^2(k)\frac{k^2+m_s^2}{k^2+m_\sigma^2}\right].$$

Here, $\Psi_I^2(k)$ is the Fourier transform of $\Psi_I^2(x)$; the singularities of the function $\Psi_I^2(k)$ begin with the square of twice the mass of the scalar particle. The result of perturbation theory is $\langle\varphi^2(p)\varphi^2(k)\rangle^{PT}=\delta(p+k)c^2/(p^2+m_s^2)$ (to avoid double counting, we have here omitted the part containing $c^4\delta(k)$). As a result, we obtain

$$\langle\varphi^2(k)\varphi^2(p)\rangle=(2\pi)^2\delta(p+k)\delta(k)(\langle\varphi^2(x)\rangle)^2+$$
$$c^2\delta(p+k)\frac{1}{p^2+m_\sigma^2}\left[1-2\pi f_s nc^2\Psi_I^2(p)+(2\pi)^2 n\frac{c^2}{4}(\Psi_I^2(p))^2(p^2+m_s^2)\right]. \tag{23}$$

We now discuss the obtained results (21) and (23). It follows from them that in the model (1) there are particles with masses m_Σ and m_σ, and these can be identified with the particles Σ and σ in the Lagrangian (14) (note that in the present paper we do not consider questions associated with renormalizations). The instanton effects have had the consequence that in the physical subspace there are no particles with masses m_V and m_S; they have been replaced by neutral particles with masses m_Σ and m_σ, and the shift of the masses has the order $\exp(-S_{inst})$ (see (18)).

The fields Σ and σ can be approximated by the operators q(x) (see (5)) and $\varphi^2(x)$. Taking the residues at the poles in the relations (21) and (23), we find

$$|\langle 0|q(0)|\Sigma\rangle|=\frac{e}{2\pi}m_v,\quad |\langle 0|\varphi^2(0)|\sigma\rangle|=c(1-\pi f_s nc^2\Psi_I^2(k)|_{k^2=-m_\Sigma^2}),\quad n\ll 1.$$

We note that the residue in (21) is negative. In the Minkowski space, the residue will be positive because of the additional imaginary units in the definition of the pseudoscalar q(x).

Thus, at large distances we have the approximate equations

$$q(x)\approx|\langle 0|q(0)|\Sigma\rangle|\Sigma(x),\quad \varphi^2(x)\approx|\langle 0|\varphi^2(0)|\sigma\rangle|\sigma(x).$$

It is clear from this that the field σ is a true scalar and Σ a pseudoscalar.

5. Conclusions

Considering the approximation of a rarefied instanton gas, we have calculated the nonperturbative contribution to the vacuum-vacuum transition amplitude in the Euclidean variant of two-dimensional scalar electrodynamics. We have found that it is possible to go over to an effective low-energy Lagrangian containing one scalar and one pseudoscalar field that are not transformed by the gauge group. The masses of these fields are shifted with respect to the masses of the fields A_μ and φ calculated in accordance with the Higgs mechanism by an amount of order $\exp(-S_{inst})$, where S_{inst} is the instanton action.

In Sec. 4 we have shown how the effective Lagrangian can be used to calculate the various vacuum expectation values of the model with allowance for the nonperturbative effects associated with the instantons.

The Fourier transforms of the propagators $\langle TF_{\mu\nu}(x)F_{\tau\rho}(y)\rangle$, $\langle T\varphi^2(x)\varphi^2(y)\rangle$ contain poles with masses equal to the bare masses of the particles contained in the effective Lagrangian. Therefore, it can be asserted that at large distances the interaction is transmitted by particles neutral with respect to the gauge group.

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