Superlattices and Microstructures, Vol. 20, No. 4, 1996

Charge sensitivity of single-electron transistor with superconducting electrodes

ALEXANDER N. KOROTKOV
Department of Physics, State University of New York, Stony Brook, NY 11794-3800, U.S.A. and Nuclear Physics Institute, Moscow State University, Moscow 119899 Russia

(Received 20 May 1996)

The noise-limited charge sensitivity of a single-electron transistor with superconducting electrodes operating near the threshold of quasiparticle tunneling, can be considerably higher than that of a similar transistor made of normal metals or semiconductors. The reason is that the superconducting energy gap, in contrast to the Coulomb blockade, is not smeared by the finite temperature. The same reason leads to the increase of the maximum operation temperature due to superconductivity.

Key words: single-electron transistor, superconductivity, noise properties.

The simplest and most thoroughly studied single-electron [1] circuit is the Single Electron Transistor [2] (SET) which consists of two tunnel junctions connected in series. At low temperatures ($T \ll e^2/C_\Sigma$, $C_\Sigma = C_1 + C_2$ where C_1 and C_2 are the junction capacitances) the current through this structure depends on the background charge Q_0 of the central electrode (the dependence is periodical with a period equal to the electron charge e). Hence, by controlling Q_0 (for example, by a capacitive gate) it is possible to control the current I through the circuit. The possibility for use of the SET as a highly-sensitive electrometer has been confirmed in numerous experiments. It has been noticed [3–5] that the superconductivity of electrodes improves the performance of the SET (operating near the threshold of quasiparticle tunneling) as an electrometer in comparison with the normal-state operation. This issue will be a subject of the quantitative analysis in the present paper (see also Ref. [6]).

There are two major characteristics of the SET operation as an electrometer. The first one is the amplitude of the output signal modulation for Q_0 variations larger than e. It was found experimentally [4] that the use of superconducting electrodes increases the modulation amplitude of current I (for fixed bias voltage V), especially at temperatures comparable to e^2/C_Σ, thus increasing the maximum temperature. The theoretical results of the present paper confirm this statement for both $NISIN$ and $SISIS$ structures.

The other, even more important characteristic of the SET operation is the noise-limited sensitivity (ability to detect variations of Q_0 much smaller than e). In the present-day technology the sensitivity is typically limited by 1/f noise which is most likely caused by random trapping-escape processes in nearby impurities. However, with technological improvement one can expect the reduction of the noise due to impurities. Then the charge sensitivity of the SET would achieve the limit determined by the intrinsic noise [7, 8] of the

0749–6036/96/080581 + 05 $25.00/0

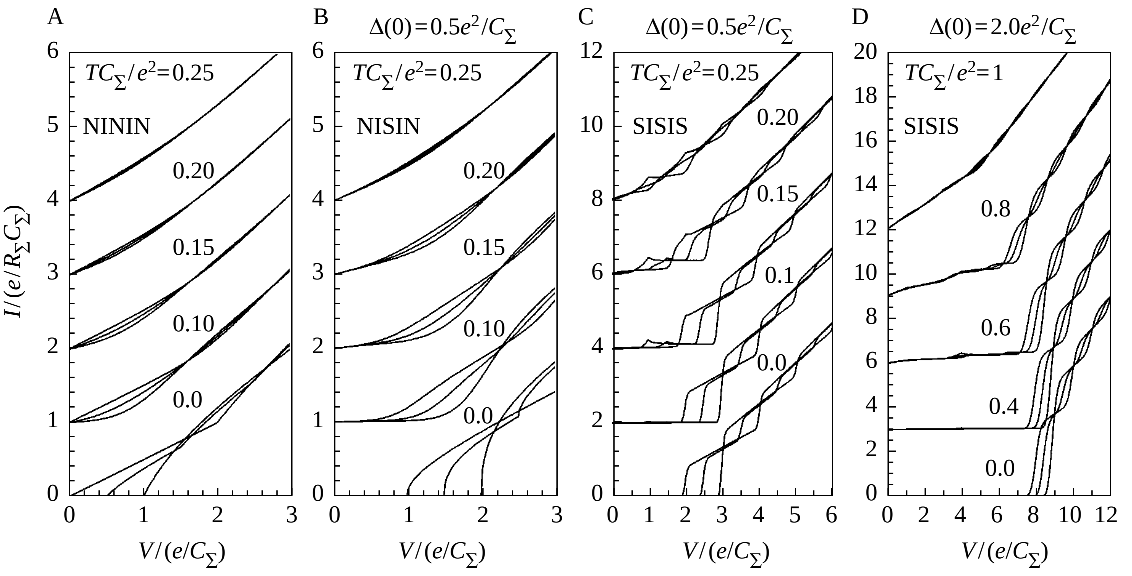

Fig. 1. I–V curves for (A) $NININ$, (B) $NISIN$ (or $SINIS$), and (C) and (D) $SISIS$ SETs for three values of Q_0 (0, $e/4$, and $e/2$) and several temperatures T. The curves for different T are offset vertically for clarity. Notice that the modulation by Q_0 survives up to higher T in the superconducting transistors.

device caused by random electron jumps through tunnel junctions (this 'white' noise has been recently measured in experiment [9]). Though the theory of the 'classical' thermal/shot intrinsic noise of the SET is applicable to the general case of one-particle tunneling (normal metals, semiconductors, quasiparticle current in superconductors, etc.), most numerical results in Refs [7] and [8] as well as in a number of subsequent papers on this subject (see, e.g. Refs [10–13]) were obtained only for SETs made of normal metals. (Recently some generalization was done [14] to include the possibility of two-particle tunneling which can be important in the superconducting case. Let us also mention Ref. [5] in which the noise in $NISIN$ SET was briefly considered.)

In the present paper we apply the theory of Refs [7] and [8] to the cases of capacitively coupled superconducting $SISIS$ and $NISIN$ SETs (the analysis of a resistively coupled SET can be done in a similar way—see Ref. [7]). We show that the noise-limited sensitivity of a SET-electrometer can be considerably improved by the use of superconducting electrodes.

We consider only the quasiparticle tunneling, neglecting the Josephson current, resonant tunneling of Cooper pairs, Andreev reflection, and cotunneling. This assumption is appropriate when the Josephson coupling is negligible and the normal state resistances R_1 and R_2 of tunnel junctions are well above the resistance quantum $R_Q = \pi\hbar/2e^2$. We use the 'orthodox' theory [1, 2] of the SET and the BCS theory [15] for the calculation of the tunneling rates.

Figure 1 shows the I–V curves at different temperatures for (A) the normal metal $NININ$ case, (B) $NISIN$ case (which is equivalent to $SINIS$ case), and (C) and (D) $SISIS$ case. SETs with $C_1 = C_2$ and $R_1 = R_2 = R_\Sigma/2$ are chosen, and we neglect the gate capacitance C_g because it can always be formally distributed between C_1 and C_2 (see, e.g. Ref. [16]). Three curves in each set represent $Q_0 = 0$, $e/4$, and $e/2$, respectively. Temperature increase decreases the superconducting energy gap $\Delta(T)$ (which is assumed to be equal in all S-electrodes) leading to the noticeable shift to the left of the positions of the current jumps in Fig. 1C and D. The pure BCS theory would lead to the abrupt jumps of the current in $SISIS$ case. To take into account the unavoidable smoothing of the jumps in reality, we assume additionally the inhomogeneous broadening of $\Delta(0)$ with Gaussian distribution characterized by the dispersion w_0. This phenomenological parameter is chosen as $w_0 = 0.05\Delta(0)$ in Fig. 1C and D (for finite temperatures $w(T) = w_0[\Delta(T)/\Delta(0) - (T/\Delta(0))(d\Delta(T)/dT)]$ was used).

One can see that in the normal metal case the current I can be considerably modulated ($I_{max}/I_{min} \gtrsim 2$) by Q_0 (V is fixed) only at $T \lesssim 0.15e^2/C_\Sigma$, while at $T = 0.3e^2/C_\Sigma$ the modulation is already negligible, $(I_{max} - I_{min})/I_{max} \simeq 5\%$. Notice that the maximum relative modulation is achieved at small voltages and does not depend on ratios C_1/C_2 and R_1/R_2.

$NISIN$ transistor with $\Delta(0) = 0.5e^2/C_\Sigma$ shows considerable modulation crudely up to $T \approx 0.2e^2/C_\Sigma$, while $SISIS$ transistors with $\Delta(0) = 0.5e^2/C_\Sigma$ and $\Delta(0) = 2.0e^2/C_\Sigma$ operate well almost up to the critical temperature T_c ($T_c/(e^2/C_\Sigma) = 0.28$ and 1.14, respectively). The case $\Delta(0) = 0.5e^2/C_\Sigma$ corresponds to the typical present-day experimental situation with aluminum junctions and $C_\Sigma \approx 0.4$ fF (see, e.g. Ref. [4]). Comparison of Fig. 1C and D shows that the increase of $\Delta(0)$ provides further improvement of the transistor performance at high temperatures. Using Fig. 1D one can predict the operation of the niobium-based SET with $C_\Sigma \approx 0.2$ fF (current state-of-the-art for aluminum junctions) at temperatures up to 7 K.

Superconductivity improves the SET performance at relatively high temperatures because, in contrast to the Coulomb blockade, the superconducting energy gap is not smeared by the finite temperature. In the normal metal case the I–V curve has a cusp at the Coulomb blockade threshold

$$V_t = \min_{i,n}\{V_{i,n} \mid V_{i,n} > 0\}, \qquad \text{where } V_{i,n} = \frac{e}{C_i}\left(\frac{1}{2} + (-1)^i\left(n + \frac{Q_0}{e}\right)\right), \tag{1}$$

and this cusp is rounded within the voltage interval proportional to the temperature. In $SISIS$ case the jump of the I–V curve at V_t, which is shifted due to the energy gap,

$$V_t = \min_{i,n}\{V_{i,n} + 2\Delta(T)C_\Sigma/eC_i \mid V_t > 4\Delta(T)\}, \tag{2}$$

remains sharp even at $T \sim \Delta(T)$, and the subthreshold current increase is only proportional to $\exp(-T/\Delta(T))$. This explains why $SISIS$ transistor shows considerable dependence on Q_0 for the temperatures almost up to T_c even if $T \gtrsim e^2/C_\Sigma$. In $NISIN$ case the I–V curve in the vicinity of

$$V_t = \min_{i,n}\{V_{i,n} + \Delta(T)C_\Sigma/eC_i \mid V_t > 2\Delta(T)\} \tag{3}$$

is rounded by the finite temperature, that makes $NISIN$ transistor worse than $SISIS$ transistor, however, it is still better than usual $NININ$ transistor.

Now let us consider the noise-limited sensitivity of the SET. The minimum detectable charge for the given bandwidth Δf is

$$\delta Q_0 = (S_I\Delta f)^{1/2}/(\partial I/\partial Q_0) \tag{4}$$

where the spectral density S_I of the current noise is taken in the low frequency limit. The ultimate low-temperature ($T \ll e^2/C_\Sigma$) sensitivity in the $NININ$ case is [7, 8]

$$\min \delta Q_0 \simeq 2.7C_\Sigma(R_{min}T\Delta f)^{1/2}, \qquad R_{min} = \min\{R_1, R_2\}. \tag{5}$$

This result can be somewhat improved in the $NISIN$ SET (with the same resistances) operating near the threshold V_t of quasiparticle tunneling. At low temperatures, $T \ll \min\{e^2/C_\Sigma, \Delta(T)\}$, and for V close to nondegenerate V_t, we can use approximation

$$S_I \simeq 2eI, \qquad I \simeq I_{0,i}((V - V_t)C_1C_2/C_iC_\Sigma), \tag{6}$$

where

$$I_{0,i}(v) = (1/eR_i)[T\Delta(T)/2]^{1/2}\int_0^\infty dy/\sqrt{y}/[1 + \exp(y + (\Delta - ev)/T))]^{-1} \tag{7}$$

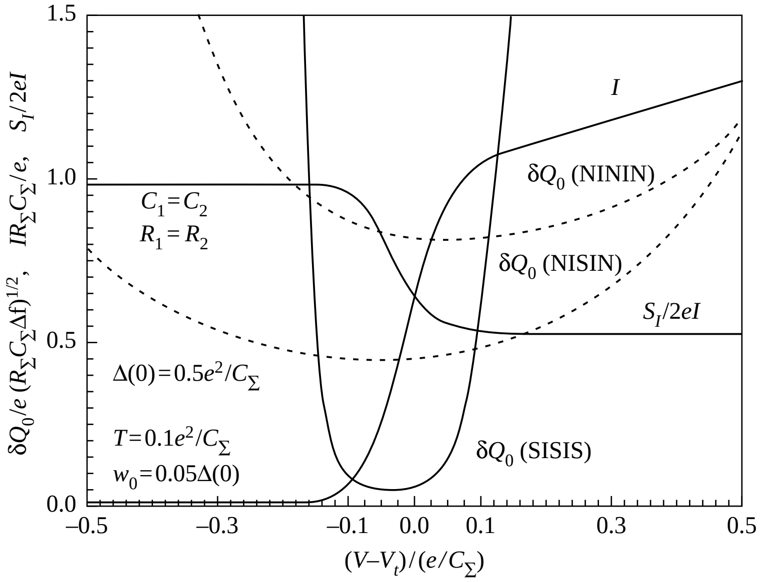

Fig. 2. The minimum detectable charge δQ_0, the current I, and the ratio $S_I/2eI$ as functions of the bias voltage V for $SISIS$ SET. Dashed lines show δQ_0 for $NININ$ and $NISIN$ SETs. The best sensitivity is achieved in $SISIS$ case.

is the 'seed' I–V curve of ith junction. Then the ultimate sensitivity is given by equation

$$\min \delta Q_0 = C_\Sigma (2e\Delta f)^{1/2} \min_v \{\sqrt{I_0(v)}/(dI_0/dv)\}, \tag{8}$$

and finally we get the result

$$\min \delta Q_0 \simeq 2.6 C_\Sigma (R_{min} T \Delta f)^{1/2} [T/\Delta(T)]^{1/4} \tag{9}$$

which is better than $NININ$ sensitivity when $T < \Delta(T)$. The main reason for the improvement is the increase [3–5] of the transfer coefficient $\partial I/\partial Q_0 \simeq V(1/C_i)(\partial I/\partial V)$, because the differential resistance R_d of the 'seed' I–V curve near the onset of quasiparticle tunneling is less than R_i. Notice that the 'orthodox' theory used here is valid only if $R_d \gtrsim R_Q$ because the cotunneling processes [17, 5] impose the lower bound for $(\partial I/\partial V)^{-1}$ on the order of R_Q [18]. For relatively high temperatures the ratio of minimum δQ_0 in $NISIN$ and $NININ$ cases is larger than $[\Delta(T)/T]^{1/4}$ (e.g., compare the dashed lines in Fig. 2) because $NININ$ sensitivity starts to deviate up from the low-temperature approximation at smaller T than $NISIN$ sensitivity.

The improvement of the ultimate sensitivity is more significant in $SISIS$ SET. For pure BCS model the 'orthodox' theory gives infinite derivative $\partial I/\partial Q_0$ at $V = V_t$ even for finite temperature leading to $\delta Q_0 \to 0$. Hence, the 'orthodox' ultimate sensitivity depends on the imperfection of the current jump which is described in our model by the energy gap spread w_0 ($w_0 \ll \min\{\Delta(T), e^2/C_\Sigma\}$).

Figure 2 shows δQ_0 together with current I and ratio $S_I/2eI$, as functions of the voltage for the symmetric $SISIS$ SET with parameters $\Delta(0) = 0.5e^2/C_\Sigma$, $w_0 = 0.05\Delta(0)$, $T = 0.1e^2/C_\Sigma$, and $Q_0 = 0.25e$ (numerical calculations are done using the method described in Refs [7] and [8]). Dashed lines show δQ_0 for similar $NININ$ and $NISIN$ SETs. One can see that the sensitivity of $SISIS$ SET is much better than for $NININ$ and $NISIN$ cases within a relatively narrow voltage range which corresponds to the jump of current.

In contrast to $NININ$ and $NISIN$ cases, the approximation $S_I \simeq 2eI$ is not accurate in the vicinity of V_t for $SISIS$ SET even at low temperatures (see Fig. 2) because the relatively large tunneling rate in the junction determining V_t, is comparable to the tunneling rate in the other junction. This approximation is valid only if $T \ll \Delta(T) \ll e^2/C_\Sigma$, and would lead to inaccuracy typically about 10% for the analytical calculation of $\min \delta Q_0$ if $T \ll \Delta(T) \sim e^2/C_\Sigma$. Nevertheless, it can be used as a crude estimate. Using eqn (8) and smoothed by w_0 low-temperature ($T \ll \Delta(T)$) 'seed' I–V curve for SIS junction [15] we get

$$\min \delta Q_0 \simeq 1.8 C_\Sigma \left(R_{min} \Delta f w_0^2/\Delta(T)\right)^{1/2}. \quad (10)$$

Notice that the numerical factor depends on the particular model describing the shape of the current jump. Comparing eqn (10) with the result for $NININ$ SET, we see that the temperature T is replaced in $SISIS$ case by $w_0^2/\Delta(T)$. Hence, the ultimate sensitivity is better in $SISIS$ SET (resistances are the same) with sufficiently narrow width of the current jump, $w_0 < (T\Delta(T))^{1/2}$.

In the case of very sharp 'seed' I–V curve, $w_0 \lesssim \Delta(T) R_Q/R_i$, the slope of the jump of the SET I–V curve is determined by cotunneling [17] and it cannot be sharper than crudely R_Q^{-1} [18]. Then $\min \delta Q_0$ is on the order of $C_\Sigma(\Delta f \Delta(T) R_Q^2/R)^{1/2}$ (we assume $\Delta(T) \gtrsim e^2/C_\Sigma$, $R_1 = R_2$), and the ultimate sensitivity is better than for $NININ$ SET if $T \gtrsim \Delta(T)(R_Q/R)^2$.

In conclusion, the superconductivity of electrodes can considerably improve the performance of the single electron transistor as an electrometer at relatively large temperatures, if the superconducting energy gap is comparable or larger than e^2/C_Σ.

Acknowledgements—The author thanks D. V. Averin and K. K. Likharev for valuable discussions and critical reading of the manuscript. The work was supported in part by ONR grant N00014-93-1-0880 and AFOSR grant 91-0445.

References

[1] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb, Amsterdam, Elsevier: p. 173 (1991).
[2] K. K. Likharev, IEEE Trans. Magn. **23**, 1142 (1987).
[3] P. Lafarge, H. Pothier, E. R. Williams, C. Urbina, and M. H. Devoret, Z. Phys. B **85**, 327 (1991).
[4] J. M. Hergenrother, M. T. Tuominen, T. S. Tighe, and M. Tinkham, IEEE Trans. Appl. Supercond. **3**, 1980 (1993).
[5] W. Krech and H.-O. Müller, Mod. Phys. Lett. B **8**, 605 (1994).
[6] A. N. Korotkov, to be published in Appl. Phys. Lett. **69**, (1996).
[7] A. N. Korotkov, D. V. Averin, K. K. Likharev, and S. A. Vasenko, *Proc. SQUID'91* (Berlin, 1991); in *Single-Electron Tunneling and Mesoscopic Devices*, edited by H. Koch and H. Lubbig Springer-Verlag, Berlin Heidelberg: p. 45 (1992).
[8] A. N. Korotkov, Ph.D. Thesis (Moscow State University, 1991); see also A. N. Korotkov, Phys. Rev. B **49**, 10381 (1994).
[9] H. Birk, M. J. M. de Jong, and C. Schönenberger, Phys. Rev. Lett. **75**, 1610 (1995).
[10] S. Hershfield, J. H. Davies, P. Hyldgaard, C. J. Stanton, and J. W. Wilkins, Phys. Rev. B **47**, 1967 (1993).
[11] L. Y. Chen, Mod. Phys. Lett. B **7**, 1677 (1993); *ibid*. **8** 841 (1994).
[12] U. Hanke, Yu. M. Galperin, K. A. Chao, and N. Zou, Phys. Rev. B **48**, 17209 (1993).
[13] W. Krech and H.-O. Müller, Z. Phys. B **91**, 423 (1993).
[14] U. Hanke, Yu. Galperin, K. A. Chao, M. Gisselfalt, M. Jonson, and R. I. Shekhter, Phys. Rev. B **51**, 9084 (1995).
[15] M. Tinkham, *Introduction to superconductivity*, New York: McGraw-Hill (1996).
[16] A. N. Korotkov, Phys. Rev. B **49**, 16518 (1994).
[17] D. V. Averin and A. A. Odintsov, Phys. Lett. A **140**, 251 (1989).
[18] D. V. Averin and A. N. Korotkov, 'Quasiparticle transport in superconducting SET transistor' to be published,