THERMODYNAMIC RESTRICTIONS ON THE MECHANISM OF 1/f NOISE*

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It is shown that thermodynamics makes it possible to reject specific equilibrium mechanisms of 1/f noise.

1. The problem of 1/f noise (flicker noise) has recently been attracting general interest [1-4]. We shall talk about electric "current noise," which is observed when a current is passed through samples. The mechanism responsible for the appearance of the excess noise is still not fully understood. The models employed for explaining 1/f noise can be divided into two classes: "degradational" and thermodynamically equilibrium.

In the "degradational" models [5] it is assumed that the sample contains "traces of rapid preparation" and slowly approaches the thermodynamically equilibrium state — the state of minimum free energy. This process (aging, degradation) is accompanied by fluctuations, which give rise to 1/f noise. For example, if a semiconductor is doped nonuniformly, then after a very long time τ_{max} diffusion leads to a uniform distribution of impurities. In degradational models there is no thermodynamic equilibrium. Thermodynamic equilibrium is established over a time τ_{max}, and the spectrum thus reaches a plateau at rates of the order of τ_{max}^{-1}. The 1/f noise in this case has a low-frequency boundary, which eliminates the question of the divergence of the integral of the spectral density of the noise. In degradational theories the noise intensity and the time τ_{max} must be related with one another, and this relationship must be given by the theory, though no one has yet derived such formulas. The time τ_{max} has a lower limit, for example, due to guaranteed device lifetimes, estimated from some physical data, and the intensity of the noise is measured. Such investigations would be of great interest.

In thermodynamically equilibrium models fluctuations about the equilibrium state are studied [6]. In this paper it will be shown that on the basis of the first and second laws of thermodynamics some assertions can be made concerning the noise properties, in particular, specific equilibrium mechanisms of 1/f noise can be rejected.

We call attention to two circumstances.

In writing this paper we tried not to use the formulas and concepts of statistical physics and thereby not to use the assumptions about ergodicity [7]. In nonergodic systems a Gibbs distribution may not exist and the fluctuation-dissipation theorem has not been proved, but the first principle of thermodynamics — the law of conservation of energy — and the second principle — heat flows from a hot body to a cold body — should be valid. We shall show below (for example, Sec. 2) that by using only these principles (the laws of thermodynamics) it is possible to make definite assertions for nonergodic systems also. On the basis of what we have said above, in this paper we preferred an exposition that did not employ the formulas for the entropy and Gibbs distribution.

In equilibrium models the time after "preparation" of the sample does not appear in the theory, and for this reason all requirements of thermodynamics, which studies the thermodynamic equilibrium for $-\infty < t < +\infty$, should be satisfied. Thermodynamics forbids any breakdown of equilibrium, however small, and in addition it is not important that the elapsed time required for observing an excess above the level of the fluctuations can exceed the age of the universe. For this reason, the cases of thermodynamic equilibrium which are studied

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below are analogous to thought experiments in quantum mechanics [8] — they cannot be performed in reality, but they carry the force of proof in the analysis of equilibrium models in which time does not enter explicitly.

2. Statistical physics makes it possible to calculate equilibrium fluctuations for ergodic conservative systems ([6], Chapter 12). For $\hbar\omega \ll kT$ the equipartition theorem is valid: energy $kT/2$ is allotted to each degree of freedom. Resistance is not a thermodynamic quantity. The noise emf of the resistance R can be calculated by considering a system consisting of a resistor and a capacitor connected in parallel. Since the charge on the capacitor must fluctuate (since this is a thermodynamic degree of freedom), energy will flow from the capacitor to the resistor. The noise emf can be calculated from the condition that in thermodynamic equilibrium there are no directed energy flows [9]:

$$\overline{\mathcal{E}^2} = 4kTR\,\Delta f, \tag{1}$$

where Δf is the frequency interval. We emphasize that in Eq. (1) $\overline{\mathcal{E}^2}$ depends only on the resistance and does not depend on its nature. In this connection there arises the following question: Is formula (1) valid for nonergodic systems, when the standard statistical theory of fluctuations cannot be used? We shall prove that since the basic laws of thermodynamics — that perpetual motion of the first and second kind is impossible — are also valid for nonergodic systems, Eq. (1) also remains valid. Consider a circuit consisting of two resistances — one resistance made of a material in which ergodicity does not hold and the other is a standard material. With the help of purely reactive loss-free elements it is possible to construct an ideal transformer and a filter, whose frequency characteristic is as close as desired to that of an ideal filter, and a prescribed characteristic impedance. From thermodynamics it follows that the spectrum of fluctuations of the noise emf is identical for any resistors with the same nominal rating, because if the spectrum is not the same, then by connecting two such resistors through a filter it is possible to get one of the resistors to heat up and the other to cool down. Thus breakdown of ergodicity within a resistor should not affect the external characteristics.

3. We now consider the question of fluctuations of the resistance of the sample. We first study the auxiliary problem of the relation between fluctuations of the resistance of the sample δR and the local fluctuations of the conductivity $\delta\sigma$ (the conductivity can be nonuniform over the sample).

We start from the fact that the electric power absorbed is equal to the Joule heat released:

$$\oint \varphi \vec{j} d\vec{S} = \int \sigma E^2 dV. \tag{2}$$

Here $\vec{E} = -\Delta\phi$, $\vec{j} = \sigma\vec{E}$, $\sigma = \sigma(\vec{r})$, and in the integration over the surface of the sample the normal is directed inward. Varying Eq. (2) with respect to σ and using the continuity of the current, it is easy to obtain, to first order in $\delta\sigma$,

$$\oint (\varphi\delta\vec{j} + \vec{j}\delta\varphi) d\vec{S} = \int \delta\sigma E_0^2 dV, \quad \vec{E}_0 = \bar{\vec{E}}. \tag{3}$$

Here E_0 is the average value of the electric field. For simplicity, we shall study a sample with two contacts. In addition, let the current I be constant and let the resistance fluctuation δR lead to a change of the potential difference u on the contacts. Then, from Eq. (3) we have

$$-\int \delta\sigma E^2 dV = I\delta u = I^2 R(\delta R/R_0), \quad R_0 = \bar{R}. \tag{4}$$

We emphasize that Eq. (4) contains a volume integral of $\delta\sigma$ with the weighting function E^2. Spatial modes of the fluctuations $\delta\sigma$ which are orthogonal to the weighting function cannot contribute to the fluctuations of the resistance. In other words, only one spatial mode is important. We would like to point out that the situation here is radically different from the heat capacity — all modes of lattice vibrations contribute to the energy of a material and for this reason the summation over all modes is performed ([6], Chapter 6). In this connection, in our opinion, the analogy between 1/f noise and heat capacity of spin glasses [10, 11] requires more detailed justification.

We note that the relation (4) can also be applied in the case of the four-contact method of measurement, if the measuring contacts are placed between the current contacts; in this case, the volume integral extends only over the region between the measuring contacts.

4. The resistance of the sample is not a thermodynamic variable which characterizes the state of the sample. This makes it possible to avoid contradictions with the infinite variance in the case of 1/f noise. However the conductivity of a material depends on the microscopic state of the material: to first order $\delta\sigma = \Sigma S_K \eta_K$, where S_K are constants and η_K are coordinates characterizing the microscopic state. If all η_K are thermodynamic variables, then the variance of each one is limited. If the number of such variables is finite, then the variance is finite, which is impossible for 1/f noise.

In order to clarify this situation we shall study a specific example. In [2] the question of the possibility that conductivity fluctuations with a 1/f spectrum are governed only by fluctuations of the number density of free carriers n_0 was discussed in detail. We shall show that thermodynamics gives an unequivocal negative answer. The conductivity is linear in the free-carrier density.

$$\sigma = e\mu n_0, \quad \delta\sigma/\sigma_0 = \delta n_0/n_0, \quad \sigma_0 = \tilde{\sigma}, \tag{5}$$

where μ is the mobility. From Eq. (4), we obtain for the uniform case

$$\frac{\overline{\delta R^2}}{R^2} = \frac{\overline{\delta N_0^2}}{N_0^2} - \frac{i}{N_0}, \quad \sigma E_0^2 = \text{const.} \tag{6}$$

Here N_0 is the total number of free carriers in the sample.

Hoog's formula for the finite frequency interval $f_1 < f < f_2$ gives the variance

$$\frac{\overline{\delta R^2}}{R^2} = \int_{f_1}^{f_2} \frac{\alpha}{N_0} \frac{df}{f} = \frac{\alpha}{N_0} \ln \frac{f_2}{f_1}, \tag{7}$$

where $\alpha = 2\cdot 10^{-3}$ is Hoog's constant. Comparing Eqs. (6) and (7) we have

$$\overline{\delta N_0^2} = \alpha N_0 \ln f_2/f_1.$$

It is obvious that 1/f noise cannot be explained at all frequencies by this model of fluctuations of the free-carrier density, since $\ln(f_2/f_1)$ is in this case an infinite quantity, while $(\delta N_0)^2$ remains finite.

5. We assumed above that 1/f-noise exists at all frequencies, but it is more correct to study frequency bands where 1/f-noise is actually measured. For example, a necessary condition for explaining 1/f-noise in five decades of frequency is

$$\delta N_0^2 \geq 2{,}3\cdot 10^{-2} N_0. \tag{8}$$

We now take into account the quasineutrality of the sample. Then fluctuations of the number of free carriers N_0 are possible only if carriers, bound by traps, appear. Assuming that carrier trapping is independent of carrier emission by each trap $(\delta N_0)^2 = (\delta N_1)^2 \cong N_1$, where N_1 is the effective number of "working" traps. By "working" we mean a trap that is filled with a probability of the order of 1/2, so that it makes a contribution of the order of unity to the variance, and in addition the characteristic carrier trapping and emission time of this trap must fall into the required range $f_2^{-1} < \tau < f_1^{-1}$). Obviously, "working" traps must have energy levels in a band of the order of kT near the Fermi level. Since kT = 0.026 eV at room temperature and the characteristic energy range of the distribution of trap levels is of the order of one electron volt, we obtain, together with Eq. (8), that in order to explain 1/f-noise in five decades the number of traps must be of the order of or greater than the number of free carriers.

For this reason, this model cannot be applied to metals, where the number of free carriers is of the order of the number of atoms, while models which explain 1/f-noise in semiconductors by fluctuations of the free-carrier density, in particular, MacWorter's model, must encounter great difficulties when quantitative comparison with experiment is made since they require too many traps.

6. We now cosider a model in which the mobility μ fluctuates because the properties of the defects which scatter the carriers, for example, electrons, change. Let a defect have two states A and B with close energies, but strongly different electron scattering cross sections $\tilde{\sigma}$, $\tilde{\sigma}_A \neq \sigma_B$. The the fluctuations of the effective number of collisions ν are equal to

$$\delta\nu \approx \nu(\sigma_A \sigma_B)\delta n_A, \quad \delta n_A + \delta n_B = 0, \tag{9}$$

where δn_A is the fluctuation of the concentration of defects in the state A, when $\delta\sigma/\sigma = \delta n_A/n_A$, which is identical in form to Eq. (5), except that instead of the carrier concentration we are studying the defect concentration. We can see that thermodynamics rejects this model as completely as the preceding model.

The formula (9) presumes that the scattering by each defect occurs independently, as is usually assumed in the theory of semiconductors. When the distance between defects is small, however, $\sigma_{A+B} \neq \sigma_A + \sigma_B$. Taking this into account, the number of states of the lattice on the surface of constant energy in phase space which are not equivalent with respect to resistance increases rapidly. Formally, thermodynamics does not reject this case, but no one has studied it yet.

The examples presented in this paper show that thermodynamic analysis of the mechanisms of 1/f-noise is undoubtedly of interest.

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