Density matrix purification due to continuous quantum measurement

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Abstract

We consider the continuous quantum measurement of a two-level system, for example, a single-Cooper-pair box measured by a single-electron transistor or a double-quantum dot measured by a quantum point contact. While the approach most commonly used describes the gradual decoherence of the system due to the measurement, we show that when taking into account the detector output, we get the opposite effect: gradual purification of the density matrix. The competition between purification due to measurement and decoherence due to interaction with the environment can be described by a simple Langevin equation which couples the random evolution of the system density matrix and the stochastic detector output. The gradual density matrix purification due to continuous measurement may be verified experimentally using present-day technology. The effect can be useful for quantum computing. © 2000 Elsevier Science B.V. All rights reserved.

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The active research on quantum computing as well as the progress in experimental techniques have motivated renewed interest in the problems of quantum measurement, including the long-standing "philosophical" questions. In contrast to the usual case of averaging over a large ensemble of similar quantum systems, it is becoming possible to study experimentally the evolution of an individual quantum system. In this paper we consider the continuous measurement of a two-level system by a "weakly responding" [1] detector which can be treated as a classical device.

While after averaging over the ensemble the continuous measurement leads to the gradual decoherence of the system density matrix, the situation is completely different in the case of an individual quantum system. In particular, the system evolution becomes dependent ("conditioned") on the particular detector output. The theory of conditioned evolution of a pure wave function was developed relatively long ago, mainly for the purposes of quantum optics (see, e.g. Ref. [2] and references therein). However, for solid state structures the problem

of continuous quantum measurement with an account of the measurement result has only been addressed recently [1], with the main emphasis on the mixed quantum states and the detector nonideality.

The evolution of the density matrix σ of a double-dot with the tunneling matrix element H and energy asymmetry ε can be described by nonlinear equations

$$
\dot{\sigma}_{11} = -\dot{\sigma}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\sigma_{12} \\
- \sigma_{11}\sigma_{22}(2\Delta I/S_I)[I(t) - I_0], \qquad (1)
$$

$$
\dot{\sigma}_{12} = \mathrm{i}(\varepsilon/\hbar)\sigma_{12} + \mathrm{i}(H/\hbar)(\sigma_{11} - \sigma_{22}) \\
+ (\sigma_{11} - \sigma_{22})(\Delta I/S_I)[I(t) - I_0]\sigma_{12} - \gamma\sigma_{12}, \qquad (2)
$$

where $I(t)$ is the particular detector output (we assume electric current), $I_0 = (I_1 + I_2)/2$, I_1 and I_2 are the average currents corresponding to two localized states of the double-dot, $\Delta I = I_2 - I_1$, S_I is the low-frequency spectral density of the detector shot noise, and the detector nonideality is described by the extra dephasing due to interaction with an "untrackable" environment $\gamma = \Gamma - (\Delta I)^2/4S_I$, where Γ is the dephasing rate in the conventional approach (after ensemble averaging). In particular, the quantum point contact (QPC) can be an

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ideal detector, $\gamma = 0$ (see, e.g. Ref. [3]), while the single-electron transistor (SET) in a typical operation point is a significantly nonideal detector, $\gamma \sim \Gamma$ [4].

Eqs. (1) and (2) allow us to calculate the evolution of the system density matrix if the detector output $I(t)$ is known from the experiment. They can be also used for the simulation, then the term $[I(t) - I_0]$ should be replaced with $[\Delta I(\sigma_{22} - \sigma_{11})/2 + \xi(t)]$ where the random process $\xi(t)$ has zero average and $S_\xi = S_I$. (We use the Stratonovich formalism for stochastic equations.)

Fig. 1 shows the result of such a simulation for a slightly nonideal detector, $\gamma = 0.1\Gamma$, in the case when the evolution starts from the maximally mixed state, $\sigma_{11} = \sigma_{22} = 0.5, \sigma_{12} = 0$. One can see that σ_{12} gradually appears during the measurement, eventually leading to well-pronounced quantum oscillations. In the case $\gamma = 0$ the density matrix becomes almost pure after a sufficiently long time. This gradual purification can be interpreted as being due to the gradual acquiring of information about the system. The detector nonideality, $\gamma \neq 0$, causes decoherence and competes with the purification due to measurement.

In contrast to QPC, the SET as a detector directly affects the two-level system asymmetry ε because of the fluctuating potential $\phi(t)$ of SETs central island. Since there is typically a correlation between fluctuations of $I(t)$ and $\phi(t)$ [5], we should add into Eq. (2) the term $i\sigma_{12}K[I(t) - (\sigma_{11}I_1 + \sigma_{22}I_2)] = i\sigma_{12}K\xi(t)$ where $K = (d\varepsilon/d\phi)S_{\phi I}/S_I\hbar$. This allows the partial recovery of coherence, so that $\gamma = \Gamma - (\Delta I)^2/4S_I - K^2 S_I/4$. The average asymmetry ε should be also renormalized to account for the backaction of $\bar{\phi}$ shift.

To observe the density matrix purification experimentally, it is necessary to record the detector output with sufficiently wide bandwidth, $\Delta f \gg \Gamma$ (possibly, $\Delta f \sim 10^9$ Hz), and plug it into Eqs. (1) and (2). Calcu-
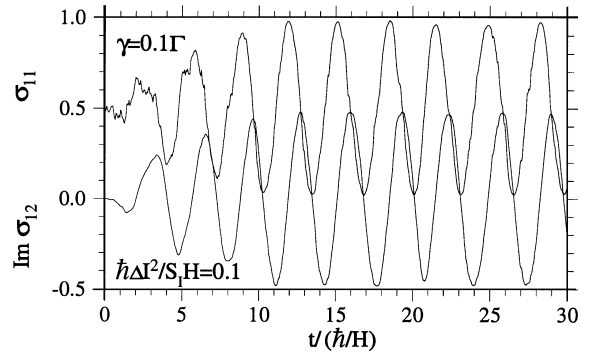

Fig. 1. Gradual purification of the two-level system density matrix $\sigma(t)$ in a course of continuous measurement.

lations will show the development of quantum oscillations with precisely known phase. Stopping the evolution by rapidly raising the barrier ($H \to 0$) when $\sigma_{11} \simeq 1$ and checking that the system is really localized in the first state, it is possible to verify the presented results.

The potential application in quantum computing is the fast initialization of the qubit state (not requiring relaxation to the ground state) after the intermediate measurements.

References

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