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Single-electron tunneling coexisting with the barrier suppression

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We have investigated the charge transport properties of the single-electron transistor in the case of interest when the charging energy of the device is comparable with the characteristic energy of the tunnel barrier suppression. One encounters the problem handling with the artificial tunnel barriers. A new type of the periodic peculiarities on the I–V curve originated from charging effects is predicted and considered in detail.

In recent years the submicron technology advances supported by the appropriate theory [1] have created an exciting possibility to manipulate with single electrons. The possibility arises from the charge effects in the systems of metallic islands connected with the tunnel junctions formed by the oxidation of metal surface. In this case high ($\gg \hbar/e^2$) tunnel resistances provide the number of electrons in each island to be a well-defined value and Coulomb interaction allows to distinguish the states with different numbers of electrons from its electrostatic energy.

Now the experimental [2, 3] and theoretical studies [4] of these single electronic effects are starting in such a system of the considerable physical interest as GaAs/AlGaAs heterostructures. The tunnel barriers in these systems are mostly artificial versus native oxide barriers on the metal surface. They may be produced by making a region with different dopant concentration [5] or simply by applying the electrostatic potential to the system in a certain way [2, 3].

The important feature of these barriers is that the typical barrier height U is much lower and the typical width L is much wider than these parameters for oxide barriers. It makes the tunneling traversal time $\tau \simeq \sqrt{UmL}$ large and as distinct from the metal systems this time begins to play a role in these single electron phenomena.

In the case of single tunnel junction interacting with the electromagnetic environment the effect of the traversal time was considered in refs. [6–8]. It was pointed out that this effect on the I–V curve Coulomb peculiarity positioned at voltage V_C is significant only if $eV_C\tau/\hbar \gg 1$. It is just a condition for a noticeable tunnel barrier suppression by the voltage applied, which makes the I–V curves essentially nonlinear on the voltage scale of $\hbar/e\tau$. From the experiments [2, 3] one can estimate this scale as $\hbar/e\tau \simeq 1$–10 mV and it is quite comparable with the scale of the Coulomb peculiarities observed in these multi-junction systems.

So that there is a time to consider quantitatively single electronic effects coexisted with strong barrier suppression in multi-junction system and we have done this for the simplest but important case of the single-electron transistor (SET).

It is just the conducting island connected by tunnel junctions with two massive electrodes (fig. 1). Its I–V curve exhibits a set of periodically positioned peculiarities ('Coulomb staircase'). The period and clearness of these peculiarities depend on the ratios of junction capacitances and resistances, C_1/C_2 and R_1/R_2. This system was investigated theoretically in refs. [9, 1] and was first manufactured in a controlled way by Fulton and Dolan [10].

Let us remind the basis of the theory men-

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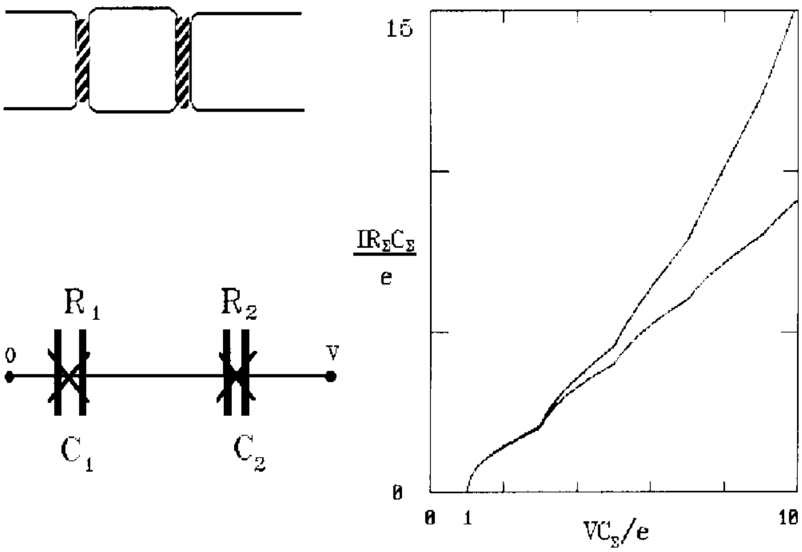

Fig. 1. Single-electron transistor. The crosses over capacitors mean the possibility of tunneling. The lower I–V curve corresponds to conventional SET, the upper one illustrates the weak non-linearity case $E_Q\tau/\hbar = 0.125$. Other parameters are identical for both curves: $C_1/C_2 = 1$, $R_1/R_2 = 10$, $T = 0$. The barriers are symmetric: $\tau_1' = \tau_2 = \tau_1 = \tau_2' = \tau/2$. The periodic Coulomb peculiarities are visible.

tioned. According to that, the SET is described completely by the probabilities σ_n to have an integer charge n in the central island. This charge number n is changed by plus or minus unity whereas the electron tunnels on the first or the second junction in a positive or negative direction (here we define the direction from the first to the second electrode as positive). So the dynamics of the device is determined by four tunneling rates $\Gamma_{1,2}^{+,-}$ that depend upon the charge n and the voltage applied. The master equation is as follows

$$\frac{\partial \sigma_n}{\partial t} = -\sigma_n(\Gamma_1^+(n) + \Gamma_1^-(n) + \Gamma_2^+(n) + \Gamma_2^-(n)) + \sigma_{n-1}(\Gamma_1^+(n-1) + \Gamma_2^-(n-1)) + \sigma_{n+1}(\Gamma_1^-(n+1) + \Gamma_2^+(n+1)) . \qquad (1)$$

The stationary distribution of σ_n obeys the following equation:

$$\sigma_n(\Gamma_1^+(n) + \Gamma_2^-(n)) = \sigma_{n+1}(\Gamma_1^-(n+1) + \Gamma_2^+(n+1)), \quad \sum_n \sigma_n = 1 . \qquad (1a)$$

When we know σ_n we are able to calculate the current:

$$I = e\sum_n \sigma_n(\Gamma_1^+(n) - \Gamma_1^-(n)) = e\sum_n \sigma_n(\Gamma_2^+(n) - \Gamma_2^-(n)) . \qquad (1b)$$

Now our task is to find the rates in the case of the strong barrier suppression. We assume the traversal time to be much more than the typical relaxation time of the electrodynamic environment. This assumption allows us not to take into account any inelastic processes and then to determine the Γs on the way of simple reasoning. Indeed, in this case after the electron moves to the tunnel barrier the charges on the electrodes are redistributed immediately, producing some voltage difference dropping on the junction. This voltage difference can be determined from the energy consideration: the additional work the electron should do to move from one electrode to another being influenced by this voltage difference must equal the difference between the electrostatic energies of the system before and after the tunneling. The last can be computed from the simplest electrostatics using the capacitances and voltage applied to the system. Thus one can express the rates needed in terms of the tunneling rates on the single voltage-biased junction at a given voltage. These rates in its turn are conventionally expressed in terms of the junction I–V curve at a given temperature T with the aid of the detailed balance principle.

Finally we obtain:

$$\Gamma_i^{\mp}(n) = \frac{\pm I_i(\pm F_i^{\mp}(n)/e)}{e(1-\exp(-F_i^{\mp}(n)/T))} \quad (i=1,2)\,, \quad (2)$$

$$F_1^{\pm}(n) = \pm\frac{e^2}{C_\Sigma}\left(\frac{C_2 V}{e} \mp \frac{1}{2} - n - \frac{q_0}{e}\right),$$
$$F_2^{\pm}(n) = \pm\frac{e^2}{C_\Sigma}\left(\frac{C_1 V}{e} \mp \frac{1}{2} + n + \frac{q_0}{e}\right). \quad (3)$$

Here the $I_{1,2}(V)$s are the single-junction I–V curves, q_0 is the so-called 'residual' charge induced on the central island by some external field [1, 10], and $C_\Sigma = C_1 + C_2$.

To include the barrier suppression into consideration we treat the effect in the WKB approximation as was done in ref. [11]. According to ref. [11] the I–V curve is as follows:

$$I(V) = R^{-1}(T) \times\left(\exp\left(\frac{2eV\tau^+}{\hbar}\right) - \exp\left(\frac{-2eV\tau^-}{\hbar}\right)\right)\frac{\hbar}{2e\tau}\,, \quad (4)$$

where

$$R(T) = R\sin\left(\frac{2\pi\tau T}{\hbar}\right)\frac{\hbar}{2\pi\tau T}\,,$$

$$\tau = \int_0^L \mathrm{d}x\left(\frac{2U(x)}{m}\right)^{-\frac{1}{2}},$$

$$\tau^+ = \int_0^L \mathrm{d}x\,\frac{x}{L}\left(\frac{2U(x)}{m}\right)^{-\frac{1}{2}}, \quad \tau^- = \tau - \tau^+\,,$$

R is the low-voltage and low-temperature junction resistance, $U(x)$ the barrier potential, and L the barrier width.

The most important parameter of the problem involved is the ratio of charging energy $E_Q = e^2/2C_\Sigma$ and the typical energy of the barrier suppression, $E_Q\tau/\hbar$. We present here the numerical results for SET I–V curves calculated from eqs. (1a) and (1b) using eqs. (2), (3) and (4) for moderate values of parameter $E_Q\tau/\hbar \simeq 1$ and for $T=0$. These results are supported by the analytical consideration at $E_Q\tau/\hbar \gg 1$ and $E_Q\tau/\hbar \ll 1$. The most evident feature of these I–V curves is that the current grows exponentially when increasing the voltage. We are interested mostly in Coulomb peculiarities on the curves and in this relation we have found the plots of the current logarithmic derivative to be most representative.

First let us consider the case of weak nonlinearity $E_Q\tau/\hbar \ll 1$. The usual Coulomb peculiarities are conserved in this case. Let us remind their origin. At $T=0$ from energy reasons the charge n can have the values from the interval restricted by $(n_{\min}, n_{\max})$. They are equal to $n_{\min} = [\frac{1}{2} - (C_1 V + q_0)/e]$, $n_{\max} = [\frac{1}{2} + (C_2 V - q_0)/e]$. When $n_{\min}$ or $n_{\max}$ are changed with increasing voltage a new possible charge state would occur. It results in the jump of the differential conductance. The peculiarity voltage positions are as follows ($m_{1,2}$ are arbitrary integers):

$$V = \left(-\frac{1}{2} + \frac{q_0}{e} + m_1\right)\frac{e}{C_2}$$

or

$$V = \left(\frac{1}{2} - \frac{q_0}{e} + m_2\right)\frac{e}{C_1}\,. \quad (5)$$

Note that the peculiarity is noticeable only if the probability to be on the edge of the possible states interval ($\sigma_{n\min}$ or $\sigma_{n\max}$) is not too small in comparison with unity. Due to this-fact usually only one series of peculiarities can be observed.

The difference between the SET described elsewhere and this one with a weak non-linearity is illustrated by the two curves in fig. 1. The upper one grows exponentially at high voltages and due to this fact the voltage offset from the Ohm law cannot be defined in contrast to the lower curve. The other difference is the intensity of peculiarities. The suppression of this intensity at high voltages is stronger in the non-linear case. The explanation of this fact we will find analyzing the case of the strong non-linearity $E_Q \tau/\hbar \gg 1$.

First let us note that in the case $T=0$, $V>0$, only the rates $\Gamma_1^+(n)$ and $\Gamma_2^+(n)$ are nonzero from the energy consideration (see eqs. (2) and (3)). The dependence of the tunneling rates Γ_1^+ and Γ_2^+ on n is shown in fig. 2, where the squares correspond to integer n. When the charge number increases one of them grows rapidly while the other drops. Due to this fact the charge number may be only k or $k+1$ (see fig. 2) in the stationary state, otherwise it would change quickly, moving to these values.

In the limit considered it is possible to reduce eq. (2) to

$$\Gamma_1^+(n) = R_1^{-1} \frac{\exp\left(\frac{e^2}{C_\Sigma}\left(\frac{C_2 V}{e} - \frac{1}{2} - n - \frac{q_0}{e}\right)\frac{2\tau_1^+}{\hbar}\right)}{\frac{2e^2\tau_1}{\hbar}},$$

$$\Gamma_2^+(n) = R_2^{-1} \frac{\exp\left(\frac{e^2}{C_\Sigma}\left(\frac{C_1 V}{e} - \frac{1}{2} + n + \frac{q_0}{e}\right)\frac{2\tau_2^+}{\hbar}\right)}{\frac{2e^2\tau_2}{\hbar}}. \quad (6)$$

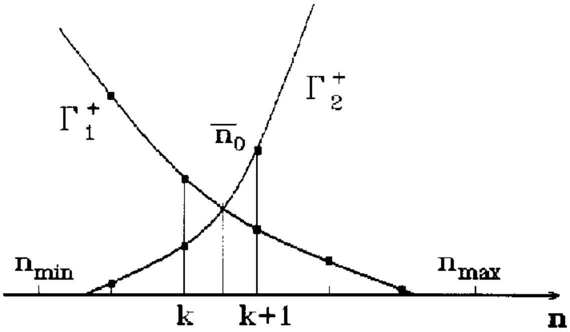

Fig. 2. Sketch of the charge-number dependence of the tunneling rates Γ (see the text).

Using the conditions $\Gamma_1^+(k) \gg \Gamma_2^+(k)$, $\Gamma_1^+(k+1) \ll \Gamma_2^+(k+1)$ we find that (see fig. 2) $k = [\bar{n}_0]$, where noninteger $\bar{n}_0$ is defined by the equation $\Gamma_1^+(\bar{n}_0) = \Gamma_2^+(\bar{n}_0)$ and is equal to

$$\bar{n}_0 = \left((\tau_2^+ - \tau_1^+) + \frac{2V(C_2\tau_1^+ - C_1\tau_2^+)}{e} + \frac{\hbar C_\Sigma}{e^2} \ln \frac{R_2\tau_2}{R_1\tau_1}\right) \frac{1}{2(\tau_1^+ + \tau_2^+)} - \frac{q_0}{e}. \quad (7)$$

If the voltage V is high enough so that $n_{min} + 1 \leq k \leq n_{max} - 2$, then the probabilities $\sigma_{n\min}$, $\sigma_{n\max}$ are negligible. This is the reason that the usual SET peculiarities (see eq. (5)) on the I–V curve, associated with the abrupt change of the n_{min} or n_{max}, are suppressed (see fig. 3).

But in the same time under these conditions there are new peculiarities due to the abrupt change of k (fig. 4). Their voltage positions can be calculated from eq. (7) assuming $\bar{n}_0$ to be integer. The voltage period of new peculiarities corresponds to $\Delta\bar{n}_0 = 1$ and equals

$$\Delta V = e\left|\frac{\tau_1^+ + \tau_2^+}{C_2\tau_1^+ - C_1\tau_2^-}\right|. \quad (8)$$

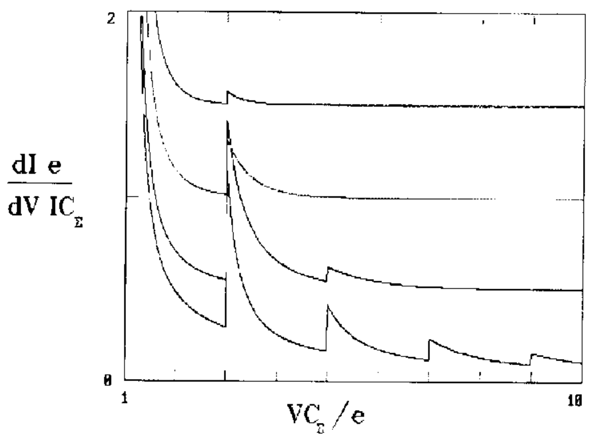

Fig. 3. The barrier suppression also suppress the usual Coulomb peculiarities. The non-linearity parameter $E_Q\tau/\hbar$ is 0, 0.5, 1 and 1.5, increasing from the lowest to the upper curve. The ratio parameters are identical for all of the curves: $C_1/C_2 = 10$, $R_1/R_2 = 1$. The barriers are symmetric: The periodic derivative jumps being clearly visible at $\tau = 0$ decrease and disappear rapidly with increasing of non-linearity.

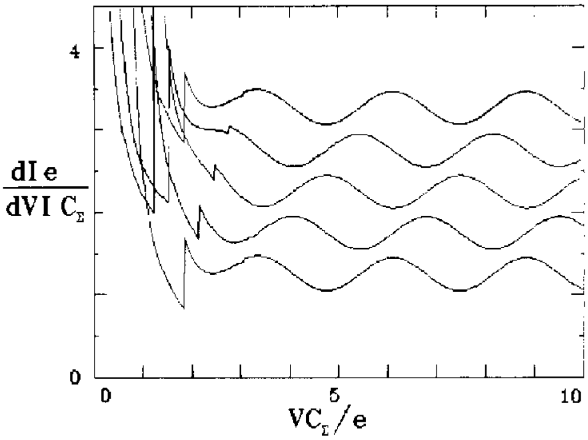

Fig. 4. New periodic Coulomb peculiarities. At moderate $E_Q\tau/\hbar$ new Coulomb peculiarities arise as weak oscillations of the current logarithmic derivative. They increase when the non-linearity grows. The ratio parameters are identical for both curves: $C_1/C_2=10$, $R_1/R_2=1$. The barriers are symmetric.

One can see from eq. (8) that there is no effect in the symmetric case when $C_2\tau_1^+ = C_1\tau_2^+$.

The second series of new peculiarities with the same period, eq. (8), corresponds the rapid increase of one of the tunneling rates in comparison to another $\Gamma_1^+(k) \gg \Gamma_2^+(k+1) \leftrightarrow \Gamma_1^+(k) \ll \Gamma_2^+(k+1)$ (in other words $\sigma_k \ll \sigma_{k+1} \leftrightarrow \sigma_k \gg \sigma_{k+1}$). The voltage positions of these peculiarities are given by the equation $\Gamma_1^+(k) = \Gamma_2^+(k+1)$. Thus, we get from eq. (6) that there is constant voltage shift between two series of peculiarities:

$$V_{\text{shift}} = \frac{e\tau_1^+}{C_2\tau_1^+ - C_1\tau_2^+}\ .$$

For the detail discussion of the new type of peculiarities we will approximate the current as:

$$I = e\,\frac{\Gamma_1^+(k)\Gamma_2^+(k+1)}{\Gamma_1^+(k)+\Gamma_2^+(k+1)}\ . \tag{9}$$

It is valid when $\bar{n}_0$ is far enough from the integer and we consider only charge numbers k and $k+1$.

Using eqs. (9) and (6) we get for the logarithmic derivative

$$\frac{\mathrm{d}I/\mathrm{d}V}{I} = \sigma_{k-1}\,\frac{2\tau_2^+}{\hbar}\,\frac{eC_1}{C_\Sigma} + \sigma_k\,\frac{2\tau_1^+}{\hbar}\,\frac{eC_2}{C_\Sigma}\ ,$$

$$\sigma_{k+1}+\sigma_k = 1\ . \tag{10}$$

One can see that the current logarithmic derivative switches between the values $(2\tau_2^+/\hbar)(eC_1/C_\Sigma)$ and $(2\tau_1^+/\hbar)(eC_2/C_\Sigma)$. In the limit $2(\tau_1^+ + \tau_2^+)e^2/\hbar C_\Sigma \to \infty$ this switching is abrupt. In the real case the width of the peculiarities V_{width} can be estimated from the voltage change when $\Gamma_1^+(n)/\Gamma_2^+(n)$ changes significantly:

$$\frac{V_{\text{width}}}{\Delta V} \simeq \left(\frac{2E_Q(\tau_1^+ + \tau_2^+)}{\hbar}\right)^{-1}\ .$$

This result can be compared to the fact that the usual SET peculiarities (see eq. (9)) have the zero width at $T=0$.

In fig. 3, where maximal ratio $\Delta V/V_{\text{width}} \simeq 3$, one can see still the rather smooth oscillations of the current logarithmic derivative but it is essential that they can be observed at moderate values of $E_Q\tau/\hbar$.

Note that eqs. (9) and (10) are not valid when $\bar{n}_0$ is near enough to integer k_0, i.e. at the voltages of the first series of new peculiarities. In this case one should substitute for eq. (9):

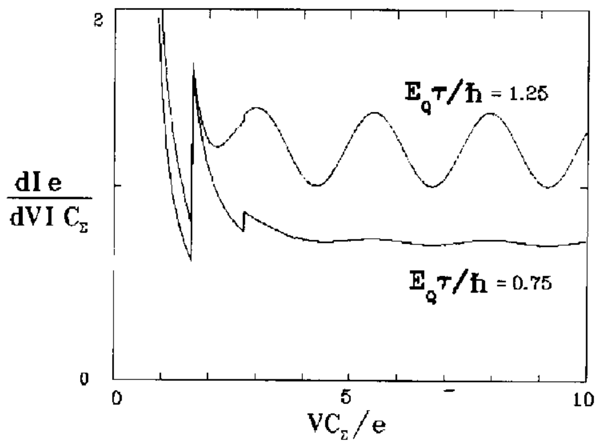

Fig. 5. Influence of residual charge on the old and new Coulomb peculiarities. The parameters are: $C_1/C_2=10$, $R_1/R_2=1$, $E_Q\tau/\hbar=1.25$. The barriers are symmetric. The charge q_0/e is 0, 0.25, 0.5, 0.75 and 1, and every curve is shifted along the vertical axis by $2q_0/e$ for the better representation.

$$I = e(\Gamma_1^+(k_0) + \Gamma_2^+(k_0)) . \quad (11)$$

For the logarithmic derivative we also obtain from (11) the same switching with the same width.

The q_0 dependence of the new peculiarity positions is shown in fig. 5. As follows from eq. (7) the shift of these positions is proportional to q_0 and the change of q_0 by the electron charge corresponds to the complete period ΔV. It emphasizes the Coulomb nature of these oscillations.

In conclusion we have investigated the single-electron transistor with the strong barrier suppression and have found a new type of the periodic Coulomb peculiarities on the I–V curve of this device.

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