Resonant Tunneling through a Macroscopic Charge State in a Superconducting Single Electron Transistor

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We predict theoretically and observe in experiment that the differential conductance of a superconducting single electron transistor exhibits a peak which is a complete analog, in a *macroscopic* system, of a standard resonant tunneling peak associated with tunneling through a single quantum state. In particular, in a symmetric transistor, the peak height is universal and equal to $e^2/2\pi\hbar$. Away from the resonance we clearly observe the cotunneling current which, in contrast to the normal-metal transistor, varies linearly with the bias voltage. [S0031-9007(97)03412-1]

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Charging effects in systems of small Josephson junctions are quite well understood by now—see, e.g., [1–3]. Interest, however, has been focused mostly on the interplay between the charging effects and Cooper pair transport, which can be described in generic terms as the quantum dynamics of the Josephson phase difference. The aim of this work is to study the quasiparticle transport in a superconducting single electron transistor (SET)—a system of two junctions connected in series (see inset of Fig. 1). We show that the BCS singularity in the density of states of superconducting electrodes of the junctions brings about several interesting new features of quasiparticle transport. Most notably, in the vicinity of the threshold voltage V_t for classical tunneling the quasiparticle transport is identical to resonant tunneling through a single macroscopic quantum state of the transistor.

In this work, we study the low voltage regime, where the quasiparticles do not have enough energy to enter the central electrode of the transistor and can traverse it only by quantum tunneling through the energy barrier created by the charging energy of the central electrode. The effects of the superconducting density of states in the classical sequential tunneling were discussed recently in Ref. [4]. The dominant contribution to the current I in the regime of quantum tunneling comes from the inelastic cotunneling, the process in which two different electrons tunnel simultaneously in the two junctions of the transistor [5], and can be written as $I = e[\Gamma(V) - \Gamma(-V)]$, where the cotunneling rate $\Gamma(V)$ can be expressed in terms of the "seed" I-V characteristics $I_j(U)$, $j = 1, 2$, of the two junctions at a fixed voltage U across a single junction and no charging effects,

$$\Gamma(V) = \frac{\hbar}{2\pi e^2} \int d\epsilon \, \frac{I_1(\epsilon/e)}{1 - \exp(-\epsilon/T)} \, \frac{I_2(V - \epsilon/e)}{1 - \exp[-(eV - \epsilon)/T]} |M|^2,$$
$$M = \frac{1}{E_1 + \epsilon - 2\Delta} + \frac{1}{E_2 + eV - \epsilon - 2\Delta}. \tag{1}$$

Here V is the bias voltage across the transistor, and we have assumed, for simplicity, that all electrodes have the same energy gap Δ.

We restrict our attention to the case of low temperatures, $T \ll \Delta$, when the nonvanishing quasiparticle current exists only at large voltages, $V > 4\Delta/e$, sufficient for the creation of quasiparticles in the two junctions. In this voltage range the energies $E_{1,2}$ of the intermediate charge states in Eq. (1) are

$$E_1 = E_C - \lambda(eV - 4\Delta) - \frac{eQ_0}{C_\Sigma}, \qquad E_2 = E_C - (1 - \lambda)(eV - 4\Delta) + \frac{eQ_0}{C_\Sigma}, \tag{2}$$

where $E_C = e^2/2C_\Sigma$ with $C_\Sigma = C_1 + C_2 + C_g$ denoting the total capacitance of the central electrode of the transistor, $\lambda = (C_2 + C_g)/C_\Sigma$ gives the fraction of the bias voltage that drops across the first junction, and $Q_0 = e\{V_g C_g/e + \Delta(2\lambda - 1)/E_C\}$ with $\{x\} \equiv x - [x + 1/2]$ can be interpreted as the charge induced by the gate voltage V_g into the central electrode.

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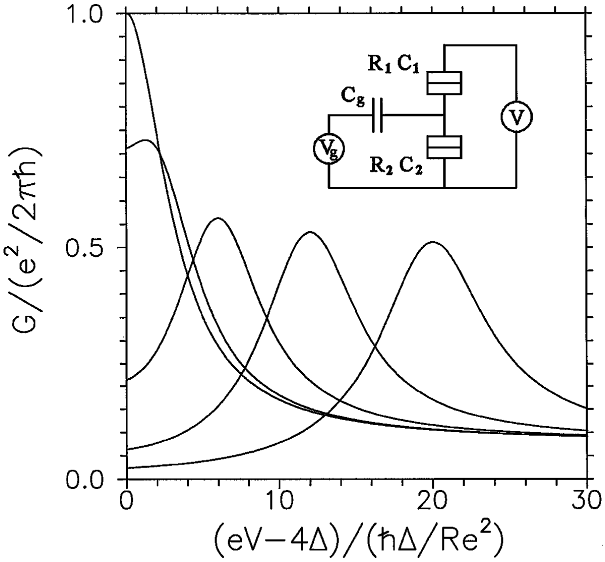

FIG. 1. Calculated bias-voltage dependence of the differential conductance of a symmetric superconducing SET transistor with junction resistance $R = 20\hbar/e^2$. The curves are plotted for several values of the gate voltage, i.e., the charge Q_0 induced on the central electrode of the transistor, that correspond to several charging energy barriers E_0 for tunneling: $E_0/(\hbar\Delta/Re^2) = 0; 1; 3; 6; 10$. The induced charge Q_0 cannot be close to 0. The inset shows the equivalent circuit of the SET transistor.

From Eq. (1) we see directly that a jump of the quasiparticle current $I_j(U)$ at $U = 2\Delta/e$ in superconducting junctions changes the voltage dependence of the cotunneling current for V close to $4\Delta/e$ from cubic [$\Gamma(V) \propto V^3$ for a normal-metal transistor [5]] to linear. Indeed, for $T \ll \Delta$ we can approximate $I(U)$ near the threshold $U = 2\Delta/e$ as (see, e.g., [6])

$$I(U) = I_j\Theta(U - 2\Delta/e), \qquad I_j = \frac{\pi\Delta}{2eR_j}, \tag{3}$$

where R_j is the normal-state tunnel resistance of the jth junction. Equations (1) and (3) give for low temperatures and $eV - 4\Delta \ll \Delta, E_C$,

$$I(V) = e\Gamma(V) = \frac{\hbar I_1 I_2}{2\pi}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)^2\left(V - \frac{4\Delta}{e}\right). \tag{4}$$

When the bias voltage approaches the threshold V_t of classical sequential tunneling, where one of the energy barriers E_j vanishes, the cotunneling current grows and crosses over into the current carried by sequential tunneling, in which quasiparticles traverse the transistor by two independent jumps across the two junctions. The energy width of the crossover region between the cotunneling and sequential tunneling is determined by the lifetime broadening of the intermediate charge states $E_{1,2}$ [7–10]. If the gate voltage is not close to the special point $Q_0 = (\frac{1}{2} - \lambda)e$ where $E_{1,2}$ vanish simultaneously [the situation that corresponds to the maximum threshold voltage $V_t = (4\Delta + 2E_C)/e$], then the current through one intermediate state, for instance, $E_1 \equiv E$, dominates near the tunneling threshold. The current in the transition region can be described in this situation by simply adding the lifetime broadening γ of the intermediate state in Eq. (1) for the cotunneling rate [11,12],

$$M = \frac{1}{E + \epsilon - 2\Delta + i\gamma},$$
$$\gamma = \frac{\hbar}{2e}\left[I_1\left(\frac{\epsilon}{e}\right)\coth\left(\frac{\epsilon}{2T}\right) + I_2\left(\frac{eV - \epsilon}{e}\right)\coth\left(\frac{eV - \epsilon}{2T}\right)\right]. \tag{5}$$

(This simple approach neglects the renormalization of E and γ significant at temperatures exponentially small on the scale of E_C [12].)

Combining Eqs. (1), (2), (3), and (5) we can calculate the differential conductance of the transistor at low temperatures,

$$G = \frac{dI}{dV} = \frac{\hbar I_1 I_2}{2\pi} \times \left[\frac{\lambda}{[E_0 - \lambda(eV - 4\Delta)]^2 + \delta^2} + \frac{1 - \lambda}{[E_0 + (1 - \lambda)(eV - 4\Delta)]^2 + \delta^2}\right], \tag{6}$$

where $E_0 = (e/2 - |Q_0|)e/C_\Sigma$ is the Coulomb energy barrier at $V = 4\Delta/e$, and $\delta = \hbar(I_1 + I_2)/2e$ is the energy width of the charge state due to tunneling. If we use the second equation in expression (3) we see that $\delta = \pi\hbar(R_1^{-1} + R_2^{-1})\Delta/4e^2$. Since the ideology of cotunneling is applicable only to junctions with small tunnel conductance, $R^{-1} \ll e^2/h$, this means that the width of the charge state is small, $\delta \ll \Delta$, and Eq. (6) describes the narrow conductance peak located at the threshold V_t of classical tunneling ($eV_t = 4\Delta + E_0/\lambda$). This peak corresponds to the rapid current rise from almost zero to $I_1 I_2/(I_1 + I_2)$ at $V = V_t$. The maximum conductance is achieved when $E_0 = 0$ (i.e., when the tunneling threshold reaches minimum) and $V = V_t = 4\Delta/e$,

$$G = \frac{dI}{dV} = \frac{e^2}{2\pi\hbar}\frac{4I_1 I_2}{(I_1 + I_2)^2}. \tag{7}$$

Equation (7) shows that in a symmetric transistor, where $I_1 = I_2$, the differential conductance reaches the absolute maximum $e^2/2\pi\hbar$ which is independent of Δ, E_C, or the junction resistance R. This universality is similar to that of the resonant tunneling through a single microscopic quantum state, and is quite remarkable in view of the fact that in the present context the quantum state is the *macroscopic* charge state of the central electrode of the transistor.

If the energy barrier E_0 is large on the scale of the width δ of the charge state, δ starts to increase with increasing E_0, i.e., increasing threshold voltage V_t. The conductance peak can be described analytically in this regime by retaining only the first, resonant, term in Eq. (6), and taking into account that the peak width δ depends then on its position V_t through the dependence on V_t of the contribution of the current I_2 through the second junction to δ: $I_2 = I_2(V_t - 2\Delta/e)$.

The shape of the conductance peak in a symmetric transistor (with $R_1 = R_2$, and $\lambda = 1/2$) calculated numerically from Eqs. (1), (2), and (5) without the approximation (3) or restrictions on E_0 is shown in Fig. 1. We see that this, more accurate, calculation preserves all the qualitative features of the simple analytical expression (6): maximum conductance is $e^2/2\pi\hbar$ when $E_0 = 0$ and decreases to approximately half this value at nonzero E_0.

For the results discussed above to be valid, the lifetime broadening of the resonant charge state should not only be much smaller than the superconducting gap Δ, but also much smaller than the typical energy distance (on the order of E_C) to the excited charge states of the central electrode of the transistor. The condition for this is

$$\alpha \equiv \frac{\Delta}{E_C} \frac{\pi\hbar}{e^2}(R_1^{-1} + R_2^{-1}) \ll 1\,. \qquad (8)$$

If this condition is violated, the charging effects are washed out by quantum fluctuations and the current rise at $V = 4\Delta/e$ becomes infinitely sharp (provided that the singularity of the density of states at the energy gap Δ is not smeared out by some internal mechanism).

To test these predictions experimentally we fabricated and measured four superconducting SET transistors with differing parameters. The transistors were fabricated by electron beam lithography on oxidized silicon by the standard shadow evaporation technique using aluminum electrodes and aluminum oxide junction barriers. The length of the central island was 1 μm, its width was 80–120 nm, and the overlap at the two ends of the island with the external electrodes was nominally 70 nm. The gate capacitance was about 0.02 fF.

Tunnel resistance R of the transistor junctions was measured from the large-voltage asymptote of the I-V characteristic of the transistor assuming equal resistances of the two junctions. Although we did not carry out any systematic study of how symmetric the transistors were, we checked from the gate voltage dependence of the threshold voltage V_t that sample 1 had equal parameters to within 30%, and we do not expect the other transistors to be worse in this respect since their dimensions were larger than in sample 1. The charging energy E_C was measured as a half of the amplitude of the V_t modulation by the gate voltage, and Δ can be obtained from the onset of the current at $4\Delta/e$. All these parameters of the four samples are shown in Table I, together with the combined parameter α defined in Eq. (8) as a small parameter of the present theory.

Measured I-V characteristics and traces of the differential conductance of sample 1 as a function of the bias voltage V are shown in Fig. 2 for several values of the gate voltage. The curves agree qualitatively with the predictions of the theory described above. The differential conductance has a narrow peak of the roughly correct width at the threshold of classical tunneling. The height of the peak away from the resonance is slightly below one-half of $e^2/2\pi\hbar$. The main discrepancy between the experimental results [Fig. 2(b)] and the simple model calculations (Fig. 1) is that at resonance the conductance does not reach the ideal maximum value $e^2/2\pi\hbar$ but rather is about one-half of this value. Although the asymmetry of junction resistances contributes according to Eq. (7) to suppression of the resonance conductance, the actual asymmetry of our transistors was too small to account for the observed magnitude of this suppression. This discrepancy can be qualitatively explained by the sensitivity of resonant tunneling to all sources of inelastic scattering. In our system, the most probable source of this scattering is the finite impedance of the voltage leads. Since we could not characterize this impedance quantitatively, we did not attempt to find a theoretical fit to the curves in Fig. 2.

The results of measurements for all four samples are summarized in Table I, which shows two characteristic values of the differential conductance in units of $e^2/2\pi\hbar$: (i) $G_{0,\mathrm{exp}}$, the conductance at bias voltage just above $4\Delta/e$ and at gate voltage that corresponds to the maximum threshold voltage V_t, and (ii) $G_{1,\mathrm{exp}}$, the peak conductance at resonance (when V_t reaches minimum). Variation of the peak conductance G_1 with the tunnel resistance R and charging energy E_C described by Table I confirms that when the relative width of the charge states of the transistor [characterized by the parameter α of Eq. (8)] becomes considerable, G_1 increases gradually beyond $e^2/2\pi\hbar$. At large α, when $G_1 \gg e^2/2\pi\hbar$, the charging effects are completely washed out by the quantum fluctuations of charge on the central electrode of the transistor and the differential conductance becomes insensitive to the gate voltage. This case is approached by sample 4 with the largest α in which V_t is practically independent of the gate voltage, and we could assign only one value of the characteristic conductance to this sample.

When α is small and the charging effects are well pronounced, the threshold conductance G_0 originates only from the process of cotunneling, and is much smaller than the peak conductance G_1. It can be calculated from Eq. (4)

TABLE I. Parameters of the four studied SET transistors. Conductances in the last three columns are shown in units of $e^2/2\pi\hbar$.

Sample	$R(k\Omega)$	E_C (meV)	Δ (meV)	α	$G_{0,\mathrm{exp}}$	$G_{0,\mathrm{theory}}$	$G_{1,\mathrm{exp}}$
1	206	0.35	0.22	0.08	3.1×10^{-3}	1.6×10^{-3}	0.5
2	152	0.15	0.21	0.24	0.032	0.014	0.9
3	65	0.15	0.20	0.55	0.096	0.086	1.6
4	52	0.08	0.23	1.44	⋯	⋯	4.0

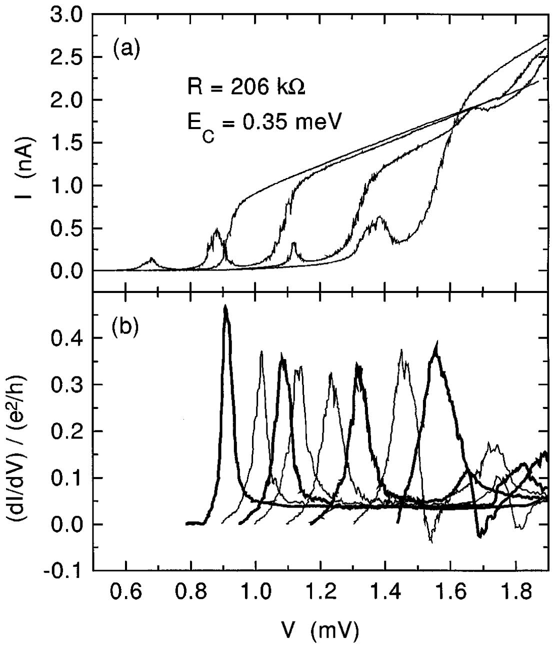

FIG. 2. Measured (a) I-V characteristics, and (b) bias-voltage dependence of the differential conductance of sample 1 for several gate voltages. The traces shown with thick lines in (b) correspond to the I-V curves presented in (a). For clarity, the features due to the current peaks associated with the Cooper-pair tunneling that are visible in (a) have been omitted in (b). For discussion see text.

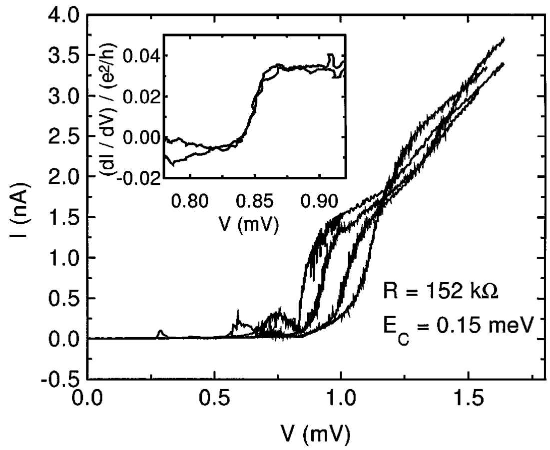

FIG. 3. Measured I-V characteristics of sample 2 for several gate voltages. The inset shows the differential conductance in the vicinity of the gap edge $V = 4\Delta/e$ for gate voltages which correspond to the two largest tunneling thresholds. The conductance jump at $V = 4\Delta/e$ is due to the cotunneling.

which predicts that at $V = 4\Delta/e$ the cotunneling conductance of the superconducting SET transistor should increase abruptly to a finite, voltage-independent level which also does not depend on temperature at low temperatures. This behavior is indeed found in our three samples with larger tunnel resistances. Figure 3 shows, for example, the data for sample 2. At gate voltages which correspond to the thresholds V_t close to maximum we see the kink in the I-V curves and the step in the dI/dV at the onset of the quasiparticle current at $V = 4\Delta/e$. [For other values of the gate voltage small current peaks due to Cooper pair tunneling that are visible in Figs. 2(a) and 3, overlap with the onset of quasiparticle current and do not allow one to identify the conductance jump.] The data shown in Figs. 2 and 3 were taken at a temperature of about 100 mK. We checked that the jump in the quasiparticle conductance is practically temperature independent for temperatures up to 0.4 K.

Table I contains a comparison between the observed cotunneling conductance $G_{0,\mathrm{exp}}$ and $G_{0,\mathrm{theory}}$ calculated from Eq. (4) under the assumption of a symmetric transistor. Taking into account that any asymmetry and/or nonuniformity of the junction resistances increases G_0 we can say that the agreement between $G_{0,\mathrm{exp}}$ and $G_{0,\mathrm{theory}}$ is reasonable.

In summary, we proposed theoretically and confirmed in experiment that the quasiparticle transport in a superconducting SET transistor in the vicinity of the tunneling threshold can be described as resonant tunneling through a macroscopic charge state of the central electrode of the transistor. The maximal differential conductance associated with this process is $e^2/2\pi\hbar$, while the width of the resonance is determined by the lifetime broadening of the charge states of the transistor. For gate voltages away from the resonance we observed very clearly the cotunneling current which exhibits linear (in contrast to cubic of the normal-metal case) dependence on the bias voltage.

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