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Combined Bloch and single-electron-tunneling oscillations in one-dimensional arrays of small tunnel junctions

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It is shown that at sufficiently low temperatures, one-dimensional arrays of ultrasmall tunnel junctions (for example, semiconductor superlattices) may exhibit an unusual type of electron transfer. This process can be considered as fast "Bloch" oscillations with frequency $f_B = \varepsilon/h$ (where ε is the electron energy change due to tunneling through one tunnel barrier), modulated with lower single-electron-tunneling frequency $f_S = I/e$ (where I is the dc electric current through the array).

One of the most important results of the field of correlated single-electron tunneling (for reviews see, e.g., Refs. 1 and 2) is the concept of the so-called "single-electron-tunneling" (SET) oscillations with frequency,[3]

$$f_S = I/e\,, \tag{1}$$

fundamentally related to the dc electric current I. Such oscillations arise due to the *particle* properties of electrons and can take place in systems with purely *classical* dynamics.[4] They can be, however, most naturally implemented[5,6] in one-dimensional (1D) arrays of small tunnel junctions.

But it is well known[7] that such systems, at sufficiently weak electron scattering, may allow another type of fundamental periodic process: so-called "Bloch" (or "Stark") oscillations with frequency

$$f_B = \varepsilon/h\,, \tag{2}$$

where ε is the electron energy change due to its tunneling through one junction (in the simplest case of negligible self-charging effects, $\varepsilon = eEd$, where E is the external electric field and d is the structure period). Recently, these oscillations were observed directly.[8] The Bloch oscillations are obviously a *quantum* phenomenon and reflect *wave* properties of electrons.

A very natural question is whether these two types of oscillations can exist simultaneously. An apparent answer is *no*, because Heisenberg's uncertainty principle forbids the electron to behave simultaneously as a wave and as a particle. The goal of this work was to show that, surprisingly enough, this apparent answer is wrong. Specifically, we calculate the spectral density $S_I(f)$ of the current through a semiconductor superlattice and show that under certain conditions it exhibits simultaneously peaks at frequencies (1) and (2) corresponding to both types of oscillations.

We have considered the following simple model of the semiconductor superlattice which consists of $N+1$ similar thin conducting layers ("quantum wells") separated by N tunnel barriers (Fig. 1). Quantization of electron motion perpendicular to the layers leads to formation of the minibands. In our analysis, energy gaps Δ_i $(i = 1, 2, \ldots)$ between the lowest minibands were supposed to be wider than ε and $k_B T$, so that the electron motion is restricted to the lowest miniband $(i = 1)$, and its energy in the kth quantum well can be presented as

$$\varepsilon_{k,p} = \Delta_0 + e\Phi_k + \frac{p^2}{2m}\,, \tag{3}$$

where Δ_0 is the 1D quantization energy, Φ_k is the background potential including the part $(-eEdk)$ due to external electric field E, and p is the electron momentum in the plane of the layer (quantization in this direction was accepted to be negligible). The Hamiltonian of the system can be written as

$$H = \sum_{k,p} [\epsilon_{k,p} a^\dagger_{kp} a_{kp} - (t a^\dagger_{kp} a_{k+1,p} + \mathrm{H.c.})] + \sum_{k,p,p'} V^{(k)}_{pp'} a^\dagger_{kp} a_{k,p'} + H_L + H_R\,. \tag{4}$$

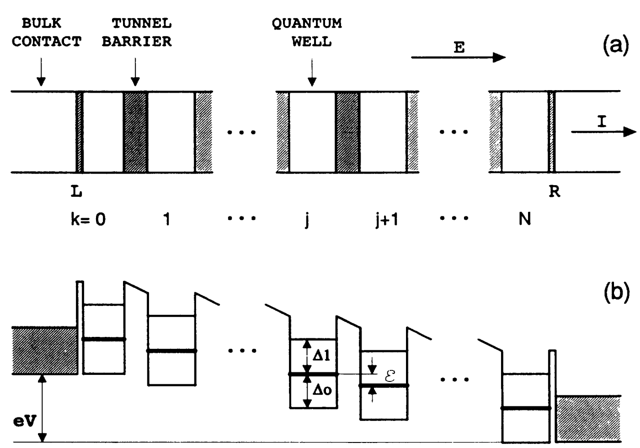

FIG. 1. Semiconductor superlattice: (a) scheme of the structure and (b) sketch of its band edge diagram. Shown energy levels in the conducting layers (quantum wells) correspond to vanishing transversal momentum p; however, due to conservation of p during the tunneling, the energy ε the electron gains at tunneling does not depend on p.

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The term in the parentheses describes tunneling between neighboring layers with conservation of the transversal momentum p. At $E \neq 0$ this tunneling alone cannot provide a finite dc current through the superlattice. Such a dc transport is provided by elastic scattering on impurities described by the term with $V_{pp'}^{(k)}$ (and by the implicit assumption of the electron energy relaxation after the tunneling, due to weak inelastic interactions). We assumed that the net rate $\Gamma = (2\pi/\hbar)\sum_{p'} \mid V_{pp'}^{(k)} \mid^2 \delta(\epsilon_{k,p} - \epsilon_{k,p'})$ of the scattering is within the range

$$t \ll \hbar\Gamma \ll \Delta_0, \tag{5}$$

so that all calculations can be carried out using the perturbation theory with respect to t. Terms H_L and H_R describe tunneling through the edge barriers L and R (Fig. 1). Without considering these terms explicitly we have assumed that the edge barriers are much more transparent than the internal barriers, so that the tunneling in the edge junctions fixes the total voltage drop across N internal barriers to be equal to the applied voltage $V = EdN$ [Fig. 1(b)].

For superlattices of large cross-section S, where the charge Q_k of each layer can be considered as a continuous variable, models similar to ours were discussed repeatedly—see, e.g., Ref. 7. For relatively small voltages $V < \Delta_1 N/e$, and in the regime when the electric field is distributed uniformly along the superlattice, the model yields the following simple expression for the dc I-V curve of the structure:

$$I(U) = GU/[1 + (U/U_0)^2], \tag{6}$$

where $U = V/N$, $U_0 = \hbar\Gamma/e$, and $G = 2et^2mS/(\pi\hbar^3 U_0)$. At $U > U_0$ the I-V curve (6) has a negative slope dI/dU—see the inset in Fig. 2. (Account of weak tunneling without momentum conservation and/or tunneling between minibands leads to small additional contribution to the current, rising with the voltage U and leading to positive slope at larger voltages.[7] We will disregard these processes, because the charge quantization effects we are going to discuss are rather insensitive to them.[9]) The negative slope leads to charge accumulation effects which result in low-frequency instability of the uniform state of the system and formation of high-field domains (see, e.g., Ref. 10).

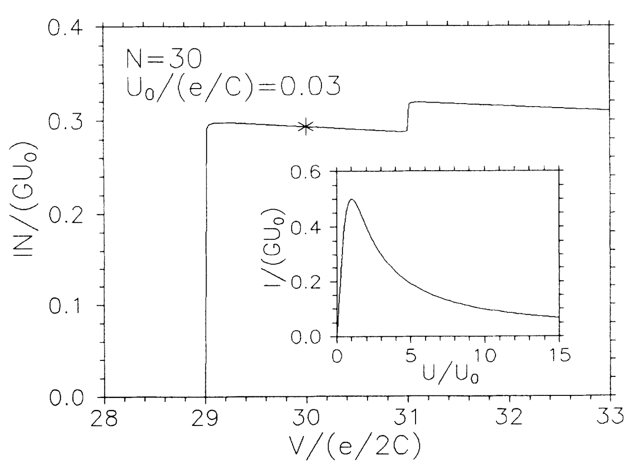

FIG. 2. A typical dc I-V curve of the "slim" superlattice, which exhibits the Coulomb staircase. The inset shows the "seed" I-V curve $I(U)$ used in the calculations. Temperature is assumed to be much lower than eU_0/k_B.

In contrast to the standard approach we have considered the case when the superlattice cross-section is so small and temperature T so low that capacitances C and conductances G of the barriers satisfy the conditions[1,2]

$$C \ll \frac{e^2}{k_B T}, \quad G \ll \frac{e^2}{h}. \tag{7}$$

As a result, single-electron charging effects become important, i.e., the electrostatic potentials Φ_k of the conducting layers become dependent on the whole charge configuration $\{n_{k'}\}$, where $n_{k'}$ is the number of electrons in the k'th layer ($Q_k = en_k$). In order to simplify this dependence we have assumed that the number $(N+1)$ of the layers is much less than $(C/C_0)^{1/2}$, where C_0 is the stray capacitance of one layer,[5] so that the stray field effects can be ignored. Then the electrostatics problem can be readily solved to give the following result:

$$\Phi_k = (\Phi_k)_{\rm int} - \frac{kV}{N},$$

$$(\Phi_k)_{\rm int} = \frac{e}{NC}\left[\sum_{k'=1}^{k} n_{k'}(N-k)k' + \sum_{k'=k+1}^{N} n_{k'}(N-k')k\right]. \tag{8}$$

Due to the second of conditions (7) the system has two quite different time scales, $\tau_1 = C/G$ and $\tau_2 = h/\varepsilon \sim hC/e^2 \ll \tau_1$. On the *longer* time scale, $\delta t \sim \tau_1$, dynamics of the system can be considered as a sequence of charge configurations $\{n_k\}$ which change due to single-electron tunneling events. The rates $\Gamma_j^{\pm}$ of the electron tunneling to the right/left direction through the jth barrier can be found using the standard theory of the single-electron tunneling:[1]

$$\Gamma_j^{\pm} = I(\varepsilon_j^{\pm}/e)[1 - \exp\{-\varepsilon_j^{\pm}/k_B T\}]^{-1}, \tag{9}$$

where $(-\varepsilon_j^{\pm})$ is the change of the electrostatic energy of the system,

$$\varepsilon\{n_k\} = \varepsilon_{\rm int}\{n_k\} - eV\sum_k kn_k - eVn_\Sigma,$$

$$\varepsilon_{\rm int}\{n_k\} = (e/2)\sum_k n_k(\Phi_k)_{\rm int}, \tag{10}$$

due to the tunneling event. Here n_Σ is the total number of electrons passed through the system, and $I(U)$ is the "seed" I-V curve (6). Knowledge of $\Gamma_j^{\pm}$ allows one to calculate both the average current flowing along the structure[5] and the spectral density $S_I(f)$ of the current.[11]

Figure 2 shows a typical dc I-V curve of the structure calculated in this way. The initial horizontal part [at $V < V_t = e(N-1)/2C$] is a manifestation of the Coulomb blockade of tunneling. The current steps (the "Coulomb staircase") are also a result of the quantization of the charge Q_k.[9] For example, if the applied field corresponds to the first current step (as indicated by the star in Fig. 2), and $e/CN \sim U_0$, each new electron passing through the superlattice tunnels rapidly through the first barrier, because for this process ε_j^+/e is very small ($\sim U_0$), and the rate Γ^+ (9) is high. For each following tunneling event ε_j^+ increases by e/NC, so that ε_j^+/e becomes larger than U_0, and the electron passes through the superlattice with a gradually decreasing speed. At the first Coulomb-staircase step, the next electron can start moving inside the superlattice only when the first one leaves it. This process repeats again and again with the average frequency given by Eq. (1).

The spectral density $S_I(f)$ at low frequencies can be found by the method described in Ref. 11. Calculating $S_I(f)$ one should make a clear distinction between the tunneling current $I_j(t)$ through the jth junction and the current $I(t)$ flowing through the external contacts [Fig. 1(a)]. For our simple case of small stray capacitances and similar junctions,

$$I(t) = \frac{1}{N}\sum_j I_j(t)\,. \tag{11}$$

Figure 3(a) shows a typical result of the calculations of $S_I(f)$. The narrow spectral peak corresponds to the SET oscillations (1). At larger observation frequencies ($f \gg f_S$) the single-electron tunneling events are practically uncorrelated and the spectral density approaches a constant value described (at low temperatures) by the modified Schottky formula

$$S_I(f) = \frac{2e}{N}\langle I\rangle\,. \tag{12}$$

The factor $1/N$ in this equation is due to the fact that the electron passes the system in N leaps, each corresponding to transfer of the charge e/N through the external circuit.

Due to the strong relation $\tau_2 \ll \tau_1$, dynamics of the system on a *shorter* time scale $\delta t \sim \tau_2$ can be considered under the assumption that the charge configuration is fixed. It means that analyzing tunneling through the jth tunnel barrier, all other charges $Q_{j'}$ ($j' \neq j$) can be assumed constant, so that the only relevant tunneling terms in the Hamiltonian are those with $j' = j$. But for such a truncated Hamiltonian, the spectral density of $I_j(t)$ can be found following, e.g., Ref. 12. Taking into account Eq. (7), the spectral density of $I_j(t)$ can be written as

$$S_{I_j}(f) = e\sum_{\pm,\pm} I([\varepsilon_j^\pm \pm hf]/e) \times[1-\exp\{-(\varepsilon_j^\pm \pm hf)/k_BT\}]^{-1}\,, \tag{13}$$

where $\varepsilon_j^\pm = \pm e(U_j \mp U_t)$, $U_j = \Phi_j - \Phi_{j+1}$. For our simple electrostatic model, $U_t = e(N-1)/2CN$.

In the high-temperature limit, $k_BT \gg e^2/C$, and/or for high voltage across the barrier ($U_j \gg U_t$), one can neglect the threshold voltage U_t, so that $\varepsilon_j^+ = -\varepsilon_j^- = eU_j$, and Eq. (13) is reduced to the usual expression for the "fluctuations" of a tunnel junction biased by dc voltage U_j.[13] In fact, with our shape (6) of the "seed" I-V curve, this formula describes a peak in the spectral density $S_{I_j}(f)$ with a center at $f_B = \varepsilon_j/h$ (where $\varepsilon_j = eU_j$) and linewidth $\delta f \sim \Gamma$, corresponding to the Bloch oscillations (2). Note that at $U_j \gg U_0$ the linewidth is much less than f_B, so that the oscillations are nearly monochromatic, despite the fact that contributions of individual electrons to the process are incoherent.

In the low-temperature limit (7), Eq. (13) is reduced to

$$S_{I_j}(f) = e\sum_{\pm} |\, I'(U_j \pm hf/e)\,|\,,$$

where

$$I'(U) = \begin{cases} I(U-U_t) & \text{for } U > U_t \\ I(U+U_t) & \text{for } U < -U_t \\ 0 & \text{for } -U_t < U < U_t\,. \end{cases} \tag{14}$$

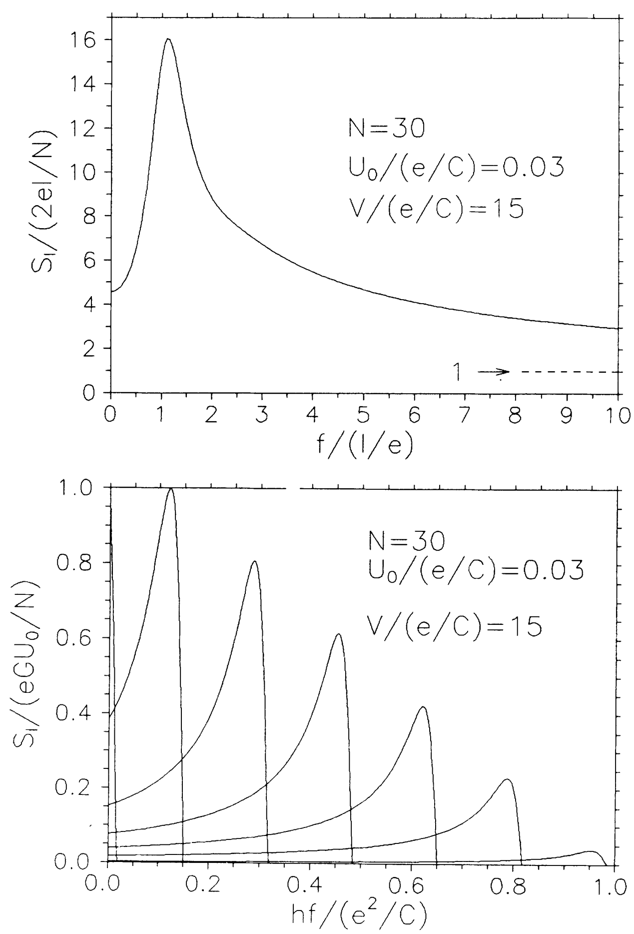

FIG. 3. Spectral power density of the current $I(t)$ in external electrodes of a "slim" superlattice: (a) at low frequencies ($f \sim f_S$) and (b) high frequencies ($f \sim f_B$) for 7 of $N = 30$ successive charge configurations (from left to right, after electron tunneling through 1, 5, 10, 15, 20, 25, and 30 barriers). In (a) the density is averaged over the SET oscillation period. Both pictures correspond to the same set of parameters, including the dc bias point (shown by the star in Fig. 2).

One can see that at $U_j \gg U_0$ the spectral density exhibits two narrow peaks at the Bloch frequencies (2) with $\varepsilon = e \mid U_j \pm U_t \mid$. These two values correspond to two possible directions of the electron tunneling through the jth barrier (from the same initial charge configuration). One should remember, however, that Eqs. (13) and (14) include contribution of the zero-point quantum fluctuations, $S_0(f) = 2hf \coth(hf/2k_BT)\mathrm{Re}G(f)$, where $\mathrm{Re}G(f)$ is the active conductance of the barrier, $\mathrm{Re}G(f) = (e/2hf)[I'(U_j + hf/e) - I'(U_j - hf/e)]$. In order to obtain the real (available) power density, this contribution should be subtracted from $S_{I_j}(f)$. At low temperature this subtraction is completely canceling the peak corresponding to tunneling with increase of the electrostatic energy ($\varepsilon < 0$), so that only the peak with $\varepsilon = \varepsilon_j^+ > 0$ remains in the spectrum.

Figure 3(b) shows a typical result of such a calculation of the real power density of the net current $I(t)$ for several sequential charge configurations (k is the number of the single-election event within the SET oscillation period). One can see that for each particular charge configuration the system exhibits the narrow-band Bloch oscillations, but their frequency (and amplitude) changes as a result of each single-electron tunneling event, because the charge distribution in general affects the energy change ε.

An important feature of $S_I(f)$ given by Eq. (14) is that in low-frequency limit $f \to 0$ (i.e., $f \ll f_B$) it coincides (after averaging over the period of the SET oscillations) with the high-frequency limit (12) of $S_I(f)$ calculated assuming classical dynamics of electrons. This means that our theory determines $S_I(f)$ consistently for all frequencies, and shows that $S_I(f)$ may contain simultaneously peaks corresponding to SET and Bloch oscillations. Hence, *the process of electron transfer as a whole can be considered as fast Bloch oscillations with frequency (2) modulated by much slower SET oscillation frequency (1).*

To summarize, we have calculated the spectral density of the current in "slim" 1D arrays of tunnel barriers separating conductors with quantized electron motion and low scattering rates. The results show that electron transport in such a system may take a form of high-frequency quantum Bloch oscillations modulated by low-frequency classical SET oscillations. In order to comprehend why this result is consistent with the general principles of quantum mechanics, one can note that the process resembles very closely the textbook description of an electron as a packet with wavelike carrier and particlelike envelope. Due to the two very different frequency scales (in our case $f_S \ll f_B$), such a coexistence of the particle and wave properties of electrons does not violate Heisenberg's uncertainty relation.

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