Single-electron transistor controlled with a RC circuit

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A single-electron transistor with a RC circuit between its central electrode and the controlling voltage source is considered. The coupling resistance is assumed to be much greater than the resistance quantum. Several metastable charge states are possible within the Coulomb blockade range. This leads to the hysteresis of the central electrode potential as a function of the gate potential. Another important feature is that the blockade threshold is nonvanishing for any gate potential. If the coupling resistance is much greater than tunnel resistances, the I-V curve can contain several branches and exhibits hysteresis.

The discreteness of the electric charge produces various effects in the systems of small capacitance tunnel junctions.[1,2] The simplest device based on these effects is the single-electron transistor (SET) which consists of two tunnel junctions in series.[3,4] The sub-single-electron charge of the central electrode of the SET controls the current through it. The most investigated case is the capacitive coupling of the central electrode to the signal source (C SET). Another possibility is the resistive coupling (R SET). The latter case is difficult to realize experimentally because the coupling resistance should be greater than the quantum unit $R_Q=\pi\hbar/2e^2\approx 6.5\ k\Omega$ and not longer than several micrometers. However, such resistors are available nowadays.[5] Hence, investigations of the single-electron circuits containing resistors are quite topical.

In the present paper, we consider the SET coupled to the controlling potential φ by the resistance R_0 and the capacitance C_0 in series (Fig. 1). (Obviously one can change the mutual position of R_0 and C_0.) Let us call this system the RC SET.

We assume that the tunnel resistances are large enough, $\min(R_1,R_2)\gg R_Q$, and the typical energy of the quantum fluctuations of the coupling circuit $\hbar/(R_0C_s)$ [where $C_s=C_0(C_1+C_2)/C_\Sigma$, $C_\Sigma=C_0+C_1+C_2$], is much less than $e^2/(C_1+C_2)$; this implies that $R_0\gg R_QC_\Sigma/C_0$.

In this case the dynamics of the RC SET can be completely calculated from the "orthodox" theory.[1] Due to the last inequality the coupling circuit does not affect the double junction system during the tunneling. However,

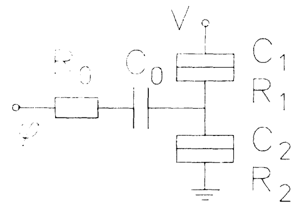

FIG. 1. The RC SET consisting of two tunnel junctions and a RC circuit.

the current through R_0 may change the effective charge Q of the central electrode of the double junction system during the time interval between the tunneling events (Q is the sum of the charges at junctions capacitances C_1 and C_2). Similar to Ref. 6, the master equation for the probability $\sigma(n,Q)$ of the charge state with definite number n of excess electrons in the central electrode and definite effective charge Q is as follows:

$$\frac{d}{dt}\sigma(n,Q)=\sum_{\pm}\sum_{j}\sigma(n\pm1,Q)\Gamma_j^{\mp}(Q)-\sigma(n,Q)\sum_j[\Gamma_j^{+}(Q)+\Gamma_j^{-}(Q)]-\frac{d}{dQ}[\sigma(n,Q)(Q_n-Q)/R_0C_S]+\frac{T}{R_0}\frac{d^2\sigma}{dQ^2}\ , \tag{1}$$

$$Q_n=(ne+Q_0+C_0\varphi')(C_1+C_2)/C_\Sigma\ , \tag{2}$$

$$\varphi'=\varphi-VC_1/(C_1+C_2)\ . \tag{3}$$

Here V is the driving voltage of the SET, Q_0 is the initial (background) charge of the central island (including the charge at the coupling capacitance), and T is the temperature. The charge Q_n corresponds to the equilibrium in the coupling circuit for the charge state n; the potential φ' determines the charge injection from the coupling circuit. The last term in Eq. (1) describes the Nyquist noise. The rate $\Gamma_j^{\pm}(Q)$ of the tunneling through the jth junction ($j=1,2$) which leads to an increase (+) or a decrease (−) of the charge number n depends on the energy gain $W_j^{\pm}(Q)$ due to tunneling and can be calculated using the same expression as for the simple double junction system[1,3,4]

$$\Gamma_j^{\pm}(Q)=(e^2R_j)^{-1}W_j^{\pm}(Q)\{1-\exp[-W_j^{\pm}(Q)/T]\}^{-1}\ , \tag{4}$$

$$W_j^{\pm}(Q)=\frac{e^2}{C_1+C_2}\left[\mp(-1)^j\frac{VC_1C_2}{eC_j}-\frac{1}{2}\mp\frac{Q}{e}\right]\ . \tag{5}$$

In the stationary state, dc current I through the RC SET coincides with the currents I_1 and I_2 through the junctions

$$I_j=(-1)^{j+1}\sum_n \int dQ[\Gamma_j^+(n,Q)-\Gamma_j^-(n,Q)]\sigma(n,Q) . \quad (6)$$

Equations (1)–(4) form a complete set of equations of the "orthodox" theory for the RC SET. Below we will consider two simple limiting cases for the relation between R_0 and $R_{1,2}$ when the calculations are simplified considerably.

SMALL COUPLING RESISTANCE

First, let us assume that the relaxation time of the coupling circuit R_0C_s is much smaller than the typical time interval between the tunneling events, which is of the order of $R_{1,2}(C_1+C_2)$. This is the case when $R_0 \ll \min(R_1,R_2)C_\Sigma/C_0$. In this case, fast relaxation of the charge Q after each tunneling event allows us to consider this charge as an inertialess function of the charge state Q_n [see Eq. (2)] with additional thermal fluctuations $(\delta Q)^2/2C_s=T/2$. Equation (1) in this case can be replaced by the usual master equation for the SET[1,3,4]

$$\frac{d}{dt}\sigma(n)=\sum_j \sigma(N+1)\tilde{\Gamma}_j^-(n+1)+\sigma(n-1)\tilde{\Gamma}_j^+(n-1) -\sigma(n)[\tilde{\Gamma}_j^+(n)+\tilde{\Gamma}_j^-(n)] , \quad (7)$$

with the tunneling rates averaged over the fluctuations around Q_n

$$\tilde{\Gamma}_j^\pm(n)=(2\pi C_S T)^{-1/2}\int \exp[-(Q-Q_n)^2/(2C_ST)] \times\Gamma_j^\pm(Q)dQ . \quad (8)$$

Hence, the only difference from the calculation procedure for the usual SET is a new expression for the tunneling rates. Note that the tunneling rates $\tilde{\Gamma}_j^\pm(n)$ can be calculated also for the arbitrary ratio R_0/R_Q as it was done in Refs. 7 and 8.

Let us consider the Coulomb blockade in the RC SET at $T=0$. Then $\tilde{\Gamma}_j^\pm(n)=\Gamma_j^\pm(Q_n)$, and the condition of the blockade for the definite charge state n is $W_j^\pm(Q_n)<0$. For voltages V close to zero it means [see Eq. (5)].

$$|Q_n|<e/2 . \quad (9)$$

From Eq. (2) it follows that several metastable charge states satisfy Eq. (9). Their number depends on the gate potential φ; and it is either $2+\mathrm{Int}[C_0/(C_1+C_2)]$ or $1+\mathrm{Int}[C_0/(C_1+C_2)]$ (here $\mathrm{Int}[x]$ means the integer part of x). The reason for the difference from the usual case of the double junction in a low impedance environment (in which only one charge state can be stable), is that the addition of the electron to the central electrode of the RC SET causes the increase of the effective charge Q_n only by $e(C_1+C_2)/C_\Sigma$. In other words, several stable states are possible because the capacitance C_1+C_2, which is charged during the tunnel event, differs from the total capacitance C_Σ of the central electrode of the RC SET.

The existence of several stable states in the single-electron circuit containing high impedance element $R \gg R_Q$ was discussed in Ref. 9 (see also Ref. 1). There the circuit consisting of one tunnel junction, resistor, and capicitor was considered. The existence of multistability in the Coulomb blockade range of the two-junction system with the resistance in series was noted in Ref. 10. One can see that multistability is the general feature of all single-electron circuits containing high impedance elements.

The number of stable Coulomb blockade states of the RC SET decreases with increase of voltage V (see the numbers between the arrows in Fig. 2), and the Coulomb blockade disappears when there is no more stable states.

The stability threshold for a certain charge state n is

$$V_t(n)=\min[(e/2-Q_n)/C_1,\ (e/2+Q_n)/C_2],\quad V_t(n)>0 \quad (10)$$

(note that this is in fact a linear equation, because Q_n is a linear function of V).

The blockade threshold for the I-V curve is determined by the maximal value

$$V_t=\max_n V_t(n) . \quad (11)$$

By changing the gate voltage, one can vary the blockade threshold. Its maximal possible value corresponds to the condition $(e/2-Q)/C_1=(e/2+Q)/C_2$ and coincides with the maximal blockade threshold for the usual double junction system (in zero impedance environment)

$$V_{t,\max}=e/(C_1+C_2) . \quad (12)$$

However, in contrast to the usual double junction case, on cannot suppress the blockade threshold to zero by varying the gate voltage (Fig. 3). Its minimal value corresponds to the condition $V_t(n)=V_t(n+1)$ which gives

$$V_{t,\min}=eC_0/[(C_1+C_2)C_\Sigma] . \quad (13)$$

Hence, the periodic peaks of the linear ($V=0$) conductance as a function of the gate potential, which exist for the C SET, are absent for the RC SET. (This result was discussed in Ref. 7. Similar result for the two-junction system with the resistance in series was obtained in Ref.

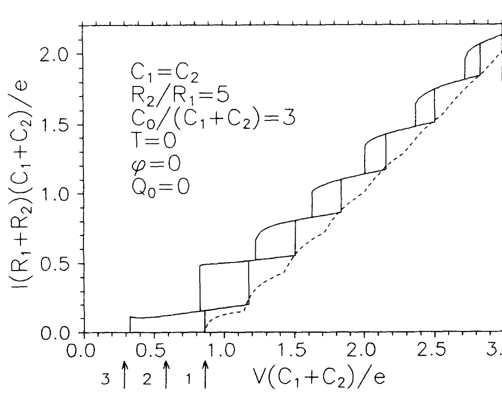

FIG. 2. The I-V curve of the RC SET for large (solid line) and small (dashed line) coupling resistance R_0. The arrows separate regions corresponding to 3, 2, and 1 metastable blockade states.

10. Obviously, this is a general consequence of the high impedance environment.)

In the case $C_0 \gg C_{1,2}$ the minimal and maximal thresholds coincide (Fig. 3), because the charge Q_n becomes quasicontinuous and "screens" the gate voltage. Obviously, in this case the I-V curve is also independent of the gate voltage, and it can be calculated from the equation $\Gamma_1^+(Q)=\Gamma_2^-(Q)$:

$$I=\begin{cases}0, & V\le V_t=e/(C_1+C_2)\\ (V-V_t)/(R_1+R_2), & V\ge V_t\ .\end{cases} \tag{14}$$

In the opposite case, $C_0 \ll C_{1,2}$, the RC SET behaves as a usual double junction with externally injected charge $C_0\varphi'$ where φ' is given by Eq. (3).

When C_0 and $C_{1,2}$ are of the same order, the I-V curve should be calculated using Eq. (8) and the standard calculation procedure for the SET. Similar to the usual SET, the I-V curve of the RC SET exhibits periodic cusps (jumps of the conductance) which create the "Coulomb staircase" when the difference between values of R_1 and R_2 is large enough (see the dashed line in Fig. 2). The cusp appears when a new charge state becomes allowed, so, their positions are determined by the equation

$$W_j^{\pm}(Q_n)=0\ . \tag{15}$$

This equation gives four series of cusps; however, only two of them are possible at $T=0$ because electrons tunnel only in the positive direction.

Similar to the C SET case when the coupling capacitance is not negligible, the corresponding periods depend on the method of the gate potential fixation. If $\varphi'=$const, Eq. (15) gives two series of the cusps with the periods

$$\Delta V_1=\frac{e}{C_2}\frac{C_1+C_2}{C_\Sigma},\quad \Delta V_2=\frac{e}{C_1}\frac{C_1+C_2}{C_\Sigma}\ . \tag{16}$$

Note that only the series corresponding to the junction with largest resistance (having the same number) can be pronounced enough.

If the effective gate potential φ' varies when the voltage V changes (for example, if $\varphi=$const or $\varphi-V=$const), then (similar to the C SET case) the cusps periods can be calculated from the same Eq. (16) using the equivalence between the following sets of parameters:

$$(C_1+\delta, C_2-\delta, V, Q_0)\leftrightarrow[C_1, C_2, V, Q_0+\delta VC_\Sigma/(C_1+C_2)] \tag{17}$$

for arbitrary δ [this transformation conserves $W_j^{\pm}(Q_n)$]. For example, using this equivalence one can reduce the case $\varphi=$const to the case $\varphi'=$const making the substitution $(C_1,C_2)\rightarrow(C_1-C_0C_1/C_\Sigma, C_2+C_0C_1/C_\Sigma)$, so the corresponding periods are

$$\Delta V_1=\frac{e(C_1+C_2)}{C_\Sigma C_2+C_0C_1},\quad \Delta V_2=\frac{e(C_1+C_2)}{C_\Sigma C_1-C_0C_1}\ . \tag{18}$$

For arbitrary linear relation between φ and V two periods of cusps satisfy the equation

$$\Delta V_1^{-1}+\Delta V_2^{-1}=C_\Sigma/e\ , \tag{19}$$

which follows from Eqs. (16) and (17).

For the fixed voltage V the current through RC SET is a periodic function of the gate potential (see Fig. 3) with a period similar to that for C SET

$$\Delta\varphi=e/C_0\ , \tag{20}$$

which corresponds to the addition of one electron to the central electrode. This fact follows from Eqs. (2) and (3).

Now let us consider the possibility of the experimental observation of different metastable charge states within the Coulomb blockage range. This can be done by an additional nearby SET working as the electrometer.[11,12] Suppose that the RC SET is biased by a small voltage V and the gate potential φ is varied. Because of the existence of several charge states, the charge number n and the potential of the central electrode $U=(VC_1+Q_n)/(C_1+C_2)$ exhibit hysteresis (Fig. 4). Hence, the output signal of the electrometer should be also hysteretic.

Different charge states can be observed only if they are stable enough. In the "orthodox" theory, transitions between them are possible only when the temperature T is

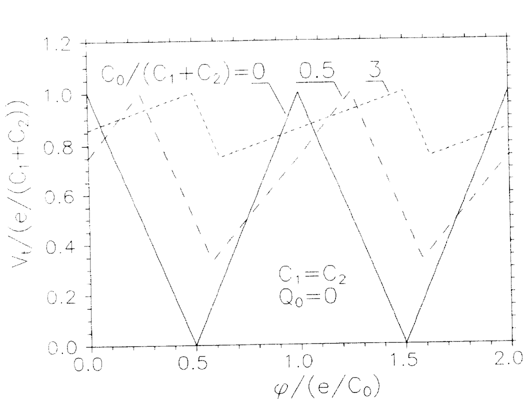

FIG. 3. The dependence of the Coulomb blockade threshold V_t on the gate potential φ for different coupling capacitances C_0.

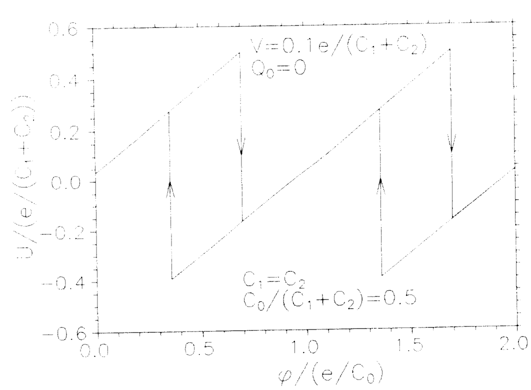

FIG. 4. The dependence of the central electrode potential U on the gate potential φ when Coulomb blockade is realized for any φ.

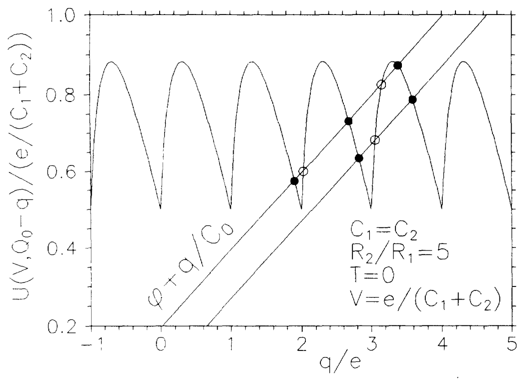

FIG. 5. The graphical solution of Eq. (21).

nonzero. Let us estimate very crudely the corresponding rate γ from the relation $-\ln(\gamma R_j C_j) \sim e^2/[(C_1+C_2)T]$. One can see that these processes are negligible for temperatures on the order of 0.1 K and capacitances $C_j \sim 10^{-16}$ F.

The processes of macroscopic quantum tunelling of charge which are not described by "orthodox" theory, are of the most importance in this case. Note that the cotunneling through both junctions does not lead to the change of the charge state (though it produces the current through the RC SET), hence, the stability of the blockade states are determined by the tunneling due to quantum fluctuations in the coupling circuit. The corresponding rate γ was calculated in Refs. 7 and 8. A very crude estimate of its magnitude is $-\ln(\gamma R_j C_j) \sim R_0 C_s e^2/[(C_1+C_2)/\hbar]$. One can see that the coupling resistance sufficiently larger than 10^5 Ω is necessary for $\gamma^{-1} \sim 1$ s.

Note that at $V=0$, the charge states corresponding to lower system energy $(Q_0+ne)^2/2C_\Sigma + \varphi(Q_0+ne)C_0/C_\Sigma$ are more probable in thermal equilibrium. In particular, at $T=0$ one state is absolutely stable, and other states are metastable due to "quantum" processes. If $V \neq 0$, all charge states are metastable. Their relative probabilities can be calculated from the master equation which takes into account "quantum" processes.

LARGE COUPLING RESISTANCE

Now let us consider the case $R_0 \gg (R_1+R_2)C_\Sigma/C_0$. In this case, for the typical current I on the order of $e/(R_1+R_2)(C_1+C_2)$, the relaxation time of the coupling circuit is much greater than I/e. Hence, the influence of the coupling circuit can be considered as a slow variation of the effective background charge Q_0-q of the double junction system due to variation of the charge q at the coupling capacitance.

Let us consider the case $T=0$. In the stationary case the average current through R_0 is zero, so that

$$\varphi + q/C_0 = U(V, Q_0 - q) , \tag{21}$$

where

$$U(V,Q_0') = \sum_n \sigma(n)(VC_1 + Q_0' + ne)/(C_1+C_2) \tag{22}$$

is the average potential of the central electrode of the double junction system calculated in the standard way.[1,3,4] An example of the graphic solution of Eq. (21) is shown in Fig. 5. One can see that for a large enough coupling capacitance, $C_0 > [\max(-dU/dQ_0')]^{-1}$, several solutions are possible. It is easy to prove that intersections marked by full circles are stable, and those marked by open circles are unstable. With variation of the gate potential φ, each intersection at first moves continuously and then disappears (it means that the system jumps to the neighboring stable state); in exchange, new intersections appear at particular potentials. It is clear that for the large variation of φ, $|\Delta\varphi| > \max(U) - \min(U)$, the state with highest q is realized when φ increases, and lowest q is realized when φ decreases (see Fig. 5).

Thus, several stable states are possible, and the functions $q(\varphi)$ and $I(\varphi)$ exhibit hysteresis. The I-V curve in this case consists of several branches and also exhibits hysteresis (Fig. 2). The number of branches for the particular bias voltage depends on C_0 (see Fig. 5) and can be more than two. The current follows the lower branch when the voltage increases, and it follows the upper one when the voltage decreases. In the case of multistability the intermediate branches can be achieved by the direction reverse at particular voltages. The stability of different charge states are restricted by processes of "quantum" tunneling, thermal noise of the coupling resistance (for $T \neq 0$), and the intrinsic noise of the central electrode potential of the double junction system.[13]

Note the small negative differential conductance of the upper branch of the I-V curve in Fig. 2 at $V(C_1+C_2)/e \approx 0.35$. This feature is not surprising, because the effective background charge Q_0-q changes with the voltage in a complicated way [see Eqs. (21) and (22)], and it can lead to the unusual I-V dependence.[14]

For high enough voltages the current through the RC SET is a single-valued function because $\max(U)-\min(U)$ tends to zero. For $V < V_t$, where V_t is given by Eqs. (10) and (11), the blockade branch exists; however, other branches can also exist (Fig. 2). The properties of the Coulomb blockade state are exactly the same as for small R_0, because the current through R_0 does not flow in both cases. This is valid for the RC SET with any ratio $R_0/(R_1+R_2)$.

DISCUSSION

The RC SET shows several features which are new in comparison with the conventional C SET or R SET and could be interesting for the possible application of the RC SET as a memory cell.[9] Several metastable charge states are possible within the Coulomb blockade range, and the potential of the central electrode exhibits hysteresis as a function of the gate potential. The blockade threshold cannot be diminished to zero by variation of the gate potential. When the coupling resistance and the capacitance well exceed those of the tunnel junctions, the I-V curve can exhibit hysteresis. The transitions between metastable states are caused mainly by the quantum fluctuations in the coupling circuit.

To conclude, let us discuss the possibility of experimental realization of the RC SET. The main difficulty in fabrication of the RC SET is to create a coupling resistor with resistance greater than R_Q but short enough to have relatively low (less than C_0) stray capacitance. However, recently there was essential progress in the solution of this problem.[5] The observations of different charge states in the case of the Coulomb blockade is quite possible with the help of the additional nearby SET working as the electrometer.[11,12] To increase the number of metastable states one should use large C_0 to increase the ratio $C_0/(C_1+C_2)$. It is quite possible to fabricate the coupling capacitance essentially greater than the junction capacitance.[15] On the other hand, for any C_0 at least two metastable blockade states are possible. Thus, the main features of the RC SET can be observed in present-day experiment.

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[14]In fact, negative differential conductance can be obtained also in the case of the R SET (having large enough coupling resistance R_0) which can be considered as the RC SET with $C_0 \rightarrow \infty$ if $\min(U) < \varphi < \max(U)$. In this case, the condition for negative differential conductance is quite simple: $dI/dV < (dI/dQ_0)(dU/dV)/(dU/dQ_0)$. It is interesting that this condition can be satisfied even for $V \gg e/(C_1+C_2)$. It means that the I-V curve of the R SET can have a decreasing part even at large voltages if φ is large (of course, the width of this part will be small for large V).

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