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Effect of the image charge on single-electron tunneling

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The existence of an image charge causes a modification of the tunnel barrier shape, which depends on the effective junction capacitance C. The tunneling rate can be calculated using the expression of the orthodox theory of single-electron tunneling, with an additional prefactor of the order of $\exp(\tau e^2/\hbar C)$ where τ is the tunneling traversal time.

Correlated tunneling in systems of ultrasmall tunnel junctions is a rapidly developing field of mesoscopic physics.[1–3] The simple orthodox theory of single-electron tunneling[1] provides the basis for theoretical analysis of these processes. There are, however, several effects not taken into account by this theory.[3] In particular, in Refs. 4–6 the influence of the finite tunneling traversal time τ was considered (in the orthodox theory, τ is assumed to be infinitesimal). It was shown that the effect becomes important when τ is of the order of $\hbar C/e^2$, where C is the junction capacitance.

The main focus of Refs. 4–6 was on the shape of the dc I-V curve. In contrast, in the present paper we calculate the multiplicative correction of order $\exp(\tau e^2/\hbar C)$ to the tunneling rate, which is independent (in our approximation) of the dc voltage applied to the system. The origin of this correction is the variation of the image charge at the edges of the tunnel junction.

First, consider a tunnel junction biased by a fixed dc voltage $V > 0$. Assume that the temperature T is zero, then tunneling is possible only in one direction (say, from left to right). The effect of the image charge on the tunneling was considered in a number of papers (see, e.g., Ref. 7 and references therein). In the simplest model we assume that the image charge follows the position of the electron inside the barrier as in the static case. This so-called "static" image[7] model is valid when the frequencies of the surface plasmons are much larger than $1/\tau$. If we also assume that the Thomas-Fermi screening length in electrodes is much less than the thickness of the barrier, the image charge can be calculated by a simple multiple reflection procedure [Fig. 1(a)], and the effective barrier shape is the sum of the initial shape $U_0(x)$ and the correction $U_{\rm IM}(x)$ due to image charges.

Note that in this case the total image charges Q_l at the left side of the junction and Q_r at the right side depend linearly on the position x of the electron inside the barrier (measured, say, from the surface of the left electrode),

$$Q_l = (x/L - 1)e, \quad Q_r = (-x/L)e, \tag{1}$$

where L is the barrier thickness. In reality these charges are located at the electrode surfaces and supplied by the voltage source.

Now consider the same tunnel junction separated from the voltage source. Let the initial voltage V be greater than $e/2C$ (after the tunneling of one electron this voltage becomes $V - e/C$). In contrast to the fixed voltage case, now the total charge of each electrode is fixed. Hence, in comparison with the previous case, there are additional charges $-Q_l - e, -Q_r$ uniformly distributed along the electrode surfaces [Fig. 1(b)]. This leads to the additional electric field $E = (Q_r - Q_l - e)/2CL = -xe/(CL^2)$ which depends on the position of the electron. Hence, the effective barrier becomes $U_0(x) + U_{\rm IM}(x) + U_{\rm SET}(x)$ where

$$\begin{aligned} U_{\rm SET}(x) &= -e\int_0^x E(x')dx' \\ &= (x^2/L^2)(e^2/2C). \end{aligned} \tag{2}$$

Let us emphasize that at the point $x = L$ this additional energy coincides with the change $e^2/2C$ of the electrostatic energy of the system after the tunneling. Because of this fact (which is valid only in the "static" image model), taking into account only the linear part of $U_{\rm SET}$,

$$U_{\rm LIN}(x) = (x/L)(e^2/2C), \tag{3}$$

we would exactly reduce the case of the separated junction to the case of the tunnel junction biased by the fixed voltage $V - e/2C$. Then the tunneling rate Γ could be calculated using the expression

$$\Gamma = I(V - e/2C)/e \quad (V > e/2C,\ T = 0), \tag{4}$$

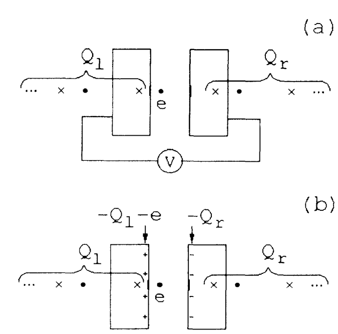

FIG. 1. (a) Image charges for voltage-biased tunnel junction and (b) additional charges in the case of the separated junction.

where $I(V)$ is the I-V curve of the junction biased by a fixed voltage. Equation (4) exactly coincides with the equation used in the orthodox theory of single-electron tunneling.[1–3]

The correction to Eq. (4) is caused by the remaining part of the barrier modification:

$$U_{\rm CORR}(x) = U_{\rm SET} - U_{\rm LIN} = -(x/L)(1 - x/L)(e^2/2C). \quad (5)$$

Assuming $U_{\rm CORR} \ll U_0 + U_{\rm IM}$ and using the WKB approximation it is easy to calculate the tunneling rate:

$$\Gamma = KI(V - e/2C)/e, \quad (6)$$

$$K = \exp\left[-\frac{1}{\hbar}\int_0^L \left(\frac{2m}{U_0(x) + U_{\rm IM}(x)}\right)^{1/2} U_{\rm CORR}(x)dx\right]. \quad (7)$$

The expression for the correction factor K depends on the barrier shape and in the general case cannot be exactly expressed in terms of capacitance C and traversal time τ. However, for an estimate let us assume $U_0(x) + U_{\rm IM}(x) = \text{const}$. This gives a simple expression

$$K = \exp\left(\frac{e^2}{6C}\frac{\tau}{\hbar}\right), \quad \tau = L[(U_0 + U_{\rm IM})/2m]^{-1/2}. \quad (8)$$

Thus, similar to the nonlinear effects considered in Refs. 4–6, the correction is essential when the traversal time τ is not too small in comparison with $e^2/\hbar C$. In our approximation this correction does not depend on the voltage.

Equations (5)–(8) can be easily extended to the case of a tunnel junction inside an arbitrary single-electron circuit containing other tunnel junctions, capacitances, voltage sources, and resistances, with the only restriction (usual for the orthodox theory) that any resistance should be either much smaller or much greater than $\hbar/e^2$ and τ/C (in the most interesting case the last two values are of the same order). Then it is straightforward[3] to introduce the effective capacitance $C_{\rm eff}$ of the tunnel junction and the only change in Eqs. (5)–(8) is the substitution $C \to C_{\rm eff}$. The effective capacitance is defined via the difference between the voltage $V_i = V$ before tunneling and the voltage V_f after tunneling,

$$C_{\rm eff} = e/(V_i - V_f). \quad (9)$$

For example, in the system of two junctions connected in series (the "single-electron transistor") the effective capacitance is the sum of the junction capacitances, $C_{\rm eff} = C_1 + C_2$.

The simple substitution $C \to C_{\rm eff}$ in Eqs. (5)–(8) is possible only if the circuit size is much less than τc (c is the speed of light), so it will not be valid for too large circuits. In this case as well as for arbitrary resistances in the circuit, a more complicated theory based on the equations of Refs. 4 and 5 is necessary.

Generalization to the case of finite temperature T is also quite simple because the barrier change $U_{\rm CORR}(x)$ does not depend on the temperature. The general expression

$$\Gamma = KI(V^*)/[1 - \exp(-eV^*/T)],$$
$$V^* = V - e/2C_{\rm eff} = \tfrac{1}{2}(V_i + V_f), \quad K = K(C_{\rm eff}) \quad (10)$$

is similar to that of the orthodox theory; the only difference is the prefactor K. The existence of this prefactor depending on the effective capacitance of the junction is the main point of the present paper.

Now let us discuss the possibility of observing the considered effect in experiment. The simplest way is to compare the dc I-V curve of the single tunnel junction biased by a fixed voltage and the dc I-V curve of the double-junction system. At $\tau \sim \hbar(C_1 + C_2)/e^2$ the current in the double-junction system should be larger than that predicted by orthodox theory. The simplest check is to compare the low-voltage resistance of one junction with the differential resistance of the double-junction system at the voltage just above the Coulomb blockade threshold. In orthodox theory these two values coincide, if the background charge is not close to zero and the temperature is low.

Let us estimate possible experimental parameters. For the tunnel junctions metal-insulator-metal the typical traversal time τ is about 3×10^{-15} s. The correction factor K in this case is essential for $e/C > 0.3$ V. Hence, in principle, the effect can be observed using the scanning tunneling microscope.[1–3] However, in this case it is practically impossible to prepare identical tunnel junctions for single-junction and double-junction experiments (note that the single-junction experiment itself is difficult in this case).

The traversal time in semiconductor tunnel junctions can be made much longer by the use of low tunnel barriers. For τ long enough the model of "static" image charge can be applicable[8] in spite of the fact that the plasmon frequencies in semiconductors are much lower than those in metals. In Ref. 8 tunnel junctions having traversal time up to 3×10^{-13} s were used. For our estimate, let us take the more moderate value $\tau = 10^{-13}$ s. Then for observation of the effect considered in the present paper, the typical voltage e/C should be of the order of $\hbar/\tau e \sim 7$ mV. Hence, the typical capacitance may be about 2×10^{-17} F. Note that the condition $\tau \sim \hbar C_{\rm eff}/e^2$ means that the typical voltage of the exponential nonlinearity of the I-V curve should be of the order of Coulomb blockade threshold.

The finite temperature affects equally the resistances in single-junction and double-junction cases.[6] Hence, the effect considered in the present paper should remain at temperatures

$$T \ll \min(e^2/C, \ \hbar/\tau). \quad (11)$$

One can find the corresponding formulas as well as generalization of the present work to the case of finite external impedance in Ref. 9.

In conclusion, we have found a correction to the tunneling rate which is used in the orthodox theory of single-electron tunneling. The correction is essential when $\tau e^2/\hbar C > 1$. It leads, in particular, to an increase of the current through the double-junction system in comparison with that calculated using the orthodox theory.

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