

Dynamics revealed by correlations of time-distributed weak measurements of a single spin

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Abstract. We show that the correlations in stochastic outputs of time-distributed weak measurements can be used to study the dynamics of an individual quantum object, with a proof-of-principle setup based on small Faraday rotation caused by a single spin in a quantum dot. In particular, the third-order correlation can reveal the 'true' spin decoherence, which would otherwise be concealed by the inhomogeneous broadening effect in the second-order correlations. The viability of such approaches lies in the fact that (i) in weak measurement the state collapse that would disturb the system dynamics occurs at a very low probability and (ii) a shot of measurement projecting the quantum object to a known basis state serves as a starter or stopper of the evolution without pumping or coherently controlling the system as otherwise required in conventional spin echo.

The standard von Neumann quantum measurement may be generalized from two aspects. One involves measurements distributed in time [1, 2], continuously or in a discrete sequence, as in the interesting Zeno [1] and anti-Zeno effects [3]. Time-distributed measurements intrinsically interfere with the evolution of the quantum object [2]. Another generalization involves weak measurement in which the probability of distinguishing the state of a quantum object by a single shot of measurement is much smaller than one [4]–[8]. On the one hand, the weak measurement has very low information yield rate; on the other hand, it only rarely disturbs the

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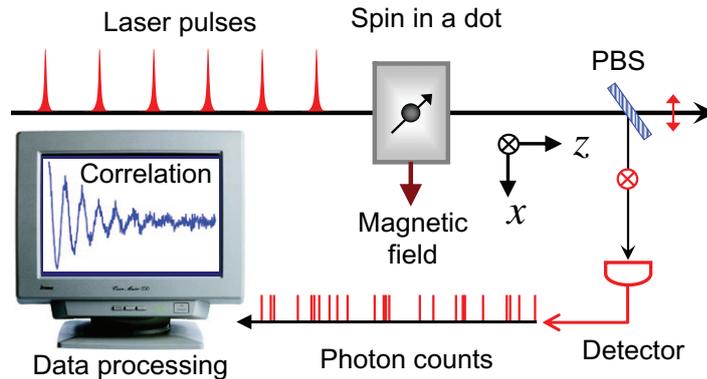


Figure 1. A proof-of-principle setup for weak measurement of a single spin in a quantum dot by Faraday rotation.

dynamics of a quantum object by state collapse. As a combination of the two generalizations, time-distributed weak measurements have been used to steer the quantum state evolution [9]. In this paper, we show that the statistical analysis of time-distributed weak measurements may be used to study the dynamics of a quantum object [8]. The outputs of time-distributed measurements bear the stochastic nature of quantum measurements, so the standard noise analysis in quantum optics [10] would be a natural method to be applied. Notwithstanding that, we should emphasize that the stochastic output of time-distributed weak measurement is not the noise in the system but an intrinsic quantum mechanical phenomenon. Revealing quantum dynamics by correlations of time-distributed weak measurements is complementary to the fundamental dissipation–fluctuation theorem, which relates correlations of thermal noises to the linear response of a system [11]–[14].

To demonstrate the basic idea, we consider the monitoring of coherent Larmor precession and decoherence of a single spin in a quantum dot, which is relevant to exploiting the spin coherence in quantum technologies such as quantum computing [15]–[18]. The difficulty of studying the spin decoherence lies in the fact that the ‘true’ decoherence due to quantum entanglement with environments is often concealed by the rapid ‘phenomenological’ dephasing caused by inhomogeneous broadening in ensemble measurements (e.g. in a typical GaAs quantum dot, the spin decoherence time is $\sim 10^{-6}$ s, but the inhomogeneous broadening dephasing time is $\sim 10^{-9}$ s [16]–[21]). Note that many single-spin experiments are still ensemble experiments with temporal repetition of measurements. To resolve the spin decoherence excluding the inhomogeneous broadening effect, spin echo [16, 19], [21]–[23] and mode locking of spin frequency [18] have been invoked. In this paper, we will show that the spin dynamics can be revealed in correlations of the stochastic outputs of sequential weak probes. In particular, the third-order correlation singles out the ‘true’ spin decoherence. Unlike conventional spin echo, the present method involves no explicit pump or control of the spin but uses the state collapse as the starter or stopper of the spin precession.

We design a proof-of-principle setup (see figure 1) based on Faraday rotation, which has been used in experiments for spin measurements [18, 20, 21, 24, 25]. The probe consists of a sequence of linearly polarized laser pulses evenly spaced in delay time τ . After interaction with a single spin (in a quantum dot, e.g.), light polarization is rotated by θ or $-\theta$ for the spin state parallel or anti-parallel to the light propagation direction (z -axis). The Faraday rotation

angle θ by a single electron spin is usually very small ($\sim 10^{-6}$ rad in a quantum dot [24, 25]), so the two polarization states of light corresponding to the two different spin states are almost identical. Thus the detection of light polarization is a weak measurement of the spin, as long as the number of photons per pulse is not too large (see the discussions following equation (3) for details). Light polarization is detected by filtering through a polarized beam splitter (PBS) that is aligned to let light with polarization rotated by θ fully pass through and light with orthogonal polarization be fully reflected. The light with Faraday rotation angle $-\theta$ is reflected with probability $\sin^2(2\theta)$. For small θ , the average number of reflected photons is much less than one, so in most cases a single-photon detector set at the reflection arm would be idle with no clicks and one cannot tell which state the spin could be in. The clicks of the detector form a stochastic sequence. Correlations in the sequence will be analyzed to study the spin dynamics, such as precession under a transverse magnetic field and decoherence. This proof-of-principle setup, being conceptually simple and adapted from existing experiments, is of course not the only possible implementation. For example, one can use continuous-wave probes instead of pulse sequences, interferometer measurement of the polarization instead of PBS filtering, polarization-selective absorption instead of Faraday rotation, and so on.

We shall derive from the quantum optics description of spin–light interaction a weak measurement theory in the formalism of positive operator value measure (POVM) [1, 26]. Consider a laser pulse in a coherent state $|\alpha, H\rangle \equiv e^{\alpha a_{H/V}^\dagger - \text{h.c.}}|0\rangle$ (where $a_{H/V}^\dagger$ creates a photon with linear polarization H or V) and a spin in an arbitrary superposition $C_+|+\rangle + C_-|-\rangle$ in the basis quantized along the z -axis; the initial spin-photon state is

$$|\psi\rangle = (C_+|+\rangle + C_-|-\rangle) \otimes |\alpha, H\rangle. \quad (1)$$

After interaction, the state becomes an entangled one as

$$|\psi'\rangle = C_+|+\rangle \otimes |\alpha, +\theta\rangle + C_-|-\rangle \otimes |\alpha, -\theta\rangle, \quad (2)$$

where $|\alpha, \pm\theta\rangle \equiv e^{\alpha a_{\pm\theta}^\dagger - \text{h.c.}}|0\rangle$ (with $a_{\pm\theta} \equiv a_H \cos\theta \pm a_V \sin\theta$) is a photon coherent state with polarization rotated by $\pm\theta$. How much the spin is measured is determined by the distinguishability between the two polarization states

$$\mathcal{D} \equiv 1 - |\langle \alpha, +\theta | \alpha, -\theta \rangle|^2 = 1 - \exp(-4|\alpha|^2 \sin^2 \theta). \quad (3)$$

When the average number of photons $\bar{N} = |\alpha|^2 \gg 1$ and the Faraday rotation angle θ is not too small, the two coherent states are almost orthogonal and $\mathcal{D} \rightarrow 1$; thus detection of the light polarization provides a von Neumann projective measurement of the spin. For a single spin in a quantum dot, the Faraday rotation angle θ is usually very small. For example, in a GaAs fluctuation quantum dot [24], $|\theta| \sim 10^{-5}$ rad for light tuned 1 meV below the optical resonance with a focus spot area $\sim 10 \mu\text{m}^2$. The number of photons in a 10 ps pulse with power 10 mW is $\bar{N} \sim 0.5 \times 10^6$. In this case, $\mathcal{D} \cong 4\bar{N}\theta^2 \sim 2 \times 10^{-4} \ll 1$, the spin states are almost indistinguishable by the photon polarization states. After interaction with the spin, the laser pulse is subject to PBS filtering, which transforms the spin-photon state to be

$$|\psi''\rangle = C_+|+\rangle \otimes |\alpha\rangle_t \otimes |0\rangle_r + C_-|-\rangle \otimes |\alpha \cos(2\theta)\rangle_t \otimes |\alpha \sin(2\theta)\rangle_r, \quad (4)$$

where $|\beta\rangle_{t/r}$ denotes a coherent state of the transmitted/reflected mode with amplitude β . Separating the vacuum state $|0\rangle_r$ from the reflected mode and keeping terms up to a relative

error $O(\theta^2)$, we write the state as

$$|\psi''\rangle = \left(C_+|+\rangle + \sqrt{1-\mathcal{D}}C_-|-\rangle \right) \otimes |\alpha\rangle_t \otimes |0\rangle_r + \sqrt{\mathcal{D}}C_-|-\rangle \otimes |\alpha\rangle_t \otimes |\alpha \sin(2\theta)\rangle'_r, \quad (5)$$

where $|\alpha \sin(2\theta)\rangle'_r$ denotes the (normalized) state of the reflected mode but with the vacuum component dropped. With a probability $P_1 = \mathcal{D}|C_-|^2 \ll 1$, an ideal detector at the reflection arm will detect a photon click and the spin state is known at $|-\rangle$, while in most cases (with probability $P_0 = 1 - P_1$), the detector will be idle and the spin state becomes $C_+|+\rangle + \sqrt{1-\mathcal{D}}C_-|-\rangle$ (up to a normalization factor), which is almost undisturbed by the measurement since the overlap between the state before the measurement and the state after the measurement is $(|C_+|^2 + \sqrt{1-\mathcal{D}}|C_-|^2)/\sqrt{1-|C_-|^2\mathcal{D}} = 1 - O(\mathcal{D}^2)$. In POVM formalism [1, 26], Kraus operators for the click and no-click cases are, respectively,

$$\hat{M}_1 = \sqrt{\mathcal{D}}|-\rangle\langle-| \quad \text{and} \quad \hat{M}_0 = \sqrt{1-\mathcal{D}}|-\rangle\langle-| + |+\rangle\langle+|, \quad (6)$$

which determine the (non-normalized) post-measurement state $\hat{M}_{0/1}|\psi\rangle$ and the probability $P_{0/1} = \langle\psi|\hat{M}_{0/1}^\dagger\hat{M}_{0/1}|\psi\rangle$.

Between two subsequent shots of measurement, spin precession under a transverse magnetic field (along the x -direction) is described by

$$\hat{U} = \exp(-i\hat{\sigma}_x\omega\tau/2), \quad (7)$$

where $\hat{\sigma}_x$ is the Pauli matrix along the x -direction and ω is the Larmor frequency. Coupled to the environment and subject to dynamically fluctuating local fields, spin precession is always accompanied by decoherence. For simplicity, we consider an exponential coherence decay characterized by a decoherence time T_2 . In the quantum trajectory picture [5, 10], decoherence can be understood as a result of continuous measurement by the environment along the x -axis, for which the Kraus operators for quantum jumps with and without phase flip are, respectively [26],

$$\hat{E}_1 = \sqrt{\gamma/2}\hat{\sigma}_x \quad \text{and} \quad \hat{E}_0 = \sqrt{1-\gamma/2}\hat{I}, \quad (8)$$

where $\gamma \equiv 1 - \exp(-\tau/T_2) \cong \tau/T_2$ is the coherence lost between two subsequent measurements. For a spin state described by a density operator $\hat{\rho}$, the decoherence within τ leads the state to $\hat{\mathcal{E}}[\hat{\rho}] \equiv \hat{E}_0\hat{\rho}\hat{E}_0^\dagger + \hat{E}_1\hat{\rho}\hat{E}_1^\dagger$.

To study the spin dynamics under sequential measurement, we generalize the POVM formalism for a sequence of n measurements. To incorporate spin decoherence in density operator evolution, we define the superoperators for the weak measurement and the free evolution as $\hat{\mathcal{M}}_{0/1}[\hat{\rho}] = \hat{M}_{0/1}\hat{\rho}\hat{M}_{0/1}^\dagger$ and $\hat{\mathcal{U}}[\hat{\rho}] = \hat{U}\hat{\rho}\hat{U}^\dagger$, in addition to the decoherence superoperator $\hat{\mathcal{E}}$ defined above. For a sequence output $X \equiv [x_1x_2\dots x_n]$ as a string of binary numbers, the superoperator,

$$\hat{\mathcal{M}}_X = \hat{\mathcal{M}}_{x_n}\hat{\mathcal{E}}\hat{\mathcal{U}}\hat{\mathcal{M}}_{x_{n-1}}\dots\hat{\mathcal{M}}_{x_3}\hat{\mathcal{E}}\hat{\mathcal{U}}\hat{\mathcal{M}}_{x_2}\hat{\mathcal{E}}\hat{\mathcal{U}}\hat{\mathcal{M}}_{x_1}, \quad (9)$$

transforms an initial density operator $\hat{\rho}$ to $\hat{\mathcal{M}}_X[\hat{\rho}]$ (not normalized) and determines the probability of the output $P_X = \text{Tr}(\hat{\mathcal{M}}_X[\hat{\rho}])$. With the POVM formalism, the spin state evolution under sequential measurement and hence the noise correlations discussed below can be readily evaluated.

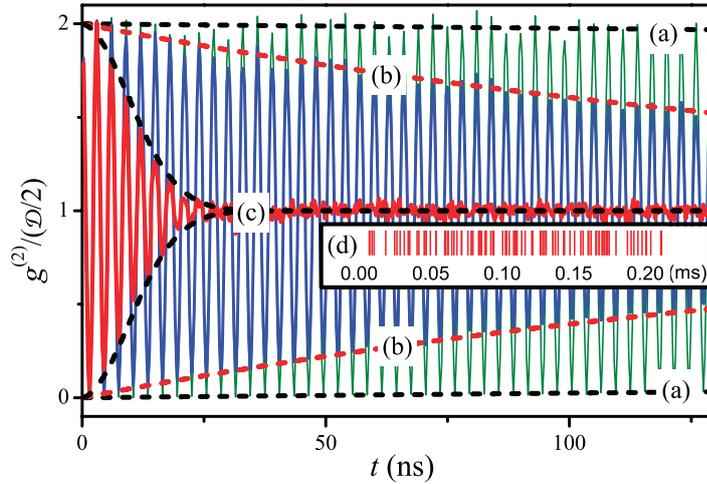


Figure 2. The Monte Carlo simulation (solid oscillating curves) and the analytical result (envelopes in dashed lines) of the second-order correlation function, calculated with distinguishability $\mathcal{D} = 3 \times 10^{-4}$, Larmor precession period $2\pi/\omega_0 = 3$ ns and the interval between two subsequent measurements $\tau = 0.3$ ns. In (a), no decoherence or inhomogeneous broadening is present ($T_2^{-1} = \sigma = 0$). In (b), $T_2 = 200$ ns but $\sigma = 0$. In (c), $T_2 = 200$ ns and $\sigma^{-1} = 10$ ns. (d) Shows the stochastic output (each line indicating a click event), obtained in the Monte Carlo simulation of about 7×10^5 shots of measurement during a real time of about 0.2 ms, with the same condition as in (a).

To illustrate how a real experiment would perform, we have carried out Monte Carlo simulations of the measurement with the following algorithm: (1) we start from a randomly chosen state of the spin $|\psi\rangle$; (2) the state after a free evolution is $\hat{U}|\psi\rangle$; (3) then the decoherence effect is taken into account by applying randomly the Kraus operator \hat{E}_0 or \hat{E}_1 to the state (with normalization) with probability $1 - \gamma/2$ or $\gamma/2$, respectively; (4) the measurement is done by randomly applying the Kraus operator \hat{M}_0 or \hat{M}_1 to the state (with normalization) corresponding to the output 0 or 1 (no-click or click), with probability P_0 or P_1 given by the POVM formalism. Steps (2)–(4) are repeated many times. The output is a random sequence of clicks, as shown in figure 2(d).

To study the correlation of the stochastic output, we first consider the interval distribution $K(n)$, defined as the probability of having two clicks separated by $n - 1$ no-clicks [10],

$$K(n) \equiv \text{Tr}(\hat{\mathcal{M}}_{[10_{n-1}]}[\hat{\rho}]) / \text{Tr}(\hat{\mathcal{M}}_1[\hat{\rho}]), \quad (10)$$

where 0_{n-1} means a string of $n - 1$ zeros. By a straightforward calculation,

$$K(n) \approx \frac{\mathcal{D} + \mathcal{D}^2}{2} e^{-n\mathcal{D}/2} \left[1 + e^{-n\tau/T_2} \cos \left(n\omega\tau + \frac{\mathcal{D}}{2} \cot \frac{\omega\tau}{2} \right) \right], \quad (11)$$

up to $O(\gamma\mathcal{D}^2)$ and $O(n\mathcal{D}^3)$, for $\gamma, \mathcal{D} \ll \omega\tau < \pi$. A successful measurement at the beginning of an interval projects the spin to the basis state $|-\rangle$ along the optical (z) axis. Then, the spin precesses under the external magnetic field about the x -axis. The interval is terminated by a second successful measurement among periodic attempts after a time lapse of $n\tau$. The decay of the oscillation is due to spin decoherence. The overall decay $e^{-n\mathcal{D}/2}$ is due to decreasing the

probability of unsuccessful measurement with increasing time. The measurement also induces a little phase shift to the oscillation. Obviously, the smaller the distinguishability \mathcal{D} , the less the spin dynamics is disturbed by the measurement.

In experiments, often the photon coincidence correlation instead of the interval distribution is measured. The second-order correlation $g^{(2)}(n\tau)$ is the probability of having two clicks separated by $n - 1$ measurements [10], regardless of the results in between,

$$g^{(2)}(n\tau) = \sum_{x_1, x_2, \dots, x_{n-1} \in \{0, 1\}} \text{Tr}(\hat{\mathcal{M}}_{1x_1x_2\dots x_{n-1}}[\hat{\rho}]) / \text{Tr}(\hat{\mathcal{M}}_1[\hat{\rho}])$$

$$= K(n) + \sum_{m=1}^{n-1} K(n-m)K(m) + \sum_{m=2}^{n-1} \sum_{l=1}^{m-1} K(n-m)K(m-l)K(l) + \dots \quad (12)$$

By Fourier transformation and summation in the frequency domain,

$$g^{(2)}(n\tau) = \frac{\mathcal{D}}{2} \left[1 + e^{-n(\tau/T_2 + \mathcal{D}/4)} \cos(n\omega\tau) + O(\mathcal{D}) \right]. \quad (13)$$

Spin precession, decoherence and measurement-induced decay are all seen in the second-order correlation function (see figure 2). Note that the overall decay of the interval distribution manifests itself as a measurement-induced dephasing of the oscillating signal in the correlation function. The Monte Carlo simulation shows that 10^{10} shots of measurement would yield a rather smooth profile of the spin dynamics, which requires a time span of about 3 s for the parameters used in figure 2.

In addition to the decoherence due to dynamical fluctuation of the local field, there is also phenomenological dephasing due to static or slow fluctuations, i.e. inhomogeneous broadening that exists even for a single spin since the sequential measurement contains many shots that form an ensemble. Inhomogeneous broadening is modeled by a Gaussian distribution of ω with mean value ω_0 and width σ . With inhomogeneous broadening included, the ensemble-averaged correlation function becomes

$$\langle g^{(2)}(n\tau) \rangle = \frac{\mathcal{D}}{2} \left[1 + e^{-n(\tau/T_2 + \mathcal{D}/4) - n^2\tau^2\sigma^2/2} \cos(n\omega_0\tau) + O(\mathcal{D}) \right]. \quad (14)$$

Since usually $\sigma \gg T_2^{-1}$, the decay of the second-order correlation is dominated by the inhomogeneous broadening effect (see figure 2(c)).

To separate spin decoherence from inhomogeneous broadening, we resort to the third-order correlation $g^{(3)}(n_1\tau, n_2\tau)$, the probability of having three clicks separated by $n_1 - 1$ and $n_2 - 1$ measurements. The idea can be understood in a post-measurement selection picture. After the first click, the second click has the peak probability appearing at an integer multiple of the spin precession period, so the coincidence of the two earlier clicks serves as filtering of the spin frequency and the third click would have a peak probability appearing at $n_2\tau = n_1\tau$, similar to the spin echo. The third-order correlation in the absence of inhomogeneous broadening is $g^{(3)}(t_1, t_2) \propto g^{(2)}(t_1)g^{(2)}(t_2)$. The ensemble average leads to

$$\langle g^{(3)}(t_1, t_2) \rangle \propto 1 + \sum_{j=1,2} e^{-(T_2^{-1} + \tau^{-1}\mathcal{D}/4)t_j - \sigma^2 t_j^2/2} \cos(\omega_0 t_j)$$

$$+ \frac{1}{2} e^{-(T_2^{-1} + \tau^{-1}\mathcal{D}/4)(t_1+t_2)} e^{-\sigma^2(t_1+t_2)^2/2} \cos(\omega_0(t_1+t_2))$$

$$+ \frac{1}{2} e^{-(T_2^{-1} + \tau^{-1}\mathcal{D}/4)(t_1+t_2)} e^{-\sigma^2(t_1-t_2)^2/2} \cos(\omega_0(t_1-t_2)). \quad (15)$$

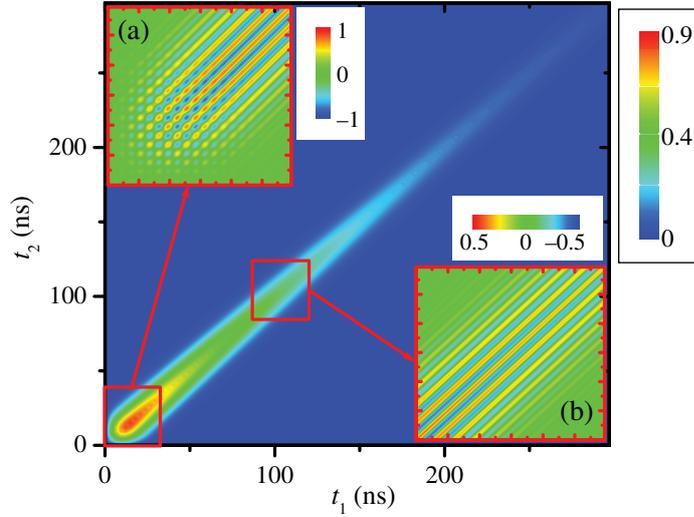


Figure 3. Contour plot of the envelope of the third-order correlation $G^{(3)}(t_1, t_2)$, with parameters the same as for figure 2(c). Insets (a) and (b) show the oscillation details of $G(t_1, t_2)$ in the range $0 \text{ ns} \leq t_{1,2} \leq 30 \text{ ns}$ and $90 \text{ ns} \leq t_{1,2} \leq 120 \text{ ns}$, respectively.

Figure 3 plots $G^{(3)}(t_1, t_2) \equiv \langle g^{(3)}(t_1, t_2) \rangle - \langle g^{(2)}(t_1) \rangle \langle g^{(2)}(t_2) \rangle$ to exclude the trivial background. Along the direction $t_1 = -t_2$, the third-order correlation oscillates and decays rapidly (with timescale σ^{-1}). But the oscillation amplitude decays slowly (with timescale T_2) along the direction $t_1 = t_2$, as expected from the last term of equation (15).

In conclusion, we have given a statistical treatment of sequential weak measurements of a single spin. The characteristics of the weak measurement consist of the negligible perturbation of the spin state except for the projective state collapse when the measurement is successful in identifying the spin state. We show that the third-order correlation reveals spin decoherence from inhomogeneous broadening. The theory presented here for sequential pulse measurement can be straightforwardly generalized to continuous weak measurement by letting the pulse separation $\tau \rightarrow 0$ while keeping the average power of the light unchanged (i.e. $\mathcal{D}/\tau = \text{constant}$). In the proof-of-principle setup based on Faraday rotation, all optical elements have been assumed to be ideal for conceptual simplicity. An investigation of the defects, e.g. in the PBS and in the photon detector, shows that they do not change the essential results presented here but only reduce the visibility of the features. Details will be published elsewhere.

Acknowledgments

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