Gain dependence of the noise in the single electron transistor

B. Starmark,[a] Torsten Henning, T. Claeson, and P. Delsing
Department of Microelectronics and Nanoscience, Göteborg University and Chalmers University of Technology, S-412 96 Göteborg, Sweden

A. N. Korotkov
Nuclear Physics Institute, Moscow State University, 119899 GSP, Russia

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An extensive investigation of low frequency noise in single electron transistors as a function of gain is presented. Comparing the output noise with gain for a large number of bias points, it is found that the noise is dominated by external charge noise. For low gains we find an additional noise contribution which is compared to a model including resistance fluctuations. We conclude that this excess noise is not primarily due to resistance fluctuations. For one sample, we find a low minimum charge noise of $q_n \approx 2\times 10^{-5}$ $e/\sqrt{\text{Hz}}$ at a frequency of 4.4 kHz. © *1999 American Institute of Physics.* [S0021-8979(99)05815-6]

I. INTRODUCTION

With the introduction of the single electron transistor (SET) one decade ago, it became possible to directly measure changes in charge below that of an electron.[1,2] Based on the Coulomb blockade, the device is the most sensitive electrometer existing today. The sensitivity of the SET is predicted to be limited by the shot noise[3] generated when electrons tunnel across the tunnel barriers.[4] Shot noise was observed in a two junction structure (without gate).[5] In most experiments involving SETs, the noise at low frequencies is dominated by the device itself, whereas external sources set the noise limit for frequencies above the kHz regime.

Several experimental studies of low frequency noise of various SET configurations have been performed.[6–19] Below 1 kHz, $1/f$ noise is observed in all SETs regardless of mode of operation.[7–19] The input equivalent charge noise at 10 Hz in all these experiments is of the order of 10^{-3} to 10^{-5} $e/\sqrt{\text{Hz}}$, with 2.5×10^{-5} $e/\sqrt{\text{Hz}}$ recently reported as the lowest figure.[17] Deviations from a $1/f$ spectrum are often observed, usually in combination with telegraph noise.[7,9] The source of the latter is believed to be random excitations of a single charge trap. Theoretically, the random trapping process of a single trap shows a Debye–Lorentzian power spectrum[20] which is also observed experimentally.[10,13,14] However, an ensemble of traps can produce a $1/f$ noise spectrum, see, e.g., Ref. 24.

There are at least three possible locations of these traps: the tunnel junction dielectric, the substrate on which the device is fabricated, and the oxide layer covering the island. The role of the substrate has been examined in at least two sets of experiments.[13,18] Those experiments did not show a strong dependence of the noise on the substrate material. The barrier dielectric has been proposed as the location of charge traps.[9–11,17] The role of the surface oxide of the island has not yet been investigated.

Several groups have found that the noise at the output of the SET varies with the gain of the SET and that the maximum noise is found at the bias point with maximum gain.[10–12,15] This indicates that the noise source acts at the input of the device, i.e., as an external fluctuating charge.

In this article, we report the low frequency current noise of one $Al/AlO_x/Al/AlO_x/Al$ SET and one $Nb/AlO_x/Al/AlO_x/Nb$ SET and make a detailed comparison with the gain. Hereafter, we will refer to the two SETs as the Al SET and the Nb SET. For the Al SET, we find that the noise follows the gain in such a manner that the noise of the SET is dominated by input noise for almost all values of bias and gate voltage. For the Nb SET however, we find a contribution from other sources when the gain is low.

II. EXPERIMENTAL TECHNIQUES

The samples were fabricated on oxidized Si substrates using electron beam lithography and the standard double-angle evaporation technique.[21,22]

The resistance of the Al SET directly after fabrication was $R_T = R_1 + R_2 \approx 3.5$ kΩ, which after a storage for six months, had increased to $R_T \approx 45$ kΩ. The Nb SET had a resistance of $R_T \approx 170$ kΩ.

We used a symmetric, current sensitive amplifier which voltage biased the SET.[12] The setup is shown in Fig. 1. To optimize the preamplifier noise performance, operational amplifiers with low $1/f$ noise were used (AD743, Analog Devices). Furthermore, the bias (feedback) resistors were chosen to $R_F = 10$ MΩ to lower the amplifier noise floor at low frequencies.

The SETs were attached to the mixing chamber of a dilution refrigerator which was cooled to a temperature below 30 mK. All measurement leads were filtered with 0.5 m of Thermocoax[23] followed by capacitors to ground. The measured total line capacitance was $C_l = 1$ nF.

Evaluating the performance of the measurement system, we found a signal bandwidth (i.e., the frequency where the output signal decreased by $1/\sqrt{2}$) of the setup consisting of

[a]Electronic mail: starmark@fy.chalmers.se

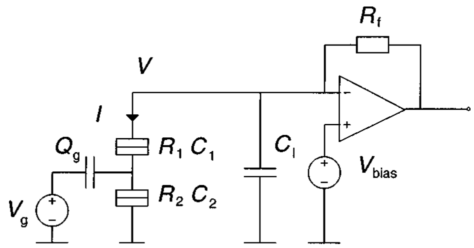

FIG. 1. A simplified scheme of the SET and the current amplifier. V_{bias} is an external bias voltage. In the experiment, we used a symmetric version of the amplifier.

the SET, preamplifier and line capacitance, of 7.5 kHz. The noise bandwidth, here defined as the frequency where the low and high frequency noise asymptotes intersect (see curve R in Fig. 3), was about 300 Hz (without any SET connected). Both these figures were set by the line capacitance and the preamplifier.[12]

The noise spectra were recorded using an HP 35665A Dynamic Signal Analyzer, which performs real-time FFT analysis of the input signal. The frequency range from 1 to 10^5 Hz was divided into four subranges to increase the resolution. The time to acquire all noise data for one bias point was 5 min.

III. RESULTS AND DISCUSSION

The current–voltage ($I-V$) characteristics for the Al SET are shown in Fig. 2, both for the normal and the superconducting state. A total island capacitance of $C_\Sigma = 0.19$ fF was deduced from the $I-V$-curves. The Nb SET had $C_\Sigma = 0.48$ fF. The output impedance, $r_o = (\partial V/\partial I)$, was calculated from the $I-V$ curves. In the superconducting state r_o was always above 20 kΩ and in the normal state, r_o was on the order of, or above R_T for both SETs. The gate coupling capacitances were $C_g \approx 4.8$ aF and $C_g \approx 0.3$ aF for the Al and the Nb SETs, respectively.

Due to its low resistance, the Al SET had high maximum gains of $\partial I/\partial Q_g = 12$ and 34 nA/e in the normal and superconducting states, respectively. The Nb SET had maximum gains of $\partial I/\partial Q_g = 1.8$ and 3.8 nA/e in the normal and superconducting states. The higher gain of the superconducting state is in accord with earlier observations.[8,9]

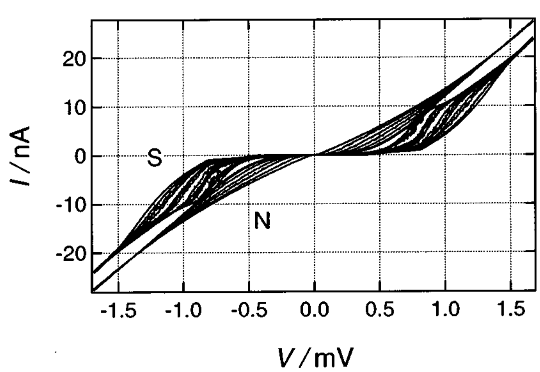

FIG. 2. Current–voltage characteristics for the Al SET in the normal and superconducting state for several different gate charges. The curves were measured at $T = 30$ mK and $B = 0.5$ T.

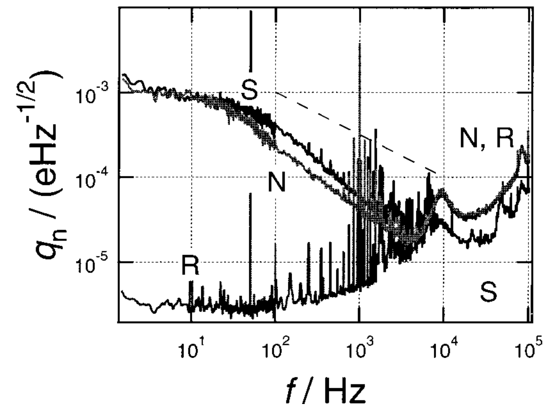

FIG. 3. Noise spectra for the Al SET at $T = 30$ mK for the normal (N) and superconducting (S) state and with no SET connected (R). The noise is input referred to the input charge. The spectrum has almost the same shape for the N and S states, and declines as $1/f^{1.6}$. The dashed line indicates $1/f$ behavior.

Noise spectra for the normal and superconducting states of the Al SET are shown in Fig. 3. Each spectrum is referred to the input of the SET by dividing by the frequency dependent gain. The spectra labeled N and S in Fig. 3 were measured at the bias points which gave maximum gain. For reference, a spectrum R, measured with no SET connected is also shown and is divided by the same gain as in the normal state to obtain the input referred noise floor set by the amplifier. Minimum charge noises of $q_n \approx 2 \times 10^{-5}\ e/\sqrt{\mathrm{Hz}}$ at a frequency of 4.4 kHz were found both in the superconducting and normal states. This limit is set by the preamplifier and mechanical resonances within the cryostat. The noise at 10 Hz was $9 \times 10^{-4}\ e/\sqrt{\mathrm{Hz}}$ for both the superconducting and normal states. A crossover from input dominated to output dominated noise can be seen as the frequency increases. Below 1 kHz, the input referred noise is almost the same in both normal and superconducting states, indicating that the noise source acts as an apparent charge noise, and thus is independent of gain. Above 5 kHz the noise is dominated by sources acting at the output of the SET, mostly preamplifier noise. When referred to the input, this noise appears as a lower equivalent charge noise in the superconducting state as compared to the normal state, due to the higher gain in the superconducting state. The spectra are very similar in the two different states, and approach a constant at low frequencies. Above 10 Hz, the power spectra, S_{Q_g}, decline as $1/f^{1.6}$. They show neither Lorentzian nor pure $1/f$ behavior. The reason for this is unknown.

We now turn our attention to the noise below 1 kHz. To determine the origin of the noise, we measured noise for 130 bias and gate voltages and compared it with the measured gain of the SET. All points were taken in the normal state at a temperature of $T \approx 30$ mK. The output current noise and the gain versus gate bias for the Al SET are shown in Fig. 4(a) for three bias voltages. To improve accuracy, the noise was integrated in the band 51–99 Hz. By using the gate charge offset as the only fitting parameter, we were able to get an excellent fit of the noise to the gain. This result clearly shows that the noise source acts at the input of the SET. It is probably due to the motion of background charges somewhere in

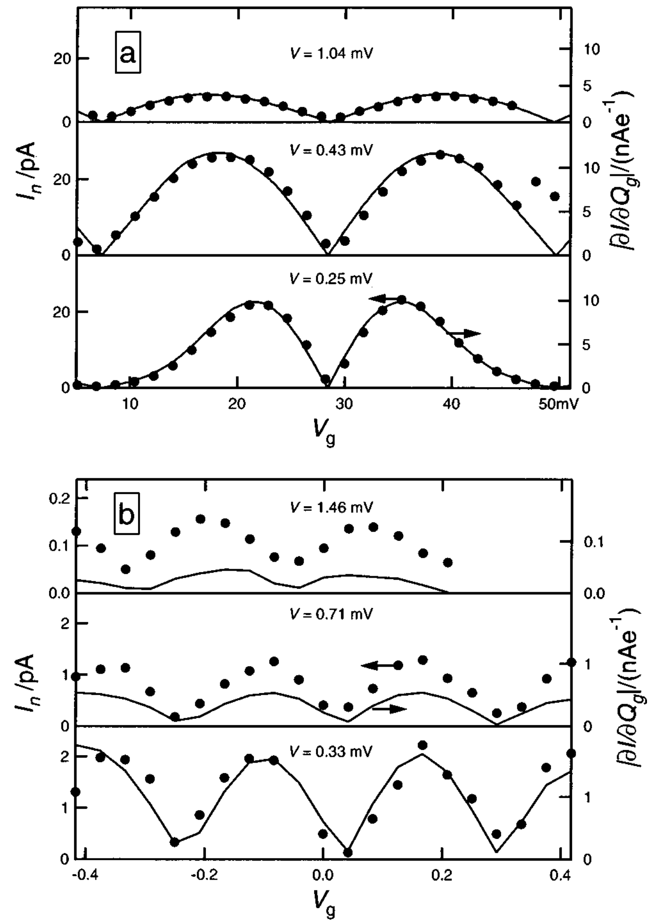

FIG. 4. (a) Integrated output noise (symbols), I_n, and gain (curves), $\partial I/\partial Q_g$, vs gate bias for the Al SET at $T=30$ mK and $B=0.5$ T. The noise and gain fits are excellent. Due to a random change in background charge, the two highest gate bias points were shifted in the $V=0.43$ mV panel. (b) Noise and gain for the Nb SET at $T=30$ mK and $B=5.0$ T. For the lowest bias voltage ($V=0.33$ mV panel), the noise and gain follows each other, but show more spread than the Al SET. This is due to lower current levels and a coarser gain determination. For high bias voltages, an excess noise contribution is clearly visible. Note that the noise and gain scales are magnified by a factor ten in the $V=1.46$ mV panel.

the vicinity of the SET. The Nb SET also showed a gain dependent output noise, but with more spread in the data [see Fig. 4(b)]. Presently, we would like to study the noise which is not caused by charge fluctuations. We call this noise *excess noise*. From the discussion above, we know that the charge fluctuations dominate and must be subtracted. For each bias point, we model the SET current noise as

$$S_I = G^2 S_{Q_g} + S_{I,\mathrm{exc}}, \tag{1}$$

where S_I is the SET current noise power spectral density (PSD), $G=\partial I/\partial Q_g$ is the gain, S_{Q_g} is the charge noise PSD, and $S_{I,\mathrm{exc}}$ is the excess noise PSD of interest. To determine the S_{Q_g} and $S_{I,\mathrm{exc}}$, at least two noise measurements are required. Good choices of bias points are one with maximum gain, $G=G_{\max}$, where S_{Q_g} dominates, and one bias point with low gain, $G=G_{\min}$, where $S_{I,\mathrm{exc}}$ may dominate. Ideally, one would like to find the bias point where the gain is zero, however, due to the finite number of bias points, the gain is not completely zeroed. The two measurements give two

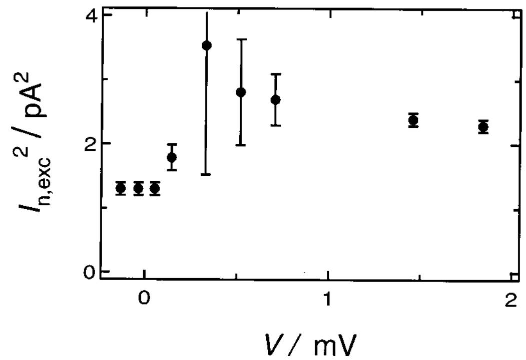

FIG. 5. The excess noise, here defined as the current noise which is not caused by charge fluctuations, as a function of bias voltage for the Nb SET.

equations which can be solved for S_{Q_g} and $S_{I,\mathrm{exc}}$. The excess noise current, $I_{n,\mathrm{exc}}$, is defined by integrating $S_{I,\mathrm{exc}}$ over the frequency band:

$$I^2_{n,\mathrm{exc}}(V) = \int_{51\ \mathrm{Hz}}^{99\ \mathrm{Hz}} S_{I,\mathrm{exc}}\, df. \tag{2}$$

Substituting the solution for $S_{I,\mathrm{exc}}$ finally yields

$$I^2_{n,\mathrm{exc}}(V) = \frac{\left(\int_{51\ \mathrm{Hz}}^{99\ \mathrm{Hz}} S_{I,\min}\, df - \frac{G^2_{\min}}{G^2_{\max}} \int_{51\ \mathrm{Hz}}^{99\ \mathrm{Hz}} S_{I,\max}\, df\right)}{\left(1-\frac{G^2_{\min}}{G^2_{\max}}\right)}, \tag{3}$$

where $S_{I,\min}$, $G_{\min}$, and $S_{I,\max}$, $G_{\max}$ are the measured current power spectra and gains at minimum and maximum gain, respectively. Note that this quantity is zero when the gain dependent noise is the only contribution. By measuring several bias points instead, an overdetermined equation system results which gives information about the accuracy of S_{Q_g} and $S_{I,\mathrm{exc}}$. Here, each set of data for constant bias voltage were used to overdetermine S_{Q_g} and $S_{I,\mathrm{exc}}$.

The Al SET showed excess noise (significant within our measurement accuracy) only at the highest bias voltage point, ($I^2_{n,\mathrm{exc}}=2.6\pm0.97$ pA^2/Hz at $V=1.57$ mV). For all other voltages the error bars are much larger than the mean values and include negative power. This is due to the large charge fluctuations dominating the noise contribution. On the other hand, the Nb SET showed excess noise at several bias voltage points (see Fig. 5). To gain more knowledge about the noise contributions we next discuss the different components of the measured noise. The excess-noise current is typically one to two orders of magnitude lower than the maximum charge–noise current.

In addition to charge noise sources in the vicinity of the transistor, current noise can also be induced by fluctuations in the tunnel barrier resistance. As we will see, these two contributions cannot generally be separated experimentally. There are also contributions from shot noise and amplifier noise. In our case, the amplifier noise is dominated by the thermal noise of two feedback resistors and the noise generated by the input equivalent voltage noise, e_n, of the amplifier. For low temperatures, the shot noise is given by S_I

$=aeI$, where $1\leqslant a\leqslant 2$, with $a=2$ for strongly correlated tunneling and $a=1$ for uncorrelated tunneling.[4] The total measured current noise in the system can be modeled by

$$S_{I_m}(f)=2k_B\frac{T_F}{R_F}+\frac{e_n^2(f)}{2r_o^2}+aeI+S_{I_{Q,R}}(f), \tag{4}$$

where T_F is the temperature of the feedback resistors and $S_{I_{Q,R}}(f)$ is the combined spectral density due to charge and resistance fluctuations. The first two terms represent amplifier noise and are frequency independent in the range of interest (51–99 Hz). The third term is the shot noise of the SET which also is independent of frequency. These terms are of order $(30\ \mathrm{fA})^2/\mathrm{Hz}$ for the worst case with lowest r_o and high I. Thus, the sum of these three terms varies only slightly between $(30\ \mathrm{fA})^2/\mathrm{Hz}$ and $(50\ \mathrm{fA})^2/\mathrm{Hz}$ for all bias points. To evaluate the last term let us start with a model in which the only fluctuating parameter is the background charge Q_g. Assuming small variations we can write

$$\delta I=\frac{\partial I}{\partial Q_g}\delta Q_g+\frac{1}{2}\frac{\partial^2 I}{\partial Q_g^2}(\delta Q_g)^2+\dots . \tag{5}$$

If only the first (linear) term is taken into account, then $S_{I_Q}(f)=(\partial I/\partial Q_g)^2 S_{Q_g}(f)$ where S_{Q_g} and S_{I_Q} are the background charge spectral density and the charge induced current spectral density. As seen in Fig. 4(a) we are close to this situation in the experiment.

Close to the operating points for which $\partial I/\partial Q_g=0$, the contribution from the quadratic term in Eq. (5) becomes important. Assuming Gaussian noise we get

$$S_{I_Q}(f)\approx\left[\left(\frac{\partial I}{\partial Q_g}\right)^2+\frac{\alpha}{4}\left(e\frac{\partial^2 I}{\partial Q_g^2}\right)^2\right]S_{Q_g}(f), \tag{6}$$

where $\alpha(f)=[\int_{-\infty}^{+\infty}S_{Q_g}(f')S_{Q_g}(f-f')df']/e^2S_{Q_g}(f)$. For the SETs, we find $\alpha(f)\sim 10^{-4}$. This is smaller than the two orders of magnitude of dynamic range we have in the noise measurement and we can thus neglect the second term.

Now let us consider a different model in which Q_g does not fluctuate, and the only fluctuating parameter is the tunnel resistance R_1 of the first junction. (The fluctuations of the tunnel junction resistance have been extensively studied in large area junctions, see, e.g., Ref. 24.) For simplicity we limit the discussion to the linear term of the series expansion, so that $S_{IR_1}(f)=(\partial I/\partial R_1)^2 S_{R_1}(f)$, where $\partial I/\partial R_1$ can be calculated.[4] Note that S_{IR_1} is asymmetric around $Q_g=e/2$ as a function of Q_g for $V\leqslant(e/2C_\Sigma)$,[25] while it becomes independent of Q_g and *increases* in proportion to V^2 for large V, since the current–voltage characteristics become linear and thus $\partial I/\partial R_1$ approaches $-V/R_1^2$. Furthermore, the fluctuations of R_1 can in principle change with V, Q_g, and T (however, a strong dependence on V and Q_g is unlikely).

On the other hand, the current noise due to Q_g fluctuations *decreases* for sufficiently large V because of the decrease in $|\partial I/\partial Q_g|$ and is symmetric around $Q_g=e/2$ for a symmetric SET transistor. Therefore, the bias dependence and this asymmetry could be used to distinguish charge fluctuations from resistance fluctuations.

In general, the noise can also be caused by simultaneous fluctuations of Q_g, R_1, and R_2. If they are uncorrelated, the corresponding current spectral densities simply add. However, fluctuators inside the tunnel barrier, which have been suggested as the source of the $1/f$ noise by several authors,[9–11,17] can be responsible for both resistance and charge fluctuations. These contributions are then correlated and we arrive at the following expression

$$\begin{aligned}S_{I_{Q,R}}(f)=&\left(\frac{\partial I}{\partial Q_g}\right)^2 S_{Q_g}(f)+\left(\frac{\partial I}{\partial R_1}\right)^2 S_{R_1}(f)+\left(\frac{\partial I}{\partial R_2}\right)^2 S_{R_2}(f)\\&+K_1\frac{\partial I}{\partial Q_g}\frac{\partial I}{\partial R_1}\sqrt{S_{Q_g}(f)S_{R_1}(f)}\\&+K_2\frac{\partial I}{\partial Q_g}\frac{\partial I}{\partial R_2}\sqrt{S_{Q_g}(f)S_{R_2}(f)},\end{aligned} \tag{7}$$

where K_i is the dimensionless correlation coefficient between Q_g and R_i fluctuations, $|K_i|\leqslant 1$.

The first term in Eq. (7) describes the dominant gain dependent noise, while the other terms contribute to the excess noise. We can now compare the bias dependence of the excess noise measured in the Nb SET to the prediction of Eq. (7). From the integrated noise spectra we calculate the measured excess current noise $I_{n,\mathrm{exc}}$, according to Eq. (3) and plot it versus V in Fig. 5. $I_{n,\mathrm{exc}}$ seems to increase with bias voltage, but there is no quadratic dependence which would be expected from Eq. (7). It thus seems likely that the excess noise is *not* due to resistance fluctuations, but has a different origin. One possible explanation is that the increasing current heats the SET and generates more noise. Increasing noise with increasing temperature has been observed by several groups.[19,26] Furthermore, at the highest bias point we can set an upper limit for the resistance fluctuations in the frequency range from 51 to 99 Hz, δR_{51-99} (which is a root-mean-square quantity). Assuming a symmetric SET ($R_1=R_2$ and $S_{R_1}=S_{R_2}$) we get $\delta R_{51-99}<31\ \Omega$ for the Nb SET and $\delta R_{51-99}<1.8\ \Omega$ for the Al SET. The correlation terms were not dominant since δR_{51-99} changed less than 10% as K_i was varied between -1 and 1.

In conclusion, we have measured the low frequency noise of two single electron transistors. In both transistors, the noise at the output closely followed the gain. This shows that low frequency noise in the SET is mainly due to external charge noise. When the gain was low, we observed an excess noise in the Nb SET for all bias voltages and in the Al SET for the highest bias voltage. From the bias dependence of the excess noise in the Nb SET we conclude that the main source of the excess noise is not resistance fluctuations. We also set an upper limit for the resistance fluctuations. The Al SET had a very high gain and showed a minimum charge noise $q_n\approx 2\times 10^{-5}\ e/\sqrt{\mathrm{Hz}}$ in both the superconducting and normal state.

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