Possible cooling by resonant Fowler-Nordheim emission

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(Received 23 February 1999; accepted for publication 23 August 1999)

A method of electronic refrigeration based on resonant Fowler-Nordheim emission is analyzed. In this method, a bulk emitter is covered with a-few-nm-thick film of a widegap semiconductor, creating an intermediate step between electron energies in the emitter and in vacuum. An external electric field tilts this potential profile, forming a quantum well at the semiconductor-vacuum boundary. Alignment of its lowest two-dimensional subband with the energy of the hottest electrons of the emitter (a few k_BT above the Fermi level) leads to a resonant, selective emission of these electrons, providing emitter cooling. Calculations show that cooling power of at least 30 W/cm^2, and temperatures down to 10 K may be achieved using this effect. © *1999 American Institute of Physics.* [S0003-6951(99)02842-9]

The idea of using thermionic transport of electrons over an energy barrier for cooling has been repeatedly discussed in the literature (see, e.g., Refs. 1 and 2). If the barrier height is a few times the thermal spread k_BT, the thermionic current may be quite substantial, with only the hot fraction of electrons being removed from the conductor. Unfortunately, the practical implementation of this idea runs into problems.

A barrier of the necessary height ($\sim$100 meV for 300 K, and proportionally lower for lower T) may be readily implemented in solid state structures, in particular using composite semiconductors. However, even if the barriers are relatively thick, the back flow of heat to the cooled conductor is prohibitively high;[1,2] multilayer structures proposed to overcome this effect[1] seem very complex and promise only a little cooling power. (Only at millidegrees Kelvin temperatures where electron-phonon coupling is very weak, has efficient cooling been demonstrated using thermionic transfer over the superconductor energy gap.[3])

Even a very narrow (submicron) vacuum gap can effectively quench the back heat flow, reducing it to radiation-limited levels of the order of 0.1 W/cm^2 (at 300 K). Unfortunately, in this case the energy barrier height is determined by the conductor work function which is too high for most materials. A natural way to enforce electron transfer through a relatively high barrier is to apply a strong electric field ($\sim$10 MV/cm), inducing Fowler-Nordheim tunneling through the barrier. However, in typical situations the tunneling through the initially uniform barrier pulls out electrons within a relatively broad energy range that results in heating rather than in cooling (the ''Nottingham effect'' [4]).

We propose to limit the energy range of transferred electrons using resonant tunneling in a simple structure (Fig. 1) where the bulk emitter (a metal or a heavily doped semiconductor) is covered with a thin (a-few-nm) layer of a widegap semiconductor. While at zero voltage the electron potential energy profile of this structure has two steps [Fig. 1(a)], its tilting by the applied electric field creates a triangular-shape potential well [Fig. 1(b)] and, hence, the discrete levels (subbands for the full energy) localized at the semiconductor film surface. If the electric field aligns the lowest subband with energy levels of the hottest electrons in the emitter (a few k_BT above the Fermi level), resonant tunneling of these electrons to vacuum may lead to very efficient heat removal, and hence to emitter cooling.[5]

Our proposal hinges on several ideas put forward earlier. Numerous experiments indicate that Fowler-Nordheim emission is frequently enhanced by resonant tunneling through localized surface states arising from unintentional contamination.[6] Cooling of the nanoclusters using this effect was proposed in Ref. 7. [Cooling of two-dimensional (2D) electron gas based on the resonant tunneling through quantum dots was proposed even earlier.[8]] However, to extend cooling to macroscopic objects, a large number of surface nanoparticles should be used in a single device. In this case, unavoidable spread of the size and shape of these particles would result in fluctuations of the resonant level positions, preventing their proper alignment with the Fermi level of the emitter, unless nanoscale fabrication with atomic precision is used. In contrast, our suggestion involves only planar structures and does not require nanofabrication.

Concerning planar structures, Fowler-Nordheim tunneling via the resonant subbands was predicted long ago[9] and

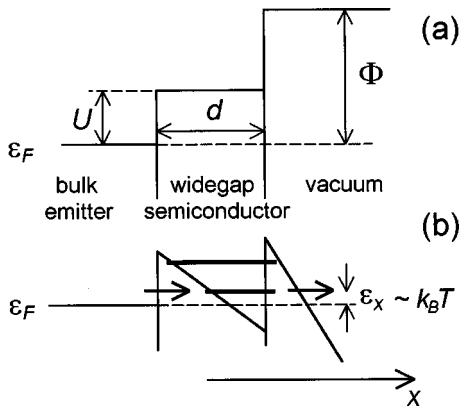

FIG. 1. The energy diagram of the proposed device: (a) in absence of bias and (b) at finite electric field. Resonant tunneling via quantized levels above the Fermi energy removes the hot fraction of electrons, thus cooling the emitter.

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then observed in several systems.[10] The Fowler-Nordheim emission through resonant subbands at the outer surface of a semiconductor in a strong electric field was predicted in Ref. 11. The emission coupled to the electron resonance in the vacuum gap was considered in Ref. 12. However, the possibility of heat removal was not mentioned in any of these publications.

To analyze the cooling effect in the system shown in Fig. 1, let us assume the interfaces to be perfectly plane, so that the electron motion in the direction of tunneling (x axis) and in perpendicular direction are separated. Neglecting band bending and assuming triangular shape of the well, the resonant energies are[13]

$$\mathcal{E}=\mathcal{E}_x+\mathcal{E}_\perp, \quad \mathcal{E}_x=U-eEd+\mathcal{E}_n, \tag{1}$$

$$\mathcal{E}_n=(-a_n)(e^2E^2\hbar^2/2m)^{1/3}, \tag{2}$$

where energies are relative to the emitter Fermi level, U is the initial energy step (Fig. 1), E is the electric field in the film, d is the film thickness, $\mathcal{E}_\perp=\hbar^2k_\perp^2/2m$, m is the electron effective mass in the film, and a_n is the sequence of Airy function zeros ($a_0=-2.34$, $a_1=-4.09$,...).

In absence of energy relaxation, the level filling probability $p=p_n(\mathcal{E}_\perp)$ may be found from the stationary solution of the usual master equation, giving $p=f\gamma_L/(\gamma_L+\gamma_R)$, where $f=f(\mathcal{E})$ is the Fermi distribution of the emitter electrons, and γ_L and γ_R are the rates of electron escape from the quantum well into conductor and into vacuum, respectively. These rates may be calculated as $\gamma_{L,R}=\nu D_{L,R}$, where ν is the ''attempt frequency,'' $\nu=[2\int dx/v(x)]^{-1}=\mathcal{E}_n/2\hbar|a_n|^{3/2}$, and barrier transparencies $D_{L,R}$ are given by Wentzel–Kramers–Brillouin (WKB) approximation (neglecting the image charge effects)

$$\ln D_L=-(4\sqrt{2m}/3eE\hbar)(eEd-\mathcal{E}_n)^{3/2}, \tag{3}$$

$$\ln D_R=-(4\sqrt{2m_0}/3eE_0\hbar)(\Phi-U-\mathcal{E}_n-\Delta\mathcal{E})^{3/2}. \tag{4}$$

Here the shift $\Delta\mathcal{E}=(\hbar^2k_\perp^2/2)(m^{-1}-m_0^{-1})$ is due to the difference between m and the electron mass m_0 in vacuum, Φ is the work function of the bulk emitter, and E_0 is the electric field in vacuum. The relation between this field and E includes the 2D charge density σ of the electrons accumulated in the well, $\epsilon_0E_0=\epsilon\epsilon_0E+\sigma$ (ϵ is the dielectric constant of the semiconductor film). The charge density σ, as well as the resonant current density j and thermal flow q, may be calculated as

$$\sigma=\sum_{n,\mathcal{E}_\perp} ep, \quad j=\sum_{n,\mathcal{E}_\perp} e\gamma_Rp, \quad q=\sum_{n,\mathcal{E}_\perp} \mathcal{E}\gamma_Rp. \tag{5}$$

When the quantized level is above the emitter Fermi level, the typical spread of $\mathcal{E}_\perp$ for the electrons in the subband is of the order of T (from this point on, $k_B=1$). Hence, assuming the barriers much higher than T, we can neglect $\Delta\mathcal{E}$ in Eq. (4). Then integrating over $\mathcal{E}_\perp$, we get

$$j=e\rho\sum_n T\ln(1+e^{-\mathcal{E}_x/T})\gamma_L\gamma_R/(\gamma_L+\gamma_R), \tag{6}$$

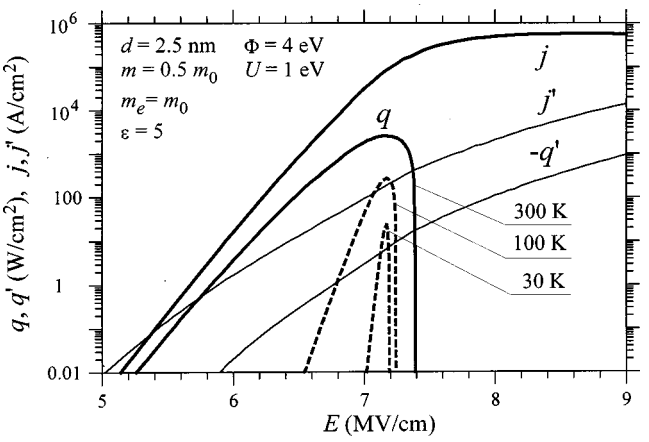

FIG. 2. Solid lines: the resonant current density j, the corresponding cooling power density q, nonresonant current j', and the corresponding heating power $-q'$ as functions of the applied electric field E for $\Phi=4$ eV, $U=1$ eV, $m=0.5m_0$, $m_e=m_0$, $\epsilon=5$, and $d=2.5$ nm, at $T=300$ K. The dashed lines show the cooling power q at $T=100$ and 30 K.

$$q=\rho\sum_n\left[\mathcal{E}_xT\ln(1+e^{-\mathcal{E}_x/T})+\frac{T^2}{2}[\ln(1+e^{-\mathcal{E}_x/T})]^2\right.$$
$$\left.+T^2\mathrm{Li}_2[(1+e^{\mathcal{E}_x/T})^{-1}]\right]\gamma_L\gamma_R/(\gamma_L+\gamma_R), \tag{7}$$

where $\mathrm{Li}_2(z)=\Sigma_{k=1}^{\infty}z^k/k^2$ is the dilogarithm function and $\rho=m/\pi\hbar^2$ is the 2D density of states per unit area.

Equations (6) and (7) do not include the components of current j' and heat flow q' due to nonresonant, ''direct'' tunneling. For this process, the barrier transparency may be calculated as $D=D_LD_R$. A standard WKB calculation yields

$$j'=\frac{e\mathcal{E}_0^2m_c}{2\pi^2\hbar^3K}D_0^LD_0^R\frac{t}{\sin t}, \quad K=\frac{m_c\mathcal{E}_0}{m\mathcal{E}_0^L}+\frac{m_c\mathcal{E}_0}{m_0\mathcal{E}_0^R}, \tag{8}$$

$$q'=-\frac{\mathcal{E}_0^3m_c}{2\pi^2\hbar^3K}D_0^LD_0^R\frac{t^2\cos t}{(\sin t)^2}, \quad t=\pi T/\mathcal{E}_0, \tag{9}$$

where $1/\mathcal{E}_0\equiv d(-\ln D)/d\mathcal{E}_x=1/\mathcal{E}_0^L+1/\mathcal{E}_0^R$, $\mathcal{E}_0^L=e\hbar E/2(2m)^{1/2}[U^{1/2}-\max(0,U-eEd)^{1/2}]$, $\mathcal{E}_0^R=e\hbar E_0/2(2m_0)^{1/2}(\Phi-eEd)^{1/2}$, $\ln D_0^L=-(4(2m)^{1/2}/3eE\hbar)\times[(U)^{3/2}-\max(0,U-eEd)^{3/2}]$, and $\ln D_0^R=-[4(2m_0)^{1/2}/3eE_0\hbar]\max(0,\Phi-eEd)^{3/2}$. (Notice that within the accuracy of WKB approximation these formulas may be used even if $U-eEd<0$.) The well-known factor[14] $t/\sin t$ shows that our approximation, based on the linear expansion of $\ln D$ near the Fermi level, can be used only at $T<\mathcal{E}_0$. At low temperatures the nonresonant tunneling always provides heating of the emitter, although it changes to cooling at $T>\mathcal{E}_0/2$.

Figure 2 shows one of the results of our calculations using Eqs. (6)–(9). The cooling power q first increases exponentially with the field, because the lowest subband is aligned with more and more populated hot electron levels, and then drops sharply when the subband crosses the Fermi level (at larger fields q becomes negative, indicating emitter heating). Just before this drop the cooling power reaches a maximum, in this case as high as 300 W/cm^2 at $T=100$ K.

The maximum values of q, as well as the corresponding values of j and q', for several other parameter sets are listed in Table I. From Fig. 2 one can see that the suitable range for

TABLE I. Maximum cooling flow density q and heating density $-q'$, as well as the corresponding electric field E and resonant electric current density j, for several parameter sets.

Φ eV	U eV	m m_0	ϵ	d nm	T K	E MV/cm	j kA/cm^2	q W/cm^2	$-q'$ W/cm^2
4	1	0.5	5	2.5	300	7.2	90	3000	8
					100	7.2	30	300	8
4	1	0.5	5	2.7	300	6.4	30	1000	0.8
					100	6.4	10	100	1.0
					30	6.4	3	9	0.9
					10	6.4	1	1	0.9
4	1	0.2	7	3	300	6.8	400	10000	900
5	1.5	0.2	7	3.5	300	7.4	20	900	10

the electric field E shrinks rapidly as emitter temperature goes down. Nevertheless, our model indicates that $q+q'$ may be positive (i.e., cooling is still possible) for temperatures as low as 10 K.

Let us discuss how realistic our model is. Equation (5) is strictly valid only if the energy relaxation in the well is much slower than γ_L, γ_R. We have also neglected the resonant subband broadening due to tunneling (but it was monitored to be negligible for our parameter sets). One more possible source of deviations from the model is electron scattering in the well and during tunneling (however, these processes can hardly affect the process of hot electron extraction). Next, we have implicitly assumed that the Fermi energy of the bulk emitter is much larger than all considered energies.

Despite the used assumptions, we expect that for smooth films the overall accuracy of our results is limited mainly by that of the WKB approximation,[15] so at least the order of magnitude is correct. Since the results show that the resonant emission cooling at temperatures above ~ 100 K may prevail over the nonresonant heating in a relatively broad range of electric field, and their ratio may be very high, we are confident that the net cooling of the emitter may be achieved. However, the estimate of the lowest achievable temperature (10 K) may be more vulnerable.

The largest problem we see with the experimental implementation of resonant emission cooling is the necessary film uniformity. In fact, Table I shows that at 300 K the effect is stable with respect to substantial ($\sim 20\%$) variations of d. However, to achieve cooling to 100 K, film thickness variations should not exceed $\sim 3\%$. Also, the electric field should be decreased below the optimal value in order to be sure that we have not stepped into the heating region on any considerable fraction of the emitter area. For the example given above ($T=100$ K), this factor reduces the average cooling power to about 30 W/cm^2 at an electric current density of ~ 1 kA/cm^2.

Another limitation of cooling power may come from unacceptably large density q_a of power release on the anode. In fact, if one uses the straightforward planar cathode-anode geometry, even for the vacuum gap d_0 as small as 10 nm the necessary voltage $V=Ed+E_0d_0$ is above 35 V, giving for our example $q_a \approx 35$ kW/cm^2. With this power, even the best imaginable radiator would not be able to keep the anode temperature below ~ 1000 K. However, the electric field may be provided by a micromachined ''grid'' electrode at a small distance d_0 from the cathode, followed by another, much more distant grid at approximately the same electric potential, and a collector at a lower potential, so that electrons are decelerated before the ''soft landing.'' Such systems (see, e.g., Ref. 16) allow to recover more than 90% of the electron energy. In this case, for our example with a feasible value $d_0=30$ nm, q_a is reduced below 10 kW/cm^2, and with a good radiator the anode temperature raise may be kept within 300 K. This may be acceptable in practice while giving negligible radiation backflow below 1 W/cm^2.

To summarize, we have proposed a method of electronic refrigeration using the resonant Fowler-Nordheim tunneling in a fairly simple planar thin-film structure. If the experiment confirms our theory, this device may be valuable for the integration of advanced low-temperature electronic devices with room-temperature circuits. Also, due to the accompanying high electric current density the effect may find applications in field emitter technologies.

Useful discussions with D. V. Averin, H. Busta, and R. Tsu are gratefully acknowledged. The work has been supported in part by DARPA's HERETIC program via ARO.

[1] G. D. Mahan, J. Appl. Phys. **76**, 4362 (1994); G. D. Mahan and L. M. Woods, Phys. Rev. Lett. **80**, 4016 (1998).
[2] A. Shakouri and J. E. Bowers, Appl. Phys. Lett. **71**, 1234 (1997); A. Shakouri *et al.*, Appl. Phys. Lett. **74**, 88 (1999).
[3] M. Nahum, T. M. Eiles, and J. Martinis, Appl. Phys. Lett. **65**, 3123 (1994); M. M. Levio, J. P. Pekola, and D. V. Averin, Appl. Phys. Lett. **68**, 1996 (1996).
[4] W. B. Nottingham, Phys. Rev. **59**, 908 (1941).
[5] Similar subbands will arise at the interface of layers of ''crested'' barriers suggested recently by one of the authors [K. Likharev, Appl. Phys. Lett. **73**, 2137 (1998)] to provide fast write/erase time in nonvolatile, floating-gate memories. The analysis presented in that paper did not take the resonant subbands into account. If this is done, the results are almost the same if the thickness of external layers of the barrier is increased (from 2 to ~ 4 nm for the example given in that paper).
[6] J. W. Gadzuk and E. W. Plummer, Rev. Mod. Phys. **45**, 487 (1973); V. T. Binh, S. T. Purcell, N. Garcia, and J. Doglioni, Phys. Rev. Lett. **69**, 2527 (1992).
[7] S. T. Purcell, V. T. Binh, N. Garcia, M. E. Lin, R. P. Andres, and R. Reifenberger, Phys. Rev. B **49**, 17259 (1994).
[8] H. L. Edwards, Q. Niu, and A. L. de Lozanne, Appl. Phys. Lett. **63**, 1815 (1993).
[9] K. H. Gundlach, Solid-State Electron. **9**, 949 (1966).
[10] J. Maserjian, J. Vac. Sci. Technol. **11**, 996 (1974); J. Maserjian and N. Zamani, J. Appl. Phys. **53**, 559 (1982); T. W. Hickmont, P. M. Solomon, R. Fisher, and H. Moroç, Appl. Phys. Lett. **44**, 90 (1984); H. Mimura *et al.*, J. Vac. Sci. Technol. **16**, 803 (1998).
[11] V. G. Litovehenko and Yu. V. Kryuchenko, J. Vac. Sci. Technol. B **11**, 362 (1993).
[12] S. Vatannia, G. Gildenblat, and J. Schiano, J. Appl. Phys. **82**, 902 (1997).
[13] See, e.g., C. Weisbuch and B. Vinter, *Quantum Semiconductor Structures* (Academic, Boston, 1991).
[14] M. Lenzlinger and E. H. Snow, J. Appl. Phys. **40**, 278 (1969).
[15] H. Y. Yang, H. Niimi, and G. Lucovsky, J. Appl. Phys. **83**, 2327 (1998).
[16] T. K. Ishii, in *The Electronics Handbook*, edited by J. C. Whitaker (Chemical Rubber, Boca Raton, FL, 1996), p. 428.