Charge sensitivity of radio frequency single-electron transistor

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A theoretical analysis of the charge sensitivity of the radio frequency single-electron transistor (rf-SET) is presented. We use the ''orthodox'' approach and consider the case when the carrier frequency is much less than I/e where I is the typical current through rf-SET. The optimized noise-limited sensitivity is determined by the temperature T, and at low T it is only 1.4 times worse than the sensitivity of conventional single-electron transistor. [S0003-6951(99)03026-0]

Single-electron devices[1] are gradually becoming useful in real applications.[2] Despite the wide variety of studied circuits, the single-electron transistor (SET)[1,3,4] remains the most important device in applied single electronics (in this letter we will discuss the new version[5] of the SET setup). At present the best reported charge sensitivity of the SET at 10 Hz is[6] $2.5\times10^{-5}\,e/\sqrt{\mathrm{Hz}}$ (the previous record figure was[7] $7\times10^{-5}\,e/\sqrt{\mathrm{Hz}}$). The low-frequency sensitivity of the SET is limited by 1/f noise, so it improves as the frequency increases. The best achieved figure so far[8] of $\sim10^{-5}e/\sqrt{\mathrm{Hz}}$ was measured at 4.4 kHz. This is still an order of magnitude worse than the limit determined by the thermal/shot noise of the SET.[9–14]

The difficulty of further frequency increase is due to the relatively large output resistance R_d of the SET. For the typical figure $R_d\sim10^5\,\Omega$ and wiring capacitance $C_L\sim10^{-9}$ F the corresponding R_dC_L time limits the bandwidth by a few kHz (the use of filters can make it even lower). The importance of potential high-frequency applications makes urgent a significant increase of the bandwidth. This can be done in several ways.

The output resistance can be reduced in superconducting (Bloch) SET based on supercurrent modulation[1,15,16] (the use of the quasiparticle tunneling threshold does not help much because R_d is limited by the quantum resistance even at the threshold[13,17]). The load capacitance C_L can be decreased placing the next amplifier close to the SET.[18,19] However, while bandwidth up to 700 kHz was demonstrated[18] using this idea, the charge sensitivity was relatively poor because of extra heating and extra noise produced by the preamplifier. Finally, a bandwidth over 100 MHz has recently been demonstrated[5] in the so-called radio frequency (rf) SET in which the SET controlled the dissipation of the tank circuit which in turn affected the reflection of the carrier wave with frequency $\omega/2\pi=1.7$ GHz. A sensitivity of $1.2\times10^{-5}\,e/\sqrt{\mathrm{Hz}}$ has been achieved[5] at 1.1 MHz. The theoretical analysis of the ultimate sensitivity of the rf-SET is the subject of the present letter.

In principle a wide bandwidth could be achieved simply by illuminating the SET with microwaves and measuring the wave reflection. The gate voltage would change the SET differential resistance R_d and thus affect the reflection coefficient $\alpha=(Z-R_0)/(Z+R_0)$, where $Z^{-1}=i\omega C_s+R_d^{-1}$, $R_0\simeq50\,\Omega$ is the cable wave resistance, and C_s is the stray capacitance. However, because of the large ratio $R_d/R_0\sim10^3$, the signal would be extremely small. To estimate the signal power $P\simeq A^2R_0/2R_d^2[1+(\omega C_sR_0)^2]$, let us use $R_d=10^5\,\Omega$ and the amplitude of the SET bias voltage oscillation $A=1$ mV (A is limited by the Coulomb blockade threshold); then $P\sim10^{-15}$ W. This figure corresponds to the noise power of the amplifier with noise temperature of 10 K within 10^7 Hz bandwidth and clearly makes such an experiment quite difficult.

To increase the signal, the authors of Ref. 5 inserted the SET into the tank circuit (see Fig. 1). Then at resonant frequency $\omega=(LC_s)^{-1/2}$ the circuit impedance is small, $Z\simeq L/C_sR_d\ll R_0$ (we assume $Q_{\mathrm{SET}}\gg Q\gg1$ where $Q_{\mathrm{SET}}=R_d/\sqrt{L/C_s}$ and $Q=\sqrt{L/C_s}/R_0$), so $\alpha\simeq-1+2L/C_sR_dR_0$. The signal power $P=[V_{\mathrm{in}}(\alpha+1)]^2/2R_0$ (V_{in} is the amplitude of the incoming wave) can be expressed via the SET bias amplitude $A\simeq2QV_{\mathrm{in}}$ as $P=Q^2A^2R_0/2R_d^2$, indicating Q^2 gain in comparison with the nonresonant case.[5]

The linear analysis above can be used only as an estimate because of the considerable nonlinearity of the SET current–voltage ($I-V$) curve. For a more exact analysis let us write the differential equation (see Fig. 1) for the voltage $v(t)$ at the end of the cable (the static component V_0 is subtracted):

$$\ddot{v}LC_s+\dot{v}R_0C_s+v=2(1-\omega^2LC_s)V_{\mathrm{in}}\cos\omega t-R_0I(t),$$

where $V_{\mathrm{in}}\cos\omega t$ is the incoming wave at the end of cable and $I(t)$ is the current through the SET while the SET bias

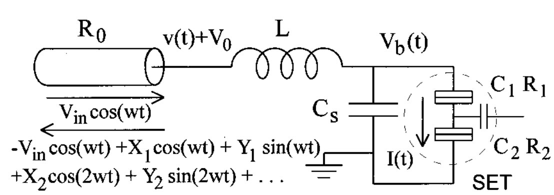

FIG. 1. The schematic of the rf-SET.

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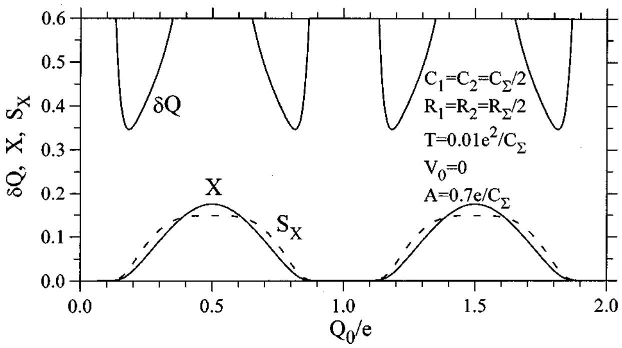

FIG. 2. The reflected wave amplitude X_1 [in units $2(L/C_s)^{1/2}e/R_\Sigma C_\Sigma$], its noise S_{X1} (in units $4L/C_s e^2/R_\Sigma C_\Sigma$), and minimal detectable charge δQ [in units $e(R_\Sigma C_\Sigma \Delta f)^{1/2}$] as functions of the background charge Q_0 for the symmetric SET at $T=0.01e^2/C_\Sigma$ for zero dc bias voltage V_0 and its rf amplitude $A=0.7e/C_\Sigma$.

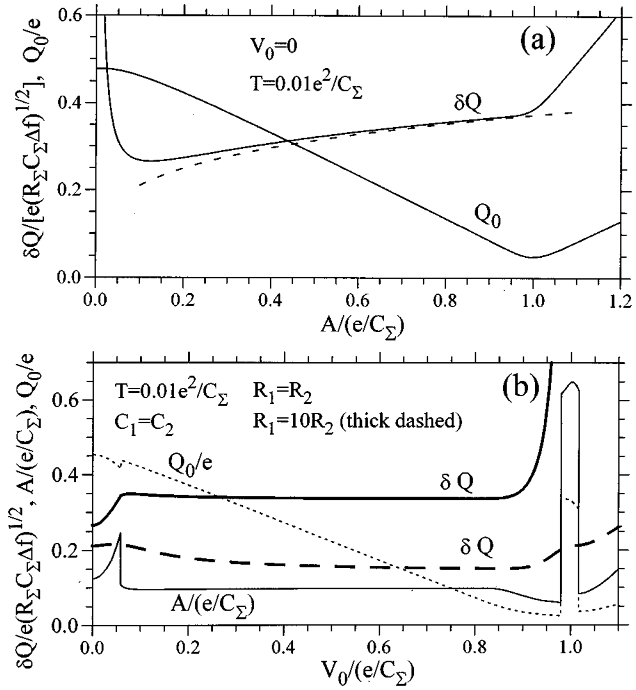

FIG. 3. (a) The sensitivity δQ optimized over Q_0 and corresponding Q_0 as functions of rf amplitude A for the symmetric SET. Dashed line shows the analytical result (see the text). (b) Dependence of δQ minimized over A and Q_0 and of the optimal operation point (A,Q_0) on the dc bias voltage V_0. Dashed line is for the asymmetric SET ($R_2/R_1=10$).

voltage is $V_b(t)=V_0+v+(2V_{\rm in}\omega \sin \omega t+\dot{v})L/R_0$. The reflected wave can be written (at the end of cable) as $v(t) - V_{\rm in}\cos \omega t = -V_{\rm in}\cos \omega t + X_1 \cos \omega t + Y_1 \sin \omega t + X_2 \cos 2\omega t + Y_2 \sin 2\omega t + \ldots$, where the coefficients X_k and Y_k should be calculated self-consistently [an obvious way is the iterative updating of $V_b(t)$ and X_k, Y_k]. While the analysis of the higher harmonics is important for the possible versions of rf-SET in which the signal is measured at the double (or triple) frequency, we will limit ourselves by the reflected wave at the basic harmonic. For simplicity we assume exact resonance, $\omega=(LC_s)^{-1/2}$, then

$$X_1=2\sqrt{L/C_s}\,\langle I(t)\sin \omega t\rangle,$$
$$Y_1=2\sqrt{L/C_s}\,\langle I(t)\cos \omega t\rangle, \tag{1}$$

where $\langle\ \rangle$ denotes averaging over time. In the first approximation (if $Q_{\rm SET} \gg Q \gg 1$) the SET bias voltage is $V_b(t)=V_0 + A\sin \omega t$ where $A=2QV_{\rm in}$.

The coefficients X_1 and Y_1 (we omit index 1 below) can be measured separately using homodyne detection and both can carry information about the low frequency signal applied to the SET gate (as usual, [1] we will describe it in terms of the background charge Q_0 induced into the SET island). If the amplifier noise and other fluctuations are negligible, then the sensitivity of the rf-SET is determined by the intrinsic noise of the SET. The minimal detectable charge δQ can be expressed as

$$\delta Q_X=\sqrt{S_X(f_s)\Delta f}/(dX/dQ_0),$$
$$\delta Q_Y=\sqrt{S_Y(f_s)\Delta f}/(dY/dQ_0), \tag{2}$$

while the simultaneous measurement of X and Y can give $\delta Q=[(1-K^2)/(\delta Q_X^{-2}+\delta Q_Y^{-2}-2K/\delta Q_X \delta Q_Y)]^{1/2}$, where $K=({\rm Re}\, S_{XY}/\sqrt{S_X S_Y})\,{\rm sign}[(dX/dQ_0)(dY/dQ_0)]$ is the correlation between two noises. Here $S_X(f_s)$ is the spectral density of $X(t)$ fluctuations at signal frequency f_s (which should be within the tank circuit bandwidth, $2\pi f_s \lesssim \omega/Q$), S_{XY} is the mutual spectral density, and Δf is the measurement bandwidth (inverse ''accumulation'' time).

In this letter we consider only the case of sufficiently low carrier frequency $\omega \ll I/e$ (where I is the typical current through the SET), so that the quasistationary state is reached at any moment during the period of oscillations. In this case the spectral density does not depend on f_s (which is even lower than ω) and

$$S_X=4(L/C_s)\,\langle S_I(t)\sin^2 \omega t\rangle, \tag{3}$$

where $S_I(t)$ is the low frequency spectral density of the thermal/shot noise of the current through the SET, which has the time dependence because of oscillating bias voltage V_b. There is no need to consider Y output in this case because $Y=0$ (so $\delta Q_Y=\infty$) and the noise correlation is absent, $K=0$ (nonzero Y and K would appear at higher ω due to delay of tunneling events).

We use the ''orthodox'' theory[1,3] for a normal SET consisting of two tunnel junctions with capacitances C_1 and C_2 and resistances R_1 and R_2 (see Fig. 1) assuming $R_j \gg R_Q = \pi\hbar/2e^2$ (as usual, the gate capacitance is distributed between C_1 and C_2 in a proper way). The effects of finite photon energy $\hbar\omega$ are neglected. We also neglect the possible rf modulation of the SET gate voltage. The low frequency thermal/shot noise of the SET current is calculated in the standard way.[9,10]

Figure 2 shows the dependence of X, S_X, and $\delta Q = \delta Q_X$ on the background charge Q_0 for a symmetric SET ($C_1=C_2, R_1=R_2$) at $T=0.01e^2/C_\Sigma$ ($C_\Sigma=C_1+C_2$), $V_0=0$, and $A=0.7e/C_\Sigma$. One can see that the minimum of δQ is achieved near the edge of Q_0 range corresponding to nonzero X, so that the amplitude A is only a little larger than the Coulomb blockade threshold V_t. For V_b close to V_t the noise of the current through the SET obeys Schottky formula, $S_I = 2eI$, with a good accuracy at low temperatures,[9,10] while the current I can be approximated as $I=W/eR_j[1-\exp(-W/T)]$ where $W=e(V_b-V_t)(C_1C_2/C_jC_\Sigma)=$

$(-1)^j e(Q_0 - Q_{0,t})/C_\Sigma$ (jth junction determines the threshold) and $|dI/dQ_0| = (dI/dV_b)C_j/C_1C_2$. (As a consequence of the Schottky formula, the dashed curve in Fig. 2 is approximately twice as high as the X-curve at small X.)

Using these equations and optimizing Q_0, one can find the minimum $\delta Q \simeq 1.2\, e\, (R_\Sigma C_\Sigma \Delta f)^{1/2} (TC_\Sigma/e^2)^{1/2} \times (eA/T)^{1/4}$ for the symmetric SET at $T \ll eA < e^2/C_\Sigma$ ($R_\Sigma = R_1 + R_2$). This dependence as a function of rf amplitude A is shown in Fig. 3(a) by the dashed line while the numerical result is shown by the solid line. The sensitivity gets worse (δQ increases) at $A > e/C_\Sigma$ because of X and S_X increase. The sensitivity also worsens rapidly when A is too small and becomes comparable to T/e, because of the contribution from the Nyquist noise of the SET at V_b close to zero. Before optimizing the amplitude A, let us notice that the results shown in Fig. 3(a) correspond to relatively small X that can be difficult to measure experimentally [in the approximation above $X \simeq 2\,(L/C_s)^{1/2} \times 15\,(T/eR_\Sigma)(T/eA)^{1/2}$]. However, as seen from Fig. 2, X can be significantly increased for the price of a few ten per cent increase of δQ.

Figure 3(b) shows δQ minimized over both A and Q_0 and the corresponding optimum values of A and Q_0 as functions of the direct current (dc) bias voltage V_0. One can see that for a symmetric SET the best sensitivity is achieved at $V_0 = 0$ and there is a long plateau of δQ which ends when V_0 approaches e/C_Σ leading to significant worsening of the sensitivity. For the asymmetric SET (dashed line) the best sensitivity can be achieved in the plateau range. At the plateau δQ can be calculated analytically using the approximations above, $\delta Q \simeq 3.34\, e\, (2R_{\min} C_\Sigma \Delta f)^{1/2} (TC_\Sigma/e^2)^{1/2}$ where $R_{\min} = \min(R_1, R_2)$. This expression can be compared with the optimized low-temperature sensitivity of the conventional SET which is given[9,10] by the same formula with the numerical factor 1.90 instead of 3.34. For the symmetric rf-SET the optimized low-temperature sensitivity (at $V_0 = 0$) is

$$\delta Q \simeq 2.65 e (R_\Sigma C_\Sigma \Delta f)^{1/2} (TC_\Sigma/e^2)^{1/2}, \tag{4}$$

only 1.4 times worse than for the conventional SET.

Figure 4 shows numerically minimized δQ for the symmetric SET and corresponding optimal A and Q_0 (while $V_0 = 0$) as functions of temperature. The result of Eq. (4) is shown by the dashed line. The sensitivity scales as $T^{1/2}$ at low temperatures while it significantly worsens at $T > 0.1e^2/C_\Sigma$, similar to the result for the conventional SET (dotted line). The "orthodox" sensitivity improves with the decrease of tunnel resistances while the optimum value (which should be comparable to R_Q) could be calculated if cotunneling[1] was taken into account.

To make a comparison with experiment,[5] let us take $C_\Sigma = 0.45$ fF, $R_\Sigma = 97$ kΩ, and $T = 100$ mK, then after optimization $\delta Q \simeq 2.7 \times 10^{-6}\, e/\sqrt{\text{Hz}}$ in the normal case (necessity of relatively large X would lead to a factor about 1.5). So, there is still an order of magnitude for possible experimental improvement. Comparison for the superconducting case is not straightforward because the sensitivity depends on the junction quality.[13]

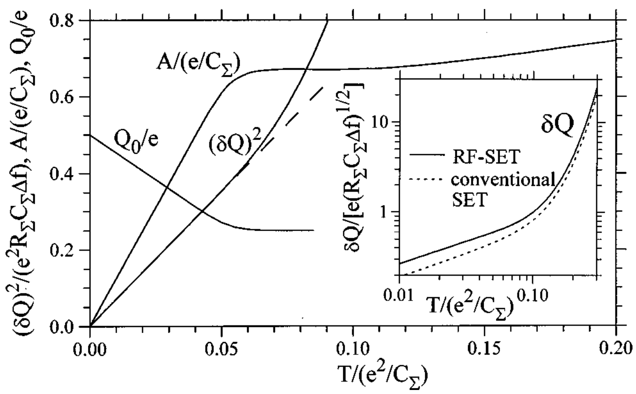

FIG. 4. The optimized δQ (squared) and corresponding A and Q_0 as functions of the temperature T for the symmetric SET at $V_0 = 0$. Dashed line represents Eq. (4). Inset shows δQ on the larger scale. For comparison, the result for conventional SET is shown by the dotted line.

In conclusion, we have shown that the price for the wide bandwidth of the rf-SET is only a little decrease of the noise-limited sensitivity in comparison with conventional SET.

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