Theoretical analysis of the resistively coupled single-electron transistor

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(Received 16 March 1998; accepted for publication 14 April 1998)

The operation of the resistively coupled single-electron transistor (R-SET) is studied quantitatively. Due to the Nyquist noise of the coupling resistance, degradation of the R-SET performance is considerable at temperatures T as small as $10^{-3}e^2/C$ (where C is the junction capacitance) while the voltage gain becomes impossible at $T \gtrsim 10^{-2}e^2/C$. © *1998 American Institute of Physics.* [S0003-6951(98)03024-1]

Single-electron tunneling[1] attracts considerable theoretical and experimental attention and can be potentially used in important applications including ultradense digital electronics.[2] The simplest and most thoroughly studied single-electron device is the single-electron transistor[3] (SET) which consists of two tunnel junctions in series. The current through this double-junction system depends on the background charge Q_0 of the central electrode (''island'') which can be controlled with an additional external electrode thus providing the transistor effect. In the usual capacitively coupled SET (C-SET) the charge Q_0 is controlled via the gate capacitance while the other possibility is to use the coupling resistance R_g (R-SET); see Fig. 1(a).

Since the C-SET can be relatively easy realized experimentally, this has also motivated numerous theoretical studies of different problems related to the C-SET. In contrast, the R-SET has almost not been studied theoretically after its initial proposal,[3] even in the simplest approximation (RC-SET with combined coupling has been considered in Ref. 4). The reason is the difficulty of experimental realization of the R-SET. In order not to smear the discreteness of the island charge by quantum fluctuations, the gate resistance should be sufficiently large,[1,3]

$$R_g \gg R_Q = \pi\hbar/2e^2 \simeq 6.5\text{k}\Omega, \tag{1}$$

and simultaneously the geometrical size of the resistor should be relatively small so that its stray capacitance does not significantly increase the total capacitance of the island. The progress in fabrication of such resistors has been achieved only recently.[5–10] (One-dimensional array of junctions instead of a resistor has been studied in Ref. 15.)

The R-SET could be a very useful element for integrated single-electron digital devices. At present the majority of the proposals for single-electron logic (see Ref. 2) are based on the capacitively coupled devices which suffer from the principal problem of fluctuating background charges (the solution is known so far only for memory devices[11]). The use of a R-SET which is not influenced by background charges would allow one to avoid this problem. Another anticipated advantage of the R-SET is the possibility of much larger voltage gain than for the C-SET. The potential importance for integrated devices and the possibility of an experimental demonstration of the R-SET in the near future makes urgent the basic theoretical analysis of R-SET operation. In this article we consider the I-V curve and the dependence on the gate potential. We also discuss the smearing of the Coulomb blockade and the reduction of the voltage gain at finite temperatures.

Assuming sufficiently large gate resistance [Eq. (1)] and tunnel resistances, $R_{1,2} \gg R_Q$, and using the ''orthodox'' theory of single-electron tunneling[1,12] we describe the internal dynamics of the R-SET by the following master equation:

$$\begin{aligned}\dot{\sigma}(Q) &= \Gamma^-(Q+e)\sigma(Q+e) + \Gamma^+(Q-e)\sigma(Q-e) \\ &\quad - [\Gamma^+(Q)+\Gamma^-(Q)]\sigma(Q) \\ &\quad + \frac{1}{R_g C_\Sigma}\frac{\partial}{\partial Q}[(Q-\tilde{Q})\sigma(Q)] + \frac{T_r}{R_g C_\Sigma}\frac{\partial^2\sigma(Q)}{\partial Q^2}.\end{aligned} \tag{2}$$

Here $\sigma(Q)$ is the density of the probability to find the total charge Q on the island, $C_\Sigma = C_1 + C_2$ is the total island capacitance, and $\tilde{Q} = UC_\Sigma - VC_2$ corresponds to the equality between the gate potential U and the island potential $\phi = Q/C_\Sigma + VC_2/C_\Sigma$. The last term in Eq. (2) describes the Nyquist noise of the gate resistance being at temperature T_r which can in principle differ from the temperature T of the electron gas in tunnel junctions (we assume $T_r = T$). $\Gamma^\pm(Q) = \Gamma_1^\pm(Q) + \Gamma_2^\pm(Q)$ where $\Gamma_i^\pm$ are the rates of tunneling through the ith junction increasing $(+)$ or decreasing $(-)$ the island charge:

$$\begin{aligned}\Gamma_i^\pm &= \frac{W_i^\pm}{e^2 R_i[1-\exp(-W_i^\pm/T)]}, \\ W_i^\pm &= \frac{e}{C_\Sigma}\left[\mp\left(Q + (-1)^i V\frac{C_1C_2}{C_i}\right) - \frac{e}{2}\right].\end{aligned} \tag{3}$$

In this letter we analyze only dc characteristics of the R-SET, so $\dot{\sigma}(Q) = 0$ is assumed in Eq. (2).

At $T = 0$ the Coulomb blockade state is realized when $\phi = U$ and the voltages across both tunnel junctions are less than the tunneling threshold,

$$|U| < e/2C_\Sigma, |V-U| < e/2C_\Sigma. \tag{4}$$

Outside the blockade range the average currents through junctions,

$$I_i = (-1)^{i+1} e \int [\Gamma_i^+(Q) - \Gamma_i^-(Q)]\sigma(Q)\,dQ, \tag{5}$$

can be different because of finite gate current $I_g = I_2 - I_1$, $I_g = [U - \int \phi(Q)\sigma(Q)\,dQ]/R_g$.

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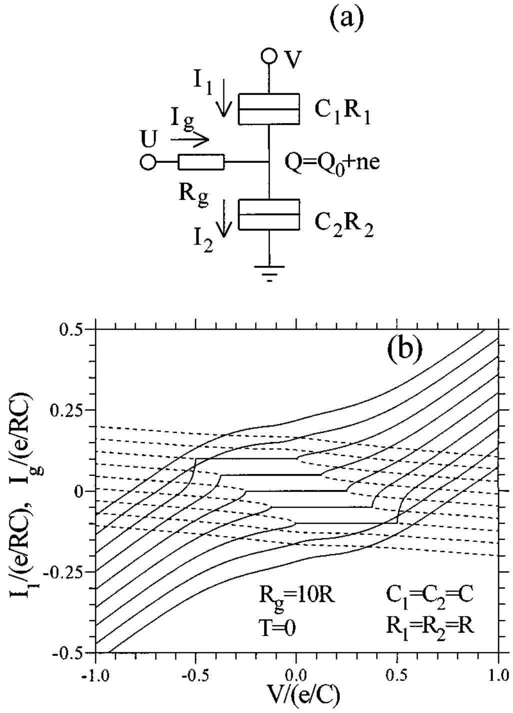

FIG. 1. (a) Schematic of the R-SET. (b) The currents I_1 (solid line) and I_g (dashed line) as functions of the bias voltage V at $T=0$. The gate voltages (from top to bottom): $U/(e/C)=-1/2$, $-3/8$, $-1/4$, $-1/8$, 0, 1/8, 1/4, 3/8, 1/2. The curves are shifted vertically (by $\Delta I=0.4U/R$) for clarity.

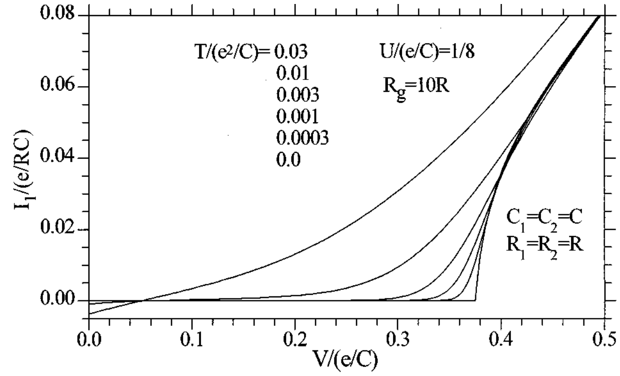

FIG. 2. The I-V curves of the R-SET for different temperatures.

The analysis can be considerably simplified in the limit $R_g \gg R_{1,2}$. Then it is useful to separate the total charge $Q = Q_0 + ne$ into part Q_0 supplied via R_g and the integer charge ne due to tunneling (initial background charge is included in Q_0). Because of $R_g \gg R_{1,2}$, the change of Q_0 is slow and the first averaging can be done over the fast tunneling events exactly like for the C-SET that gives e-periodic dependencies $\overline{\phi}(Q_0)$ and $\overline{I}(Q_0)$ (the currents through junctions are equal in this approximation).

If the Nyquist term in Eq. (2) can be neglected ($T_r = 0$), then $\dot{Q}_0 = (U - \overline{\phi})/R_g$. In the case when $\min_{Q_0}\overline{\phi}(Q_0) < U < \max_{Q_0}\overline{\phi}(Q_0)$, the stationary state with $I_g = 0$ will be eventually reached. (This condition is satisfied by two values of Q_0 per period with the stable state determined by $\partial\overline{\phi}/\partial Q_0 > 0$.) It is interesting that in this case the I-V curve of the R-SET can have negative differential conductance (see also Ref. 4) which is realized when $(\partial\overline{I}/\partial V) < (\partial\overline{I}/\partial Q_0)(\partial\overline{\phi}/\partial V)/(\partial\overline{\phi}/\partial Q_0)$.

If the gate voltage U is outside the range $(\min\overline{\phi}, \max\overline{\phi})$, then the stationary state for Q_0 is impossible and the current through the R-SET will perform single-electron oscillations[1] with the period $\tau = \int_0^e R_g / |U - \overline{\phi}(Q_0)| \, dQ_0$ while the average gate current $I_g = e/\tau$. The average output current does not depend on R_g and can be easily calculated using the numerical solution for $Q_0(t)$.

When the ratio $R_g/R_{1,2}$ is finite, the stationary solution of the full Eq. (2) can be found numerically (we will discuss the numerical methods elsewhere). Figure 1(b) shows the currents I_1 (solid line) and I_g (dashed line) for the symetric R-SET ($C_1 = C_2 = C$, $R_1 = R_2 = R$) as functions of the bias voltage V for $T = 0$, $R_g/R = 10$, and different gate voltages U (actually, the currents do not depend on the ratio C_1/C_2 if C_Σ is kept constant). Notice the strong asymmetry of the I-V curve shape near two thresholds of the Coulomb blockade for $U \neq 0$. The slope of the step-like feature grows with the increase of R_g/R (the perfect step is realized for $R_g/R = \infty$ as follows from the analysis above). In the large-bias limit ($V \gg e/C_\Sigma$, $V - U \gg e/C_\Sigma$) the currents can be found analytically using simple Kirchhoff analysis and taking into account the effective voltage shift $e/2C_\Sigma$ (opposite to the current direction) in each tunnel junction: $I_1 = [V(R_2 + R_g) - UR_2 - (e/2C_\Sigma)(2R_g + R_2)]/A$ and $I_g = [U(R_1 + R_2) - VR_2 + (e/2C_\Sigma)(R_2 - R_1)]/A$ where $A = (R_1R_2 + R_1R_g + R_2R_g)$. The voltage offset between the positive and negative asymptotes of $I_1(V)$ is equal to $(e/C_\Sigma)(2R_g + R_2)/(R_g + R_2)$.

Figure 2 illustrates the effect of the temperature on the I-V curve of the R-SET. One can see that in contrast to the C-SET, even a small temperature significantly smears the Coulomb blockade threshold. The finite temperature changes the tunneling rates [Eq. (3)] and also causes the Nyquist noise of the gate resistance. The effect of the tunneling rates change is similar to that in the C-SET and leads to the smearing of sharp features within a voltage range on the order of T/e; hence, it is quite small at $T \lesssim 0.01e^2/C_\Sigma$. The effect of the Nyquist noise is much more important at relatively low temperatures. In the absence of the tunneling current in the Coulomb blockade, even for arbitrary large R_g [that reduces the noise, see Eq. (2)] the fluctuations of Q_0 should satisfy the thermal distribution leading to root mean square (rms) values of

$$\delta Q_0 = (TC_\Sigma)^{1/2}, \qquad \delta\phi = (T/C_\Sigma)^{1/2}. \tag{6}$$

The scaling as $T^{1/2}$ makes the effect significant even for $T \sim 10^{-3}e^2/C_\Sigma$ and thus creates a serious problem for practical use of the R-SET. [We note that Nyquist noise was also the main obstacle for widespread use of resistively coupled superconducting quantum interference devices (SQUIDs).[13]]

For the ith junction biased below the blockade threshold, the noise-induced tunneling rate can be estimated as $\Gamma_i \simeq \int_0^\infty (x/eR_i)(C_\Sigma/2\pi T)^{1/2}\exp[-(x+\Delta_i)^2 C_\Sigma/2T]\,dx$, where $\Delta_1 = e/2C_\Sigma - (V - U)$ and $\Delta_2 = e/2C_\Sigma - U$ ($\Delta_i \gg T/e$). However, the numerical results show that the leakage current is typically a few times larger (and can be much larger) than this estimate. The reason is the positive feedback from the gate resistance. For example, when the positive charge tunnels to the island through the first junction, it causes some

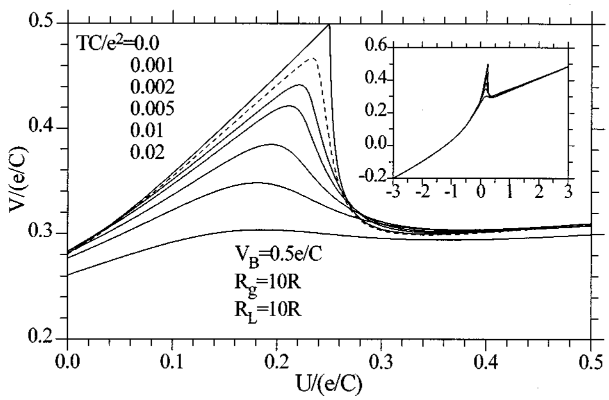

FIG. 3. The control curves of the resistively loaded R-SET (inverter) at different temperatures. The dashed line shows the result for $T = 0.005e^2/C_\Sigma$ neglecting Nyquist noise. The inset shows the same curves on a larger scale.

negative gate current. Hence, after the charge escapes through the second junction, the voltage across the first junction is increased in comparison with the situation before tunneling. This effect enhances the ''clustering'' of tunneling events above the level determined by Nyquist random walk and further increases the shot noise (which in this case is considerably higher than the Schottky level). The leakage current typically grows with R_g because at relatively small R_g the train of tunneling events can be stopped by the single charge escape through the gate resistance.

The strong smearing of the Coulomb blockade at finite temperatures significantly reduces the R-SET voltage gain. Figure 3 shows the control curves at different temperatures of the inverter made of a symmetric R-SET ($R_g = 10R$) loaded with resistance $R_L = 10R$ and biased by $V_B = 0.5e/C$. The voltage $V = V_B - I_1 R_L$ is the output of the inverter while U is the input voltage. One can see that the voltage gain $K_V = |dV/dU|$ becomes less than unity at the negative slope of the V-U dependence at temperatures as low as $\sim 10^{-2}e^2/C$ (while K_V can be arbitrarily large at $T = 0$). To check that the main reason for low K_V is the Nyquist noise of the gate resistance, we also performed calculations for $T_r = 0$ while T is nonzero. The dashed line in Fig. 3 shows such a result for $T = 0.005e^2/C$. For this curve the maximum $K_V \simeq 7$ can be compared with $K_V \simeq 1.2$ for the corresponding curve with $T_r = T$.

The inset in Fig. 3 shows the control curves on a larger scale. The asymptotes of V-U dependence can be calculated similar to that for the I-V curve, $V = [V_B A + (U \mp e/2C_\Sigma) R_2 R_L]/[A + R_L(R_2 + R_g)]$. However, in the case $R_g \gg R_i$ the V-U asymptotes are reached only at very large U because it requires sufficiently large junction currents, $|I_i| \gtrsim 2e/R_i C_\Sigma$.

In Fig. 3 the inverter bias voltage $V_B = e/C_\Sigma$ is equal to the maximum Coulomb blockade threshold. The increase of V_B destroys the Coulomb blockade even for $T = 0$ leading to additional smoothing of the negative slope range. The decrease of V_B creates the plato on the control curve when V is limited by V_B.

Figure 4 illustrates the dependence of inverter control curves on the load and gate resistances. At finite temperature the increase of R_L shifts the negative slope range to lower input voltages and also decreases the output voltage both before and after this range. An increase of R_g for fixed R_L produces similar effects. Notice that the maximum voltage gain typically grows with the increase of R_g and R_L.

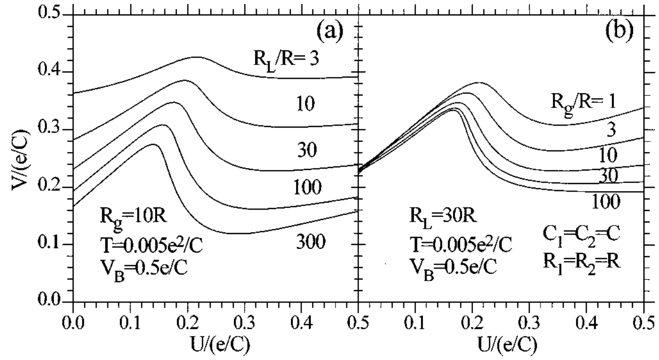

FIG. 4. The control curves of the inverter at $T = 0.005e^2/C$ for different (a) load resistances R_L and (b) gate resistances R_g.

The optimal loading and the voltage symmetry are provided by complementary R-SETs.[3] In this case (similar to the case $R_L \to \infty$) the maximum temperature T_{max} at which $K_V > 1$ is still achievable is close to $0.011e^2/C$ for $R_g/R = 10$ ($0.010e^2/C$ for $R_g/R = 3$ and $0.012e^2/C$ for $R_g/R = 30$). This value is less than one half of $T_{max} = 0.026e^2/C$ for the inverter based on the C-SETs[14] (moreover, for the C-SET it is achieved at a twice larger total island capacitance).

In conclusion, while the R-SET outperforms the C-SET at $T = 0$ (in terms of the voltage gain), its characteristics degrade with temperature much faster than do those of the C-SET due to the Nyquist noise of the gate resistance (because of $T^{1/2}$ scaling). As a result, at $T \gtrsim 10^{-2}e^2/C_\Sigma$ the R-SET performance becomes comparable to or even poorer than that of the C-SET. Nevertheless, insensitivity to the background charge and the nonoscillatory dependence on the gate voltage can still be the principle advantages of the R-SET for some applications.

The author thanks D. V. Averin, K. K. Likharev and V. I. Safarov for fruitful discussions. The work was supported in part by the French MENRT (PAST), the Russian RFBR, and the Russian Program on Nanoelectronics.

[1] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991), p. 173.
[2] A. N. Korotkov, in *Molecular Electronics*, edited by J. Jortner and M. Ratner (Blackwell, Oxford, 1997), p. 157.
[3] K. K. Likharev, IEEE Trans. Magn. **23**, 1142 (1987).
[4] A. N. Korotkov, Phys. Rev. B **49**, 16 518 (1994).
[5] L. S. Kuzmin, Yu. V. Nazarov, D. B. Haviland, P. Delsing, and T. Claeson, Phys. Rev. Lett. **67**, 1161 (1991).
[6] L. S. Kuzmin and Yu. A. Pashkin, Physica B **194-196**, 1713 (1994).
[7] T. Henning, D. B. Haviland, and P. Delsing, Supercond. Sci. Technol. **10**, 727 (1997).
[8] Sh. Farhangfar, J. J. Toppari, Yu. A. Pashkin, A. J. Manninen, E. B. Sonin, and J. P. Pekola (unpublished).
[9] W. Zheng, J. R. Friedman, D. V. Averin, S. Han, and J. E. Lukens (unpublished).
[10] P. Joyez, D. Esteve, and M. H. Devoret, Phys. Rev. Lett. **80**, 1956 (1988).
[11] K. K. Likharev and A. N. Korotkov, in Proceedings of ISDRS'95, Charlottesville, VA, 1995, p. 355.
[12] I. O. Kulik and R. I. Shekhter, Sov. Phys. JETP **41**, 308 (1975).
[13] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits* (Gordon and Breach, New York, 1986), Chap. 7.
[14] A. N. Korotkov, R. H. Chen, and K. K. Likharev, J. Appl. Phys. **78**, 2520 (1995).
[15] P. Delsing, T. Claeson, G. S. Kazacha, L. S. Kuzmin, and K. K. Likharev, IEEE Trans. Magn. **27**, 2581 (1991).