Charge sensitivity of superconducting single-electron transistor

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It is shown that the noise-limited charge sensitivity of a single-electron transistor using superconductors (of either SISIS- or NISIN-type) operating near the threshold of quasiparticle tunneling, can be considerably higher than that of a similar transistor made of normal metals or semiconductors. The reason is that the superconducting energy gap, in contrast to the Coulomb blockade, is not smeared by the finite temperature. We also discuss the increase of the maximum operation temperature due to superconductivity and the peaklike features on the $I-V$ curve of SISIS structures. © *1996 American Institute of Physics.* [S0003-6951(96)05143-1]

Electron transport in the systems of small-capacitance tunnel junctions shows a variety of single-electron effects.[1] The simplest and most thoroughly studied circuit revealing these effects is the single electron transistor[2] (SET) which consists of two tunnel junctions in series. At low temperatures ($T \ll e^2/C_\Sigma, C_\Sigma = C_1 + C_2$ where C_1 and C_2 are the junction capacitances) the current I through this structure depends on the background charge Q_0 of the central electrode (the dependence is e periodical) which can be controlled by a capacitive gate. The possibility to use the SET as a highly sensitive electrometer has been confirmed in numerous experiments.

The most developed technology of the SET fabrication uses the narrow aluminum films with a typical junction capacitance about 10^{-16} F (see, e.g., Refs. 3–5). Consequently, the operation temperature is typically less than 1 K, and the electrodes are in the superconducting state unless it is intentionally suppressed by the magnetic field. It has been noticed[3,4,6] that the superconductivity of electrodes improves the performance of the SET electrometer operating near the threshold of quasiparticle tunneling. However, we are not aware of quantitative theoretical analysis of this issue, which will be the subject of this letter.

There are two major characteristics of the SET operation as an electrometer. The first one is the amplitude of the output signal modulation for Q_0 variations larger than e. It was found experimentally[4] that superconductivity increases the modulation amplitude of current I (for fixed bias voltage V), especially at T comparable to e^2/C_Σ, thus increasing the maximum temperature. The results of this letter confirm that for both NISIN and SISIS structures.

The other, even more important characteristic of the SET operation is the noise-limited sensitivity (ability to detect variations of Q_0 much smaller than e). The best achieved sensitivity so far (by the normal state SET is $7\times 10^{-5} e/\sqrt{\text{Hz}}$ at 10 Hz.[5] In the current technology this figure is limited by $1/f$ noise which is most likely caused by random trapping-escape processes in nearby impurities. It is unlikely that superconductivity of electrodes can significantly affect these processes. Hence, present sensitivities of superconducting and normal SETs with similar parameters should not differ much for reasonably low temperatures when both SETs show sufficient modulation amplitude.

With the reduction of the noise due to impurities or at higher frequencies, the charge sensitivity of the SET achieves the limit determined by the intrinsic noise[7,8] of the device caused by the randomness of tunneling events (this "white" noise has been recently measured in the experiment[9]). Though the theory of the "classical" thermal/shot intrinsic noise of the SET is applicable to the general case of one-particle tunneling (normal metals, semiconductors, quasiparticle current in superconductors, etc), most numerical results in Refs. 7 and 8 as well as in a number of subsequent papers on this subject (see, e.g., Refs. 10–13) were obtained only for SETs made of normal metals. (Recently some generalization was done[14] to include the possibility of two-particle tunneling which can be important in the superconducting case. Let us also mention Refs. 6 and 15 in which the noise in NISIN SET was considered.)

In this letter we apply the theory of Refs. 7 and 8 to the cases of capacitively coupled superconducting SISIS and NISIN SETs (the analysis of a resistively coupled SET can be done in a similar way; see Ref. 7). We show that the noise-limited sensitivity of a SET electrometer can be considerably improved by the use of superconducting electrodes.

For simplicity we consider only the single-quasiparticle tunneling. Cotunneling[16] and Andreev reflection[17] (in NISIN case) can be neglected if the normal state resistances R_1 and R_2 of tunnel junctions are well above the resistance quantum $R_Q = \pi\hbar/2e^2$, because of scaling as $(R_Q/R)^2$ in contrast to R_Q/R for single-particle tunneling. Josephson current[1] and resonant tunneling of Cooper pairs[18,19] (both in the SISIS case) can be neglected for small Josephson coupling E_J (notice that magnetic field can suppress E_J before suppressing superconductivity); even if this is not the case, these effects are not important near the threshold of quasiparticle tunneling which is the voltage range most interesting for us. We use the "orthodox" theory[1,2] of the SET and the BCS theory[20] for the calculation of the tunneling rates.

Figure 1 shows the $I-V$ curves for $Q_0 = 0$, $e/4$, and $e/2$ at different temperatures for (a) the normal metal NININ case, (b) NISIN case (which is equivalent to the SINIS case), and (c)–(d) SISIS case. SETs with $C_1 = C_2$ and $R_1 = R_2 = R_\Sigma/2$ are chosen, and we neglect the gate capacitance C_g because it can always be formally distributed between C_1 and C_2 (see, e.g., Ref. 21). The superconducting energy gap

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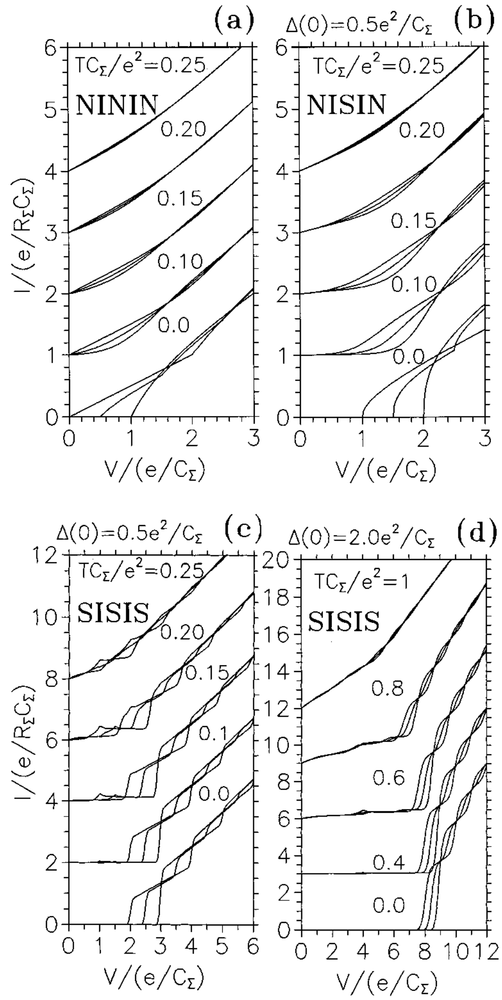

FIG. 1. $I-V$ curves for (a) NININ, (b) NISIN (or SINIS), and (c)–(d) SISIS SETs for three values of Q_0 (0, $e/4$, and $e/2$) and several temperatures T. The curves for different T are offset vertically for clarity. The modulation by Q_0 survives up to higher T in the superconducting SETs.

$\Delta(T)$ depends on T shifting the position of the steps in Figs. 1(c)–(d). The pure BCS theory would lead to the abrupt current steps in the SISIS case. To describe the smoothing of the steps, we assume the inhomogeneous Gaussian broadening of $\Delta(0)$. The dispersion $w_0=0.05\Delta(0)$ is chosen in Figs. 1(c)–1(d); for finite temperatures $w(T)=w_0\{\Delta(T)/\Delta(0) - [T/\Delta(0)][d\Delta(T)/dT]\}$ is used.

In NININ case the current I can be considerably modulated ($I_{\max}/I_{\min}\gtrsim 2$) by Q_0 (V is fixed) only at $T \lesssim 0.15e^2/C_\Sigma$, while at $T=0.3e^2/C_\Sigma$ the modulation is already negligible, $(I_{\max}-I_{\min})/I_{\max}\simeq 5\%$. (The maximum relative modulation is achieved at small V and does not depend on ratios C_1/C_2 and R_1/R_2). The NISIN transistor with $\Delta(0)=0.5e^2/C_\Sigma$ shows considerable modulation crudely up to $T\approx 0.2e^2/C_\Sigma$, while SISIS transistors with $\Delta(0)=0.5e^2/C_\Sigma$ and $\Delta(0)=2.0e^2/C_\Sigma$ operate well almost up to the critical temperature $T_c[T_c/(e^2/C_\Sigma)=0.28$ and 1.14, respectively]. Using Fig. 1(d) one can predict the operation of the niobium-based SET with $C_\Sigma\approx 0.2$ fF (current state-of-the-art for aluminum junctions) at temperatures up to 7 K.

Superconductivity improves the SET performance at relatively high temperatures because, in contrast to the Coulomb blockade, the superconducting energy gap is not smeared by the finite temperature. In the normal metal case the $I-V$ curve has a cusp at the Coulomb blockade threshold $V_t=\min_{i,n}(V_{i,n}|V_{i,n}>0)$, where

$$V_{i,n}=(e/C_i)[1/2+(-1)^i(n+Q_0/e)], \tag{1}$$

and this cusp is rounded within the voltage interval proportional to the temperature. In the SISIS case the $I-V$ curve step at V_t which is shifted due to the energy gap, $V_t=\min_{i,n}[V_{i,n}+2\Delta(T)C_\Sigma/eC_i|V_t>4\Delta(T)]$, remains sharp even at $T\sim\Delta(T)$, and the subthreshold current is only proportional to $\exp[-T/\Delta(T)]$. This explains why the SISIS transistor shows considerable dependence on Q_0 for the temperatures almost up to T_c even if $T\gtrsim e^2/C_\Sigma$. In the NISIN case the $I-V$ curve in the vicinity of $V_t=\min_{i,n}[V_{i,n}+\Delta(T)C_\Sigma/eC_i|V_t>2\Delta(T)]$ is rounded by the finite temperature, that makes the NISIN SET worse than the SISIS SET (but still better than the NININ SET).

Let us briefly discuss the origin of small peaks of the current at moderate temperatures visible in Figs. 1(c)–1(d) (SISIS case) at voltages close to the middle of the subthreshold region (see also Fig. 2 in Ref. 22). The position of a peak satisfies Eq. (1) and corresponds to zero energy gain, $W=0$, for a particular tunneling process (hence, it coincides with the position of one of the $I-V$ cusps in the corresponding NININ SET). In this case the singularities in the density of states of two electrodes match, leading to an increase of tunneling of thermally excited quasiparticles. Hence, the origin of peaks is similar to that of well-known peaks[20] at $V=[\Delta_1(T)-\Delta_2(T)]/e$ in the single junction made of superconductors with different gaps $\Delta_1(T)$ and $\Delta_2(T)$. In our case energy gaps are the same but the Coulomb blockade provides the relative shift of the singularities in the density of states. The analysis of the master equation[1,2] shows that the singularity-matching peaks are more significant within the voltage range $2\Delta(T)<V<2\Delta(T)+e/C_\Sigma$.

The steplike features of the $I-V$ curve also exist at $V<V_t$ at finite temperatures [they are not well-noticeable in Figs. 1(c)–1(d)]. The positions of the steps satisfy equation $V=2\Delta(T)C_\Sigma/eC_i+V_{i,n}$ (similar to V_t) and correspond to the energy gain $W=2\Delta(T)$ for a particular tunneling process. The step height decreases with the decrease of voltage and becomes negligible at $V<2\Delta(T)/e$.

Now let us consider the noise-limited sensitivity of the SET. The minimum detectable charge for the given bandwidth Δf is $\delta Q_0=(S_I\Delta f)^{1/2}/(\partial I/\partial Q_0)$ where the spectral density S_I of the current noise is taken in the low frequency limit. The ultimate sensitivity at $T\ll e^2/C_\Sigma$ in the NININ case is (see Refs. 7 and 8) $\min\delta Q_0\simeq 2.7C_\Sigma (R_{\min}T\Delta f)^{1/2}$, $R_{\min}=\min(R_1,R_2)$. This result can be somewhat improved in the NISIN SET (with the same resistances) operating near V_t. At $T\ll\min[e^2/C_\Sigma,\Delta(T)]$ and for V close to nondegenerate V_t, we can use approximations $S_I\simeq 2eI$, $I\simeq I_{0,i}[(V-V_t)C_1C_2/C_iC_\Sigma]$, where $I_{0,i}(v)=(1/eR_i)[T\Delta(T)/2]^{1/2}\int_0^\infty dy/\sqrt{y}/\{1+\exp[y+(\Delta-ev)/T]\}^{-1}$ is the "seed" $I-V$ curve of ith junction. Then

$$\min\,\delta Q_0=C_\Sigma(2e\Delta f)^{/12}\min[\sqrt{I_0(v)}/(dI_0/dv)], \tag{2}$$

and finally we get the result

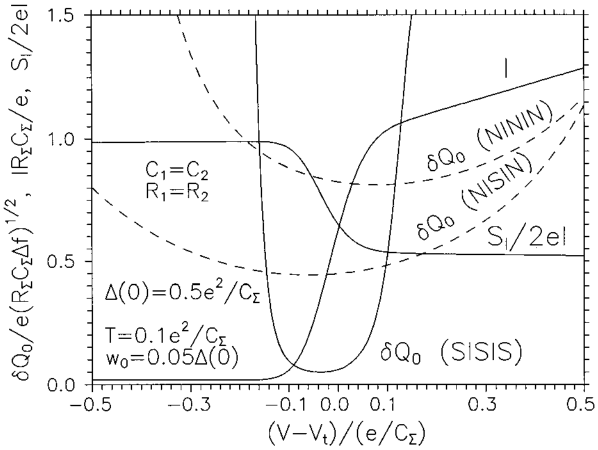

FIG. 2. The minimum detectable charge δQ_0, the current I, and the ratio $S_I/2eI$ as functions of the bias voltage V for SISIS SET. Dashed lines show δQ_0 for NININ and NISIN SETs. The best sensitivity is achieved in SISIS case.

$$\min\ \delta Q_0 \simeq 2.6 C_\Sigma (R_{\min} T \Delta f)^{1/2} [T/\Delta(T)]^{1/4}, \quad (3)$$

which is better than the NININ sensitivity when $T<\Delta(T)$. The main reason for the improvement is the increase[3,4,6] of the transfer coefficient $\partial I/\partial Q_0 \simeq (1/C_i)(\partial I/\partial V)$, because the differential resistance R_d of the ''seed'' $I-V$ curve near the onset of quasiparticle tunneling is less than R_i. Notice that the ''orthodox'' theory used here is valid only if $R_d \gtrsim R_Q$ because the cotunneling processes[16,6] impose the lower bound for $(\partial I/\partial V)^{-1}$ on the order of R_Q.[23] For relatively high temperatures the ratio of minimumδQ_0 in NISIN and NININ cases is considerably larger than $[\Delta(T)/T]^{1/4}$ (e.g., compare the dashed lines in Fig. 2).

The improvement of the ultimate sensitivity is more significant in the SISIS SET. For the pure BCS model the ''orthodox'' theory gives infinite derivative $\partial I/\partial Q_0$ at $V=V_t$ even for finite temperature leading to $\delta Q_0 \to 0$. Hence, the ''orthodox'' ultimate sensitivity depends on the imperfection of the current step which is described in our model by the energy gap spread $w_0\{w_0 \ll \min[\Delta(T), e^2/C_\Sigma]\}$.

Figure 2 shows δQ_0 together with current I and ratio $S_I/2eI$, as functions of the voltage for the symmetric SISIS SET with $\Delta(0)=0.5e^2/C_\Sigma$, $w_0=0.05\Delta(0)$, $T=0.1e^2/C_\Sigma$, and $Q_0=0.25e$. (The numerical method is described in Refs. 7 and 8.) One can see that the sensitivity of the SISIS SET is much better than for similar NININ and NISIN SETs (dashed lines) within a relatively narrow voltage range close to V_t.

In contrast to NININ and NISIN cases, the approximation $S_I \simeq 2eI$ is not accurate in the vicinity of V_t for the SISIS SET even at low T (see Fig. 2) because the tunneling rates in both junctions are comparable. This approximation is valid only if $T \ll \Delta(T) \ll e^2/C_\Sigma$, and it leads to inaccuracy for min δQ_0 typically about 10% if $T \ll \Delta(T) \sim e^2/C_\Sigma$. Nevertheless, it can be used as a crude estimate. Using Eq. (2) and smoothed by w_0 ''seed'' $I-V$ curve for the SIS junction[20] at $T \ll \Delta(T)$, we get

$$\min\ \delta Q_0 \simeq 1.8 C_\Sigma [R_{\min} \Delta f w_0^2/\Delta(T)]^{1/2}. \quad (4)$$

(The numerical factor depends on the particular model describing the step shape.) Comparing Eq. (4) with the result for the NININ SET, we see that the temperature T is replaced by $w_0^2/\Delta(T)$. Hence, the ultimate sensitivity is better in the SISIS SET (resistances are the same) with sufficiently narrow width of the current step, $w_0<[T\Delta(T)]^{1/2}$.

In the case of very sharp ''seed'' $I-V$ curve, $w_0 \lesssim \Delta(T) R_Q/R_i$, the slope of the step of the SET $I-V$ curve is determined by cotunneling[16] and it cannot be sharper than roughly R_Q^{-1}.[23] Then min δQ_0 is on the order of $C_\Sigma[\Delta f \Delta(T) R_Q^2/R]^{1/2}$ [we assume $\Delta(T) \gtrsim e^2/C_\Sigma$, $R_1 = R_2$], and the ultimate sensitivity is better than for the NININ SET if $T \gtrsim \Delta(T)(R_Q/R)^2$. The sensitivity of such an ideal SISIS SET is even better than the ''quantum'' ($T=0$) sensitivity of a symmetric ($R_1=R_2$) NININ SET operating at $V_t \sim e/C_\Sigma$ [in that case[7] min $\delta Q \sim (\hbar C_\Sigma \Delta f)^{1/2}$], if $R/R_Q \gtrsim \Delta(T) C_\Sigma/e^2$. However, notice that the quantum-noise-limited min δQ_0 of a NININ SET can be made arbitrarily small using either small V_t (and large resistances)[7] or large ratio R_1/R_2;[8] hence, in this sense the superconductivity cannot further improve the ultimate sensitivity.

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