Arrays of normal metal tunnel junctions in weak Coulomb blockade regime

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Universal features of $I-V$ characteristics of one-dimensional arrays of normal metal tunnel junctions have been tested against inhomogenities in the junction parameters, number of junctions in the array, and magnetic field. We find that the differential conductance versus bias voltage obeys the analytic form to within 1% if the fabrication errors are smaller than 10% in junction areas, and if the array has more than ten junctions. Furthermore, the universal relation is insensitive to magnetic field at least up to 8 T. © *1995 American Institute of Physics.*

Single electron tunneling (SET) effects have been intensively studied for the past few years.[1] The properties of SET components, in particular of one-dimensional (1D) arrays, in the low-temperature regime, $k_BT \ll E_c$, where E_c is the charging energy of the system, have been widely discussed,[2,3] whereas the opposite limit $k_BT \gg E_c$ has been largely overlooked. In the low-temperature limit, SET components exhibit full Coulomb blockade (CB), i.e., the electric current is prohibited at voltages below a threshold value related to E_c, but in the opposite limit the $I-V$ characteristics reflect competing thermal and charging effects. In our previous work we have declared the suitability of this effect in 1D junction arrays for absolute thermometry: This conclusion is based on the universal dependence of the differential conductance on bias voltage across the array.[4] In the present letter we take a close look at the experimental limiting factors of the accuracy of this universal behavior in different realistic situations, and we also present related theoretical results for arbitrary 1D arrays in the high-temperature limit. In particular, we point out the main limitations, which originate from inhomogeneity of junction arrays and their finite length. We have also investigated the susceptibility of the $I-V$ curves to magnetic field.

We start with a theoretical description of a 1D array of N normal metal tunnel junctions, which is schematically shown in Fig. 1. The resistance of the ith junction is denoted by $R_{T,i}$, and its capacitance by C_i. The stray (ground) capacitance of the ith island, between the ith and $(i+1)$th junction, is $C_{0,i}$. In general, we may allow nonequal values for $R_{T,i}$, C_i, and $C_{0,i}$ at different i, i.e., inhomogeneities in the array; this, in fact, is one of the main objects of the present letter. The array is symmetrically based at $\pm V/2$ at its ends.

In Ref. 4 we derived simple formulas for the $I-V$ curve and its easily measurable derivative, i.e., the differential conductance, G, in the case of two junctions. For an arbitrary N-junction array we have an analogous result

$$\frac{G}{G_T}=1-2\sum_{i=1}^{N}\frac{R_{T,i}}{R_\Sigma}\frac{\Delta_i}{k_BT}\,g\left(\frac{R_{T,i}}{R_\Sigma}\,eV/k_BT\right), \tag{1}$$

using a similar high-temperature expansion at $\Delta_i \ll k_BT$. The function g was introduced in Ref. 4 and defined by

$$g(x)=[x\,\sinh(x)-4\,\sinh^2(x/2)]/8\,\sinh^4(x/2), \tag{2}$$

and we denote the total tunnel resistance by R_Σ, i.e., $R_\Sigma=\Sigma_{i=1}^{N}R_{T,i}$. Δ_i, the Coulomb blockade threshold for the ith junction, originates from the inverse capacitance matrix, C^{-1}, of the array:[5] $\Delta_i=(C^{-1}_{i-1,i-1}+C^{-1}_{i,i}-2C^{-1}_{i,i-1})e^2/2$. G_T is the asymptotic value of G when $V\rightarrow\pm\infty$.

In the fully symmetric case with $R_{T,i}\equiv R_T$, $C_i\equiv C$, and $C_{0,i}\equiv 0$ we obtain

$$G/G_T=1-2w[(N-1)/N]g(eV/Nk_BT), \tag{3}$$

with $w=(e^2/2C)/(k_BT)$. This represents the same nearly bell-shaped dip in conductance as for the case of two junctions, but now the full width at half-maximum, $V_{1/2}$, scales by N, i.e., $V_{1/2}\simeq 5.439Nk_BT/e$, and the depth by $(N-1)/N$, i.e., $\Delta G/G_T=(w/3)(N-1)/N$.

To check the validity of the analytic method, e.g., at finite temperatures, and with nonzero background charges, we had to resort to stochastic Monte Carlo (MC) methods. This basic method could, however, be accelerated significantly at high temperatures over the method used, e.g., in Ref. 2 by a "hybridlike" method, where just the charge configuration is obtained by means of MC, but the current through the array is calculated as a sum of tunneling rates weighted by the corresponding probabilities of configurations.

We fabricate samples by the standard shadow evaporation of aluminum on oxidized silicon substrates. The junctions suitable for this work are large in area, typically 1×0.2 μm^2. Tunnel barrier is formed in pure oxygen at room temperature typically at 40 mbar pressure for 30 s yielding typical capacitances on the order of 10 fF and resistances on the order of 100 kΩ. Measurements are carried out either in our

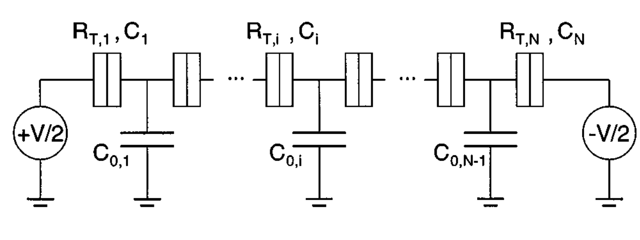

FIG. 1. Schematics of an N-junction array.

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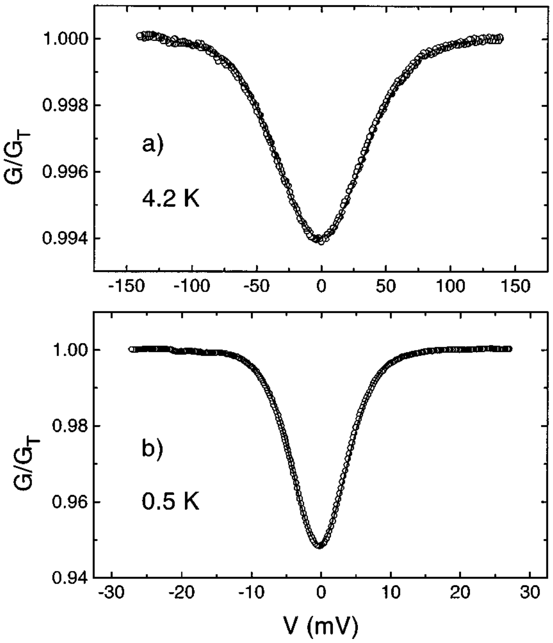

FIG. 2. Differential conductance of a junction array ($N=40$) as measured at two different temperatures of $T=4.2$ K and $T=0.5$ K.

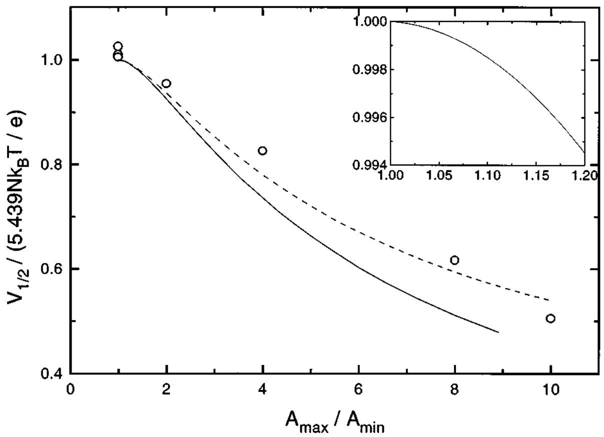

FIG. 3. The effect of inhomogeneity in junction parameters on $V_{1/2}$, as scaled by that of a homogeneous array, $5.439Nk_BT/e$. The data in open circles represents experiments as a function of the width in the distribution of junction areas. For further definition of $A_{\max}/A_{\min}$ see text. The solid and dashed lines are the corresponding theoretical lines, with assumptions explained in the text. The inset with a solid line shows the magnification of the upper left-hand side corner.

compact homemade plastic dilution refrigerators,[6] though at relatively high temperatures in this case, or in a helium dipstick at 4.2 K.

Figure 2 shows our primary data of G/G_T against V across a chain of $N=40$ nominally equal junctions ($R_{T,i}=170$ kΩ) at two temperatures, $T=4.2$ K and $T=0.5$ K. The solid line through the data points is the result of fitting the analytical formula of Eq. (3) assuming $N=40$ identical junctions of $C_i\simeq 12$ fF. The agreement between the simple theory and the experimental data is noteworthy.

One of the more important factors affecting the applicability of junction arrays for precise measurements of absolute temperature is the tolerance of $V_{1/2}$ to inhomogeneities in the junction parameters. Figure 3 shows a set of measurements where arrays of varying deviations from a uniform chain were intentionally fabricated; experimental data are shown in open circles. The measurement was taken at $T=4.2$ K with arrays where $\Delta G/G_T$ varied between 0.8% and 1.6%. A random type of distribution of junction areas in a chain of $N=10$ was generated. All the chains possessed a similar distribution, but with a varying amplitude characterized by the parameter $A_{\max}/A_{\min}$, which is the ratio of the maximum and minimum areas of junction within a chain. The width of each junction in the chain was nominally 0.2 μm, and, as an example, their lengths from one end of the chain to the other were 1, 1.4, 0.9, 1.1, 1.2, 0.75, 1, 1.3, 1.5, and 0.8 μm, respectively, for the case $A_{\max}/A_{\min}=2$. Up to $A_{\max}/A_{\min}=10$ we observe a drop of $V_{1/2}$ by a factor of 2, at most, which already demonstrates weak dependence of this thermometric parameter on fabrication errors.

If we take a close look in the dependence of $V_{1/2}$ using Eq. (1) on varying parameters within a chain, we may compare the experiment in Fig. 3 with the high T approximation. Assuming uniform thickness of the aluminum oxide barrier throughout, we may suppose that $R_{T,i}C_i=$constant for all junctions, because $R_{T,i}\propto A_i^{-1}$ and $C_i\propto A_i$, where A_i is the tunneling area of junction i. Using this approximation and the distribution of junction parameters as set in the experimental layout we obtain the solid line in Fig. 3 in fair agreement with the experiment. We believe that the slight deviation is due to the fact that the real areas of the junctions deviate by a constant additional value from that of the lithographic pattern. The dashed line is the theoretical result assuming that there is an additional area in each junction, which is 10% as compared to the area of the first one. An important conclusion involves the left-hand side upper corner of the figure, shown also by an inset. The theoretical curve (drawn for $R_{T,i}C_i=$constant) starts with zero slope and the dependence is quadratic such that $V_{1/2}/(5.439Nk_BT/e)\simeq 1-k[\delta R/R_0]^2_{\mathrm{rms}}$, where $[\delta R/R_0]_{\mathrm{rms}}$ is the rms deviation of the junction resistances from their mean value $R_0\equiv R_\Sigma/N$ in the array. The numerical factor $k=1.63$ for $N=10$. In Fig. 3 this means that a 10% deviation, i.e., $A_{\max}/A_{\min}=1.1$ induces a drop of 0.2% in $V_{1/2}$ only. With junction areas of nominally 0.2 μm^2 we can easily reach such homogeneity, as supported by the small variation in the experimental values at $A_{\max}/A_{\min}=1$ in Fig. 3.

To our mind the most important and least explained deviation from the straightforward theory is observed in fairly short arrays of $N\leqslant 10$. Figure 4 shows measurements taken with two different sets of chains at $T=4.2$ K; in both sets the arrays had $N=1$, 2, 4, and 8, and $A_i\simeq 0.2$ μm, each set on just one chip near to each other. The two sets were similar in geometry but they differed by a factor of 10 in mean junction resistance due to different oxidation; these numbers were 300 and 30 kΩ/junction, for the upper and lower sets of data, respectively. In both cases the deviation from $5.439Nk_BT/e$ shrinks down to about 2% for $N=8$. For the short chains there could be at least two sources of deviation. First, the impedance of the environment at frequencies of the order of $k_BT/\hbar$ may be of importance, because the calculations pre-

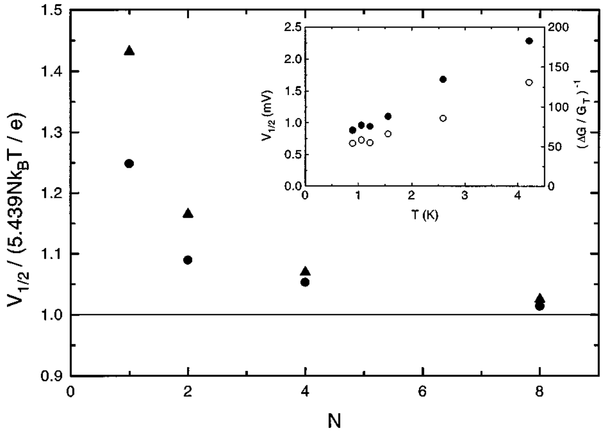

FIG. 4. Experimental results on the deviation of the width of the conductance dip from the analytic value of $5.439Nk_BT/e$ with different values of the array length, N. The two sets of data, by triangles and circles, respectively, present measurements on two different chips. In the former case $R_{T,i} \simeq 300$ kΩ, and in the latter case $R_{T,i} \simeq 30$ kΩ, otherwise the samples are nominally similar. The inset shows temperature dependence of $V_{1/2}$ and $(\Delta G/G_T)^{-1}$ of a single junction.

sented assume that the array is ideally voltage biased at its ends, which is an approximation to within an impedance of the order of thin line impedance ($\simeq 100$ Ω).[5,7] Second, the tunnel barriers are always nonideal with anomalies due to inclusions and conducting islands, and sometimes magnetic impurities inside. These phenomena lead to charging-like effects in small *single* junctions as well. In single junctions the $I-V$ curve should be ohmic without a conductance dip in the basic "orthodox" theory. The temperature dependences of $V_{1/2}$ and $(\Delta G/G_T)^{-1}$ of a single junction ($R_T = 170$ kΩ, $A = 0.2$ μm^2) are shown as an inset. Note that neither $V_{1/2}$ nor $(\Delta G/G_T)^{-1}$ is strictly proportional to T in this case, unlike for long arrays.

The intolerance to magnetic field restricts the use of almost all thermometers available for cryogenic use. Features in Coulomb blockade should supposedly not depend on magnetic field when $E_F \gg \mu_B B$, inequality which is always satisfied in practice. Here, E_F is the Fermi energy of the metal, μ_B is the Bohr magneton, and B is the magnetic flux density. Figure 5 shows measurements of magnetic field dependence of $V_{1/2}$ at three different temperatures. To within the 2% reproducibility of temperature no field dependence at any of the three temperatures of 4.2, 1.6, and 0.7 K can be observed. The only limitation as to magnetic field when using aluminum junctions seems to be the required suppression of superconductivity at $T < 1$ K.

In summary, we have found that the simple theoretical description which is a follow up of the orthodox theory with a fixed bias at the ends of a chain of normal metal tunnel junctions describes the experimental results to a high accuracy, provided that certain conditions, experimentally investigated in this letter, are fulfilled. We find that it is most important not to fabricate too short arrays; increasing N improves the agreement, although we do not fully understand the origin of the discrepancy in short arrays with $N \ll 10$. The effect of inhomogeneities in the junction parameters is easy to push down to below 1% level in the error. A magnetic field up to 8 T affects our results by an undetectably small amount. Our conclusions are for junctions of $R_{T,i} = 30$–300 kΩ and $C_i = 5$–15 fF, and for temperatures of $T = 0.3$–10 K.

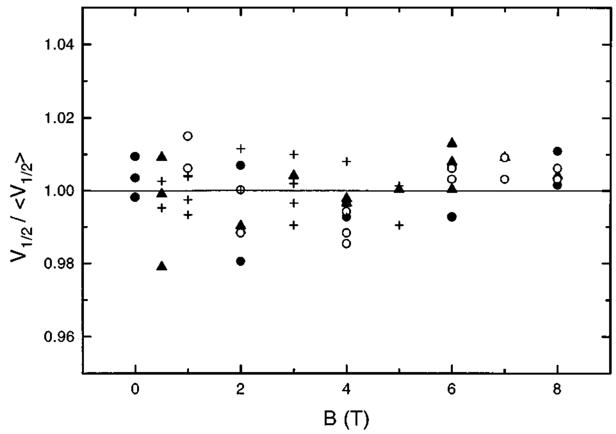

FIG. 5. Magnetic field dependence of $V_{1/2}$ divided by its mean value $\langle V_{1/2} \rangle$ within each set of data. The data are for $T = 4.2$ K, $N = 10$ (solid circles); $T = 1.6$ K, $N = 10$ (solid triangles); $T = 0.7$ K, $N = 10$ (open circles); and $T = 1.6$ K, $N = 40$ (crosses). At $T = 0.7$ K a field of ~0.5 T was necessary to suppress superconductivity of aluminum.

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