Single-Electron Transistors as Ultrasensitive Electrometers

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Abstract. We have calculated intensity of thermal and shot noise, and estimated that of the quantum noise, of single-electron transistors, capacitively and resistively coupled to the signal sources. Results of the calculations have been used to estimate the ultimate performance of those devices as ultrasensitive (sub-single-electron-charge) electrometers.

I. Introduction

Probably the simplest devices based on the correlated single-electron tunneling (for reviews of this new field, see Refs. 1-3, as well as the first papers of this collection), which have a considerable practical potential, are the so-called *Single-Electron Transistors* [2,4]. Figures 1a,b show equivalent circuits of two simplest members of the family, the capacitively-coupled transistor (C-SET, Fig. 1a) and the resistively-coupled transistor (R-SET, Fig. 1b).

Both devices are built around a system of two ultrasmall tunnel junctions connected in series (Fig. 1c). The "orthodox" theory of the correlated tunneling (for a review, see Ref. 2) says [4] that if capacitances C_k ($k = 1, 2$) and tunnel conductances G_k of the junctions are small enough,

$$e^2/C_k \gg k_B T, \tag{1}$$

$$e^2/G_k \gg \hbar, \tag{2}$$

the single-electron tunneling events in the junctions 1 and 2 are mutually correlated, and that this correlation results in several specific features of the dc $I-V$ curves of the device (all these features have been observed experimentally, see Ref. 3 for a review). For our present purposes, the most important feature is that the $I-V$ curve is quite sensitive to the background charge Q_0 of the central electrode of the structure. In particular, Q_0 controls the threshold voltage V_t of the Coulomb blockade range, i.e. the part of the $I-V$ curve with a vanishing current (Fig. 2). Note that even a sub-single-electron variation δQ_0 of the charge leads to quite a considerable change δI of the dc current I. Hence, measuring I at fixed V (or vice versa) one can register very small variations $\delta Q_0 \ll e$, limited only by noise of the device. In experiments [7] the r.m.s. noise (within one-Hertz bandwidth) was equivalent to δQ_0 as small as

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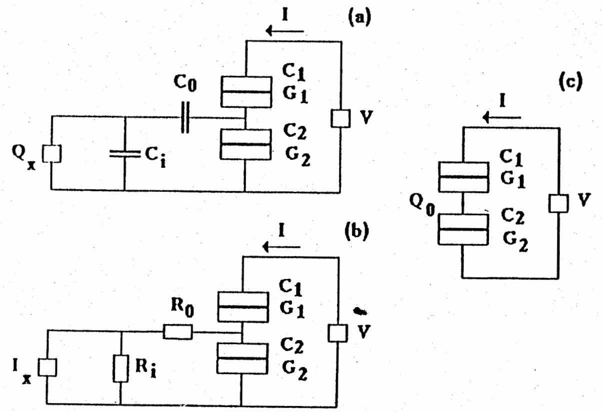


Figure 1: Equivalent circuits of the SET transistors: (a) C-SET, coupled via capacitance C_0 to a signal source Q_x with internal capacitance C_i, and (b) R-SET, coupled via resistance R_0 to a signal source I_x with internal resistance R_i. Figure (c) shows the two-junction system which is the basis of both devices.

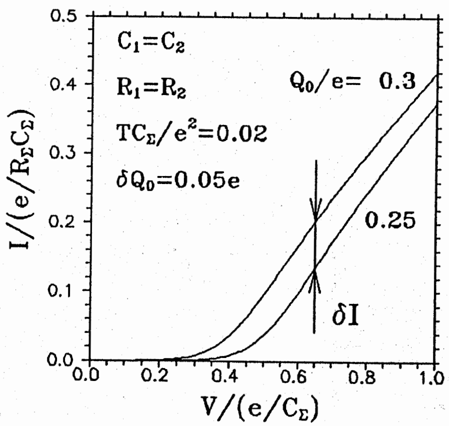


Figure 2: Modulation of the dc $I-V$ curve of the two-junction system by sub-e variation of the charge Q_0 on its central electrode as calculated from the quasiclassical theory (see Eqs. (4)-(7)); $R_k \equiv G_k^{-1}$, $R_\Sigma = R_1 + R_2$, $C_\Sigma = C_1 + C_2$.

$1.5 \times 10^{-4}e$, the figure to be compared with $\sim 10^2 e$ for the best electrometers available commercially [8].

In order to use this high sensitivity for measurement of small signals, one should couple the two-junction system to the signal source via either a small coupling capacitance $C_0 \sim C_{1,2}$ (Fig. 1a), or a large resistor $R_0 \gg R_Q$, $R_Q \equiv \pi\hbar/2e^2 \cong 6.5$ kΩ (Fig. 1b). Basic signal properties of these circuits have

been studied in Ref. 4, but their noise intensity has been only estimated there. The purpose of this paper is to present results of the noise analysis of these two single-electron transistors.

II. Basic relations

Let us start with analysis of the core system, two tunnel junctions connected in series, with a fixed value of Q_0 (Fig. 1c). At a given moment t, the state of this system can be completely characterized by the only integer variable n defined as

$$n(t) = n_1(t) - n_2(t), \tag{3}$$

where $n_k(t)$ is a number of electrons passed through the kth junction until the moment t. Statistical character of the single-electron tunneling in the normal-metal junctions [1,2] does not allow one to write any deterministic (dynamic) equation of motion for $n(t)$; instead one can write down either the *stochastic* (Langevin-type) equation for $n(t)$ [1,4] or *deterministic* (Fokker-Planck-type) equations for probabilities $p(n,t)$ [4,6]. For our present circuit with its small number of junctions the second approach seems preferable.

We will start with the quasiclassical approximation which is adequate if Eq. (2) is well fulfilled. In this approximation the probabilities $p(n,t)$ satisfy the following linear system of equations [5]

$$\dot{p}(n,t) = \sum_{k=1,2}\sum_{\pm}[\Gamma_k^{\mp}(n\pm 1)p(n\pm 1) - \Gamma_k^{\pm}(n)p(n)], \tag{4}$$

where $\Gamma_k^{\pm}(n)$ is the rate of the tunneling events in the kth junction leading to increase/decrease of n by one. In the quasiclassical approximation, each rate is determined solely by the change ΔG of the Gibbs energy G, resulting from the tunneling event:

$$\Gamma_k^{\pm}(n) = \frac{1}{e}I_k(U)[1-\exp(-\frac{U}{k_BT})]^{-1}, \quad U \equiv -\frac{\Delta G}{e}. \tag{5}$$

Here $I_k(U)$ is the dc $I-V$ curve of the kth junction when biased by a fixed voltage U; for our case of the normal-metal junctions one can accept $I_k(U) = G_kU$. The Gibbs energy of our system is a function of both n_1 and n_2:

$$G(n_1,n_2) = \frac{1}{2C_\Sigma}(Q_0+en)^2 - \frac{eV}{C_\Sigma}(n_1C_2+n_2C_1), \tag{6}$$

but ΔG for any tunneling event is a function of n alone, so that Eqs. (4)-(6) form a complete system of equations for $p(n,t)$.

When the system is solved, it is straightforward to calculate the statistical (ensemble) averages of the currents $I_{1,2}$, using the standard formula:

$$\langle I_k(t)\rangle = (-1)^{k+1}e\sum_n[\Gamma_k^{+}(n) - \Gamma_k^{-}(n)]p(n,t). \tag{7}$$

(Here and later the summation is carried out within infinite limits until indicated otherwise. Besides this, in order to simplify all expressions, we use the convention that electron charge e is positive; it does not change the final formulas.)

One should be very careful, however, in using standard textbook formulas for calculations of the intensity of fluctuations of $I_{1,2}$. In particular, one would get a *wrong* result if he tried to apply to $I_{1,2}(t)$ the standard formula (see, e.g., Ref. 9) for the autocorrelation function $K_X(t-t') \equiv \langle X[n(t')]X[n(t)]\rangle$ of a variable $X[n(t)]$:

$$K_X(t-t') = \sum_{n,n'} X(n)p(n,t \mid n',t')X(n')p(n'), \qquad (8)$$

where $p(n,t \mid n',t')$ is the partial solution of Eq. (4) for $t > t'$ with the special initial condition

$$p(n,t' \mid n',t') = \delta_{n,n'}. \qquad (9)$$

and $p(n') \equiv p(n', t\to\infty)$. On the other hand, Eq. (8) yields *correct* results for some other variables, for example for the potential U of the central electrode,

$$U(t) = \frac{1}{C_\Sigma}[C_1V + Q_0 + en(t)]. \qquad (10)$$

The reason of this difference is that the currents $I_{1,2}(t) = e\dot{n}_{1,2}(t)$ are *not* functions of n (while U *is*), so that their correlation functions should be calculated anew from the general principles (see, e.g., Ref. 9), taking into account that only the tunneling events changing n (by ± 1) give a finite contribution to the currents $I_{1,2}$. Such a calculation (for details, see Ref. 10) yields the following result

$$K_{I_k}(t-t') = K_{I_k}(t'-t) = e^2 \sum_{n,n'} [\Gamma_k^+(n) - \Gamma_k^-(n)] \times$$

$$[p(n,t \mid n'+1,t')\Gamma_k^+(n') - p(n,t \mid n'-1,t')\Gamma_k^-(n')]p(n')]. \qquad (11)$$

This formula is valid for $t > t'$ alone, because the correlation function contains an additional term $A_k\delta(t-t')$. The easiest way to find the constant A_k is to calculate the spectral density of the current fluctuations

$$S_{I_k}(\omega) = 4\int_0^\infty d\tau [K_{I_k}(\tau) - K_{I_k}(\infty)] cos(\omega\tau), \qquad (12)$$

and require that the noise approaches the Shottky value

$$S_{I_k}(\omega) \to 2A_k = 2e(\langle I_k^+\rangle + \langle I_k^-\rangle), \quad \langle I_k^\pm\rangle = e\sum_n \Gamma_k^\pm(n)p(n) \qquad (13)$$

at large frequencies $\omega \gg (R_kC_k)^{-1}$.

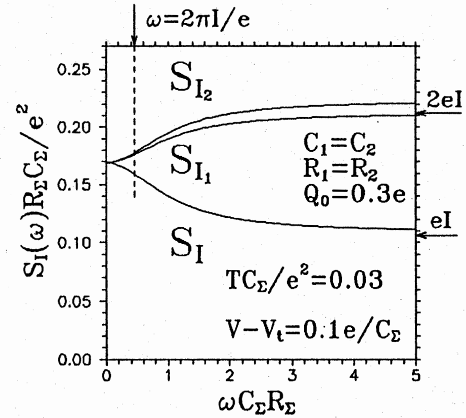


Figure 3: Spectral densities of the currents I_1, I_2 in junctions of the two-junction system, and spectral density of the total current I through the system (Fig. 1c), as functions of observation frequency ω. For a discussion, see text.

Figure 3 shows a typical behavior of the functions $S_{I_k}(\omega)$ and $S_I(\omega)$, where I is the "total" or "external" current [4]

$$I(t) = \frac{C_2}{C_\Sigma}I_1(t) + \frac{C_1}{C_\Sigma}I_2(t). \qquad (14)$$

One can see that these functions can grow with frequency. This unusual property is a consequence of electron "anti-correlations" (tunneling of an electron decreases for some time the probability of tunneling through the same junction), which make the correlation function (11) negative at $t > t'$. Note also that the functions $S_{I_k}(\omega)$ and $S_I(\omega)$ do *not* show peaks around the frequency $f = I/e$ (in other words, the single-electron transistor does *not* exhibit the SET oscillations), in accordance with the general principles of the single-electronics [1-3]. In practice, we will need to know the spectral density in the low-frequency limit $\omega \ll (R_kC_k)^{-1}$ alone, where spectral densities of I_1, I_2, and I coincide (due to conservation of charge of the middle electrode):

$$S_I(0) \equiv \lim_{\omega\to 0} S_{I_1}(\omega) = \lim_{\omega\to 0} S_{I_2}(\omega). \qquad (15)$$

Generally, noise properties of the single-electron transistor as a two-port device are determined not only by intensity of two noise sources $S_I(\omega)$ ("output noise") and $S_U(\omega)$ ("back-action noise"), but also by their mutual spectral density

$$S_{IU}(\omega) = 2\int_{-\infty}^{+\infty} d\tau [K_{IU}(\tau) - K_{IU}(\infty)]\exp(i\omega\tau) = \frac{C_2}{C_\Sigma}S_{I_1U}(\omega) + \frac{C_1}{C_\Sigma}S_{I_2U}(\omega). \qquad (16)$$

This density can be found from the correlation functions

$$K_{I_kU}(t-t') = e\sum_{n,n'}(-1)^{k+1}[\Gamma_k^+(n) - \Gamma_k^-(n)]p(n,t\mid n',t')U(n')p(n'),$$

$$\text{for} \quad t > t',$$

$$= e\sum_{n,n'}(-1)^{k+1}U(n')[p(n',t'\mid n+1,t)\Gamma_k^+(n) - p(n',t'\mid n-1,t)\Gamma_k^-(n)]p(n)$$

$$\text{for} \quad t < t'. \quad (17)$$

One can see that generally K_{I_kU} are asymmetric functions of $\tau = t - t'$, and thus the mutual density (16) is a complex function of frequency. In the low-frequency limit, however, this function approaches a real value $S_{IU}(0)$.

Analytically, the noise intensity can be calculated in an important limit when the temperature is low, $k_BT \ll e^2/C_\Sigma$, and the bias voltage V is close to the Coulomb blockade threshold V_t [4]. In this limit, the tunneling rate is determined by that in one of the junctions, and if there is no large asymmetry in junction resistances, a rare leap of an electron through this junction is almost immediately followed by the similar leap through the second junction, with a long pause after this double leap. Hence, the double leaps are virtually not correlated, and the low-frequency noise of current I obeys the Shottky formula $2e\langle I\rangle$, i.e. is completely determined by $\langle I\rangle$. The last quantity can be readily calculated from the master equation (4), because in this limit only two states (say, $n = 0$ and $n = 1$) have non-vanishing probabilities $p(n)$. For the case when electron first passes through the junction 1 (e.g., for $C_1 \le C_2$, $-e/2 < Q_0 < 0$), the result is:

$$\langle I\rangle = \frac{C_2(V - V_t)}{C_\Sigma R_1(1 - \exp\{-C_2e(V - V_t)/C_\Sigma k_BT\})}. \quad (18)$$

For arbitrary parameters, the noise can be calculated numerically, using the formulas given above (a method of considerable acceleration of these calculations will be described elsewhere [10]). Figure 4 shows typical plots of low-frequency current and voltage noise intensities, and their correlation factor $S_{IU}/(S_IS_U)^{1/2}$ as functions of the dc bias voltage V.

At very low temperatures, $k_BT \ll (G_kR_Q)e^2/C_\Sigma$, the quasiclassical current (18) in the most important voltage range $V \cong V_t$ becomes smaller than the current due to the macroscopic quantum tunneling of charge [11] and the above quasiclassical formulas should be changed for those following from a more general approach. For $G_kR_Q \ll 1$ and $T \to 0$ one can get [12] an analytical expression

$$\langle I\rangle = \frac{\hbar G_1G_2V}{2\pi e^2}\sum_{i=1,2}[\frac{\varepsilon_i(1+\varepsilon_i)}{\gamma_i}(\mathrm{arctg}\{\frac{\varepsilon_i}{\gamma_i}\} - \frac{\pi}{2}) + \varepsilon_i + (\frac{1}{2} + \frac{\varepsilon_1\varepsilon_2}{1+\varepsilon_1+\varepsilon_2})\ln\{\frac{(1+\varepsilon_i)^2}{\gamma_i^2+\varepsilon_i^2}\}], \quad (19)$$

where

$$\gamma_i \equiv g_j(1+\varepsilon_i), \quad i,j = 1,2, \quad i \ne j,$$
$$\varepsilon_i \equiv (C_j/C_\Sigma)[(V_{ti} - V)/V], \quad g_j \equiv \hbar G_j/2e^2,$$

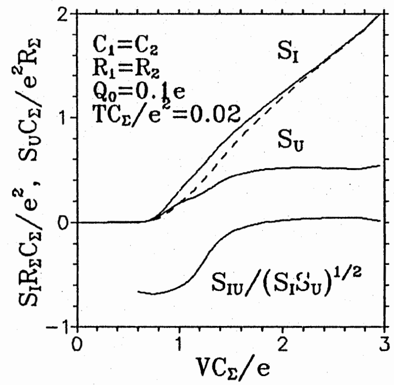


Figure 4: Low-frequency spectral densities of the current I and electric potential U of the middle electrode of the two-junction system, and their correlation factor as functions of the bias voltage. Dashed line shows the dc $I - V$ curve of the system.

and V_{ti} is the threshold of tunneling in the ith junction. Equation (19) describes transition from quantum tunneling at $V < V_t$, $V_t \equiv \min\{V_{t1}, V_{t2}\}$ to quasiclassical tunneling at $V > V_t$.

III. C-SET

Now let us consider the single-electron transistor, capacitively coupled to an external signal source which is characterized by the signal amplitude Q_x and the intrinsic capacitance C_i (Fig. 1a). It is straightforward to get convinced [4] that the Gibbs energy of the complete system is similar to that (6) of the effective bare two-junction system (Fig. 1c) with the following parameters

$$C_1' = C_1, \quad C_2' = C_2 + C_c, \quad C_\Sigma' = C_1 + C_2 + C_c, \quad C_c^{-1} = C_0^{-1} + C_i^{-1},$$
$$Q_0' = Q_0 + Q_x(C_c/C_i) = Q_0 + Q_x[C_0/(C_0 + C_i)]. \quad (20)$$

Hence, the change of the output variable, the dc current $\langle I\rangle$, due to a small signal $\delta Q_x \ll e$ can be calculated as

$$\delta I_s = \frac{\partial I'}{\partial Q_0'}\delta Q_x', \quad \delta Q_x' \equiv [1 + \frac{C_i}{C_0}]^{-1}\delta Q_x, \quad (21)$$

where (′) refers to the effective two-junction system. On the other hand, r.m.s. value of the low-frequency fluctuations of the current through the system is

$$\delta I_N = [S_{I'}(0)\Delta f]^{1/2}, \quad (22)$$

where Δf is the output bandwidth of the device (determined by the post-

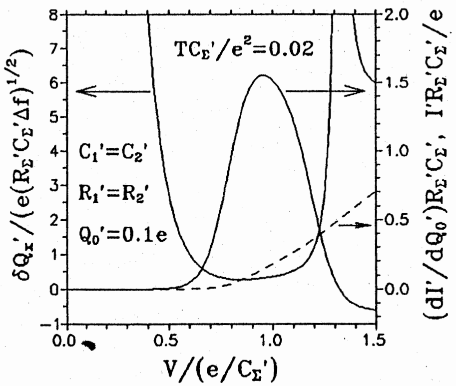


Figure 5: The responsivity $\partial I'/\partial Q_0'$ of the dc current through the two-junction system and its noise reduced to the input, as functions of the bias voltage V; $\delta Q_x'$ is defined by Eq. (21). Dashed line shows the dc $I-V$ curve of the system.

transistor filtering and directly related to the effective measurement time $\tau_m \simeq 1/\Delta f$).

The charge resolution δQ_x of the device can be obtained by equating signal $|\delta I_s|$ to noise $|\delta I_N|$:

$$\delta Q_x = [S_{I'}(0)\Delta f]^{1/2}\, |\partial I'/\partial Q_0'|^{-1}\, (1 + \frac{C_i}{C_0}). \tag{23}$$

Hence, in this particular case the device sensitivity is determined exclusively by the "output" noise (of the effective system).

Figure 5 shows a typical dependence of $\delta Q_x'$ on the bias voltage V. The noise (reduced to input) is high both inside the Coulomb blockade range (due to small responsivity $\partial I'/\partial Q_0'$) and far beyond the blockade threshold (due to the growing current fluctuations) and reaches its minimum slightly above the threshold. This trend is well seen in Fig. 6 which shows levels of fixed noise $\delta Q_x'$ on the parameter plane $[Q_0', V]$. One can see that the minimum of $\delta Q_x'$ with respect to voltage V is quite a weak function of Q_0', except for vicinities of special points where straight boundaries of the Coulomb blockade meet. If one selects the point $[Q_0', V]$ corresponding to a minimum of $\delta Q_x'$, the resulting noise is a decreasing function of C_k', R_k', and T. In practice, the minimum value C_{min} of junction capacitances is determined by technology of fabrication of ultrasmall tunnel junctions, and should be considered as a fixed parameter, while the coupling capacitance C_0 should be considered as an adjustable parameter and a subject of optimization.

Figure 7 shows the resulting minimum value of the transistor noise as a function of the source capacitance, for several values of temperature. If C_i is smaller than some value $C_t(T)$ (see Fig. 7) the optimum value of the coupling capacitance C_0 is much larger than C_i. For $C_i \to 0$, relatively low temperatures

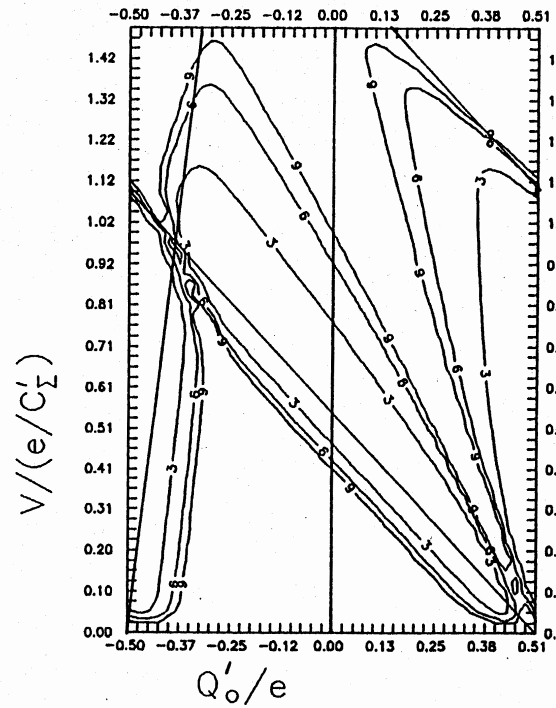


Figure 6: Levels of the fixed noise $\delta Q_x'$ on the parameter plane $[Q_0', V]$. The noise decreases in the vicinity of the Coulomb blockade threshold (indicated by the straight lines) except for the points where the thresholds of tunneling in two junctions coincide.

($k_B T \ll e^2/C_{min}$), and for $C_1 = C_2 = C_{min}$, $R_1 = R_2 = R$ the minimum noise is

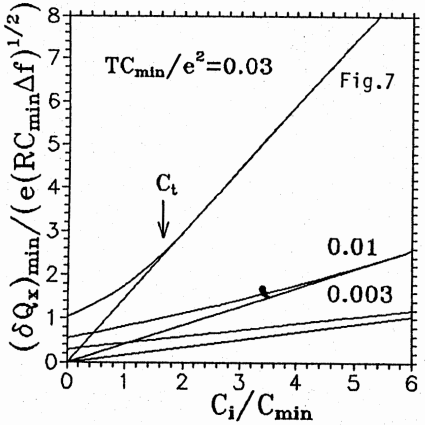
$$(\delta Q_x)_{min} \simeq 5.4e[\frac{k_B T}{e^2/C_{min}}]^{1/2}(RC_{min}\Delta f)^{1/2}. \tag{24}$$

If C_i is increased beyond $C_t(T)$, one should decrease the coupling capacitance in order to limit values of C_Σ and thus prevent the system from the destructive effect of the thermal fluctuations. In this case the optimum value of C_0 is determined by the condition $C_\Sigma' =$ const, and δI_s (21) decreases, while the effective noise δQ_x (reduced to the device input) grows linearly with C_i. At $C_i > C_t$, the device is better characterized by its voltage sensitivity,

$$\delta V_x = \delta Q_x / C_i, \tag{25}$$

which is independent of C_i in this limit. At low temperatures

$$(\delta V_x)_{min} \simeq 2.7(k_B T R \Delta f)^{1/2}. \tag{26}$$



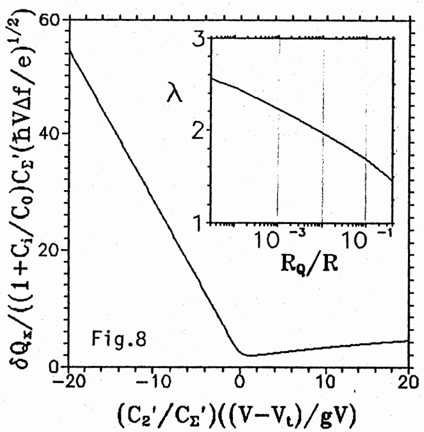


Figure 7: Minimum noise $(\delta Q_x)_{min}$ of the C-SET (Fig. 1a) at various temperatures as a function of the signal source capacitance C_i, for $C_1 = C_2 = C_{min}$ and $R_1 = R_2 = R$. Straight lines show the linear asymptotes of $(\delta Q_x)_{min}$ for large C_i.

Figure 8: Noise of the C-SET at vanishing temperature as a function of the bias voltage (for $R_Q/R = 0.01$). On the chosen voltage and charge scales the curve is almost independent of the junction resistance R. The insert shows the minimum value of this curve as a function of R; λ is defined by Eq. (27).

One can see that the charge and voltage noise (24), (26) decrease with decreasing temperature T and junction resistance R. This implies that at sufficiently small T and R the noise is dominated by the quantum fluctuations which are not accounted for by the quasiclassical approximation used to obtain Eqs. (24), (26). For $T \to 0$, one can calculate such a quantum noise inserting Eq. (19) into Eq. (23), and using the fact that near Coulomb blockade threshold, $S_{I'}(0) = 2eI'$.

The noise δQ_x obtained in this way is shown in Fig. 8 for $R_1 = R_2 \equiv R$, and for Q_0 for which $V_{t1} \neq V_{t2}$. For such values of Q_0 the noise is practically independent of Q_0, and similar to the noise in the classical regime, reaches its minimum value $(\delta Q_x)^*_{min}$ at some voltage just above V_t:

$$(\delta Q_x)^*_{min} = \lambda[1 + \frac{C_i}{C_0}](\hbar V_t C_\Sigma'^2 \Delta f/e)^{1/2}, \qquad (27)$$

where the factor λ can be calculated numerically. As shown in the insert of Fig. 8 this factor is a very slowly varying function of the resistance R. Equation (27) shows that $(\delta Q_x)^*_{min}$ decreases with decreasing threshold voltage V_t, i.e. with

[1]One should note that this is the absolute minimum of quantum noise for symmetric transistor with $R_1 = R_2$. Asymmetry of the junction resistances presumably can further decrease this value.

Q_0 approaching e/2, and reaches its absolute minimum $(\delta Q_x)_{min}$ at $Q_0 \to e/2$. Since Eqs. (19), (27) are no longer valid when $eV \simeq \hbar/RC'_\Sigma$, we can get only the estimate of the absolute minimum of the quantum noise:[1]

$$(\delta Q_x)_{min} \simeq (\hbar C'_\Sigma \Delta f R_Q/R)^{1/2}, \qquad \text{for} \quad C_i \to 0. \qquad (28)$$

This estimate implies that the minimum value of the natural energy measure of the noise

$$\delta E_x \equiv (\delta Q_x)^2/2C'_\Sigma, \qquad (29)$$

can be smaller than that given by a naive estimate $(\delta E_x) \sim \hbar\Delta f$.

Comparison of Eqs. (24) and (28) shows that a transition between regimes with sensitivity limited by thermal and quantum noise should take place at $k_B T \simeq \hbar R_Q/C'_\Sigma R^2$. Since the quantum noise decreases with increasing R (while the thermal noise increases), there is an optimum value of junction resistance at any finite temperature T:

$$R \simeq (\hbar R_Q/k_B T C'_\Sigma)^{1/2}. \qquad (30)$$

In order to make a numerical estimate, let us take parameters corresponding to the experiment [7]: $C_k \simeq 1$ fF, $R_k \simeq 100$ kΩ, $T \simeq 50$ mK. For these parameters $k_B T \gg (R_Q/\pi R)(e^2/C_\Sigma)$, so that sensitivity δQ_x should be estimated from quasiclassical equation (24), which yields the value

$$(\delta Q_x)_{min}/(\Delta f)^{1/2} \simeq 10^{-5} e/\text{Hz}^{1/2}, \qquad (31)$$

for the low-impedance-source measurements ($C_i \ll C_k$). This value is a factor of 10 smaller than that obtained in the experiments [7]. Possible reasons of this difference include:

- $1/f$ noise contribution in experiments (in our theory, this noise is disregarded);
- imperfect optimization in V and Q_0 during the measurements;
- heating of the middle electrode by the dc current.

Further detailed experiments are certainly necessary to identify the most important reasons of this discrepancy.

For a large-capacitance source with $C_i = 1$ pF, and for the same transistor parameters as above, Eq. (26) yields the estimate

$$(\delta V_x)_{min}/(\Delta f)^{1/2} \simeq 10^{-9} V/\text{Hz}^{1/2}, \qquad (32)$$

which corresponds to charge sensitivity $\delta Q_x/(\Delta f)^{1/2} \simeq 0.01 e/\text{Hz}^{1/2}$. Although not as impressive as Eq. (31), this sensitivity is still much higher than that available using conventional solid-state electrometers.

IV. R-SET

Another possible way to couple the system of two small-area tunnel junctions (Fig. 1c) to signal source is to use resistance R_0 (Fig. 1b). Such a galvanic coupling enables one to use the single-electron transistor as a galvanometer. If the source capacitance is large ($C_i \gg e^2/k_BT$), the coupling resistance should be large enough ($R_0 \gg R_Q$) to cut off C_i; otherwise only the sum $R_0 + R_i$ should be much larger than R_Q. If the "external resistance" $R_0 + R_i$ is also much larger than $R_{1,2}$, equations describing this system can be reduced to those of the bare two-junction system, complemented by the self-consistency equation for the background charge Q_0:

$$\dot{Q}_0(t) = I_0(t) = [I_xR_i + \delta U_{0N} - \langle U\rangle]/(R_0 + R_i), \qquad (33)$$

where $\langle ...\rangle$ means *statistical* averaging (or, equivalently, *time* averaging on the short time scale $R_kC_k \ll (R_0 + R_i)C_k$), and δU_{0N} is the e.m.f. describing Johnson-Nyquist noise of the coupling resistance.

Equation (33) shows that the R-SET can operate in two modes. In the first ("dc") mode realized at

$$\min_{Q_0}\{U(V, Q_0)\} < I_xR_i < \min_{Q_0}\{U(V, Q_0)\} \qquad (34)$$

the input is on the average compensated by variations of the middle electrode potential,

$$\langle U\rangle = I_xR_i, \qquad (35)$$

so that $\langle Q\rangle$, $\langle n\rangle$, and $\langle I_k\rangle$ are constant in time, and $\langle I_0\rangle = 0$. Note that the range (34) can be extended considerably by a negative-feedback circuit. This circuit, if implemented properly, does not change the system sensitivity.

In order to find the sensitivity in the dc mode, one can linearize Eq. (33) for small signal/fluctuation perturbations, and get for $\omega \ll (\partial U/\partial Q_0)/(R_i + R_0)$:

$$\delta Q_0 = (\delta I_xR_i + \delta U_{0N} + \delta U_N)(\partial U/\partial Q_0)^{-1}, \qquad (36)$$

where δU_N is the fluctuations of the middle electrode potential in absence of coupling. Uniting this equation with that for the current perturbation,

$$\delta I = \delta I_N + (\partial I/\partial Q_0)\delta Q_0, \qquad (37)$$

one gets the following result for the signal current resolution:

$$\delta I_x = \frac{1}{R_i}[S''_U(0)\Delta f]^{(}1/2), \qquad S''_U(0) = S'_U(0) + 4k_BTR_0,$$

$$S'_U(0) = S_I(0)\kappa^2 + S_U(0) - 2S_{IU}(0)\kappa, \qquad \kappa \equiv (\frac{\partial U}{\partial Q_0})_V/(\frac{\partial I}{\partial Q_0})_V. \qquad (38)$$

Figure 9 shows a typical dependence of $S'_U(0)$ on the bias voltage V. One can see that $S'_U(0)$ is minimum just at the Coulomb blockade threshold. Numerical optimization with respect to Q_0, V shows that

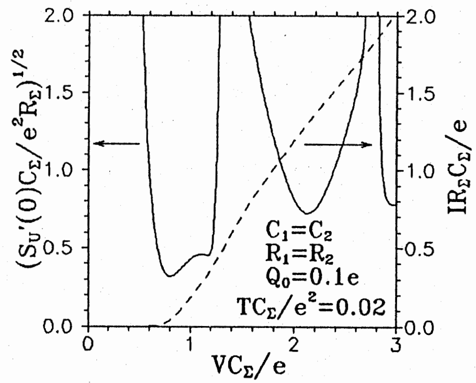


Figure 9: Internal noise $S'_U(0)$ of the R-SET (defined by Eqs. (38)) as a function of the bias voltage V (solid line), and dc $I - V$ curve of the system (dashed line).

$$\min S'_U(0) = \alpha k_BTR, \qquad \alpha \simeq 8, \qquad (39)$$

where $R_1 = R_2 = R$.

According to Eqs. (38), (39), sensitivity of the R-SET electrometer as a *voltmeter* is nearly the same as that of its C-SET counterpart (cf. Eq. (26)), and not extremely high. Moreover, as long as $R_0 \gg R$, the noise of the device is dominated by the Johnson-Nyquist noise of the coupling resistor. Thus, the resistively-coupled SET does not promise an outstanding performance for high-C_i signal sources, for which large coupling resistance ($R_0 \gg R_Q$) is necessary in order not to increase effective junction capacitance and thus to protect transistor from destructive thermal and quantum fluctuations.

On the other hand, if the signal source is small in size (say, few tens of microns or less), so that $C_i \ll e^2/k_BT$, and can be located close to the transistor, there is no need in the coupling resistance R_0. Our formulas (38), (39) show that in this case the dc current sensitivity of the electrometer can be extremely high. For example, for the same parameters as in the previous section and $R_i = 10^{12}\ \Omega$ we get

$$\delta I_x/(\Delta f)^{1/2} \simeq 10^{-21}\text{A/Hz}^{1/2}, \qquad (40)$$

which is some five orders of magnitude better than for any solid-state galvanometer we know about.

Note also the second ("ac") mode of operation attainable in the R-SET without feedback outside the range (34). In this mode, $\langle I_0\rangle \neq 0$, and all variables (including I) oscillate with the frequency $f_{SET} = \langle I_0\rangle/e$. In this regime the device can serve as an absolute ampermeter, although its noise is somewhat larger than that in the dc mode.

V. Conclusion

Our noise analysis of the capacitively- and resistively-coupled single-electron transistors shows that under certain conditions these devices can have an extremely high sensitivity as electrometers. The most important condition of this high performance is that the signal source admittance (including both conductance R_i^{-1} and capacitance C_i) is very small - see Eqs. (2), (2). For such sources, the charge/current resolution of C-SET/R-SET electrometers can be some five to six orders of magnitude better than that of the commercially available solid-state electrometers.

Unfortunately, many room-temperature sources cannot meet this condition, notably because C_i includes a relatively large contribution of the source connections to the transistor (which should be cooled to at least liquid-helium temperatures at the present-day values of the tunnel junction capacitances). For such sources, advantages provided by the single-electronics for the electrometry are not so apparent, although still considerable.

The situation could be improved significantly if an electrostatic analog of the superconducting dc transformer (widely used with Josephson-junction magnetometers called SQUIDs, see, e.g. Ref. 13) were available. Unfortunately, despite a considerable duality between the single-electronic and Josephson-junction devices [1-4], such an analog is not feasible for dc signals. This is due to the fact that electrostatics is described by the single-component electric potential $\vec{\varphi}(r)$, rather than by three-component vector-potential $\vec{A}(r)$ as magnetostatics (magnetic transformers are essentially based on the possibility to twist the vector $\vec{A}$ in space).

For non-vanishing signal frequencies, however, the last argument is not valid, and possibly a solution can be found for an effective matching of high-capacitance sources to low-capacitance single-electron transistors. Until that happens, the single-electronic electrometry will probably be restricted mostly to measurements of small-size helium-cooled objects, first of all, other single-electronic systems. The first experiment of this kind has been already carried out [13].

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