Toward Practical Weakly Hard Real-Time Systems: A Job-Class-Level Scheduling Approach

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***Abstract*—Recent applications of the Internet of Things and Cyber-Physical Systems require the integration of many sensing and control tasks into resource-constrained embedded devices. Such tasks can often tolerate a bounded number of timing violations. The concept of *weakly-hard* real-time systems can effectively improve resource efficiency without sacrificing system safety. However, existing studies have limitations on their practical use due to the restrictions imposed on task timing behavior, high analysis complexity, and the lack of multicore support. In this paper, we propose a new *job-class-level fixed-priority preemptive scheduler* and its schedulability analysis framework for weakly-hard real-time tasks. Our proposed scheduler employs the *meet-oriented classification* of jobs of a task in order to reduce the worst-case temporal interference imposed on other tasks. Under this approach, each job is associated with a "job-class" that is determined by the number of deadlines previously met (with a bounded number of consecutively-missed deadlines). This approach allows decomposing the complex weakly-hard schedulability problem into two sub-problems that are easier to solve: (1) analyzing the response time of a job with each job-class, which can be done by an extension of the existing task-level analysis, and (2) finding possible job-class patterns, which can be modeled as a simple reachability tree. We also present a semi-partitioned task allocation method for multicore platforms, which enhances the schedulability of weakly-hard tasks under the proposed scheduling framework. Experimental results indicate that our scheduler outperforms prior work in terms of task schedulability and analysis time complexity. We have also implemented a prototype of a job-class-level scheduler in the Linux kernel running on Raspberry Pi with acceptably-small runtime overhead.**

***Index Terms*—real-time systems, cyber-physical systems, scheduling, weakly-hard constraints**

I. Introduction

The performance and stability of the Internet of Things (IoT) and Cyber-Physical Systems (CPS) applications depends not only on the precision of computation but also on the time instant at which the output is generated [2], [3]. Since Liu and Layland's seminal work [4], real-time systems with hard deadlines have been extensively studied and have shown their effectiveness in satisfying all deadlines under any circumstance. However, in practical systems, there are many components that are tolerant to some deadline misses without affecting their functional correctness, if the number of misses is predictably controlled and bounded.

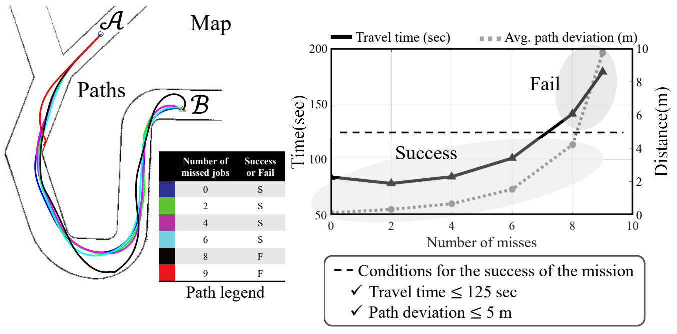

Fig. 1. An autonomous driving of a vehicle from point $\mathcal{A}$ (start) to point $\mathcal{B}$ (destination). Paths that the vehicle has driven are depicted in different colors.

Motivational example. We illustrate in Fig. 1 the impact of the number of deadline misses of a periodic task on a self-driving application that is implemented in the Robot Operating System (ROS). In this example, the mission is that the vehicle drives from point $\mathcal{A}$ (start) to point $\mathcal{B}$ (destination) safely. The success of this mission is determined by whether the vehicle arrives at the destination while satisfying the given conditions of the total travel time[1] and the average deviation from the path[2]. We focused on the control-loop task that sends angular and throttle commands to the vehicle's base at a fixed rate, i.e., 20 Hz by default. For experiment purpose, a specific number of deadline misses were injected to this task every 10 periods. As depicted in Fig. 1, the vehicle was able to achieve its mission safely as long as the task missed no more than 6 deadlines out of 10. However, when 8 deadlines were missed (black line in the left part of the figure), the vehicle failed to arrive at the destination within the required travel time (125 sec). Furthermore, the vehicle even failed to arrive at the destination when 9 deadlines were missed (red line; failure for both criteria). This example motivates the development of *weakly-hard real-time systems* [5] for practical IoT and CPS applications in order to enhance resource efficiency within safety boundaries. We believe that the (m, K) notation of weakly-hard constraints, which specifies that at most m task instances (jobs) can miss their deadlines among any K consecutive jobs, can best describe the above system compared with conventional *hard* and *soft* real-time systems

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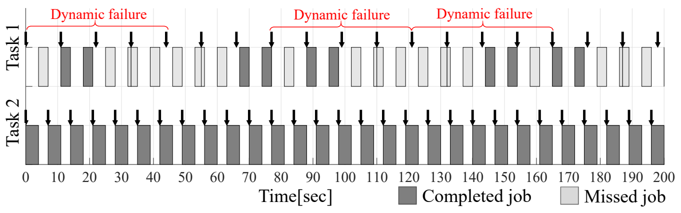
[1]The travel time is defined as the time from the departure to the complete stop of the vehicle.

[2]This is computed by the average difference between the position data of the vehicle's traveled route and the command route.

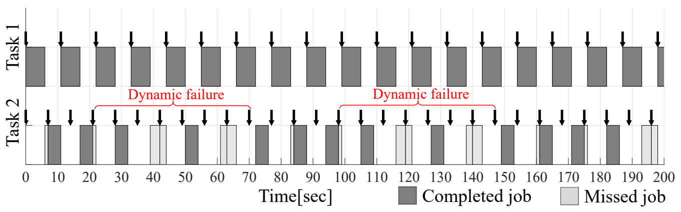
due to the following reasons: (1) it is acceptable to miss some deadlines of a task occasionally in the above system but the use of hard real-time constraints strictly disallows any deadline misses, and (2) the occurrence of deadline misses needs to be bounded for safety and mission success whereas soft real-time constraints do not provide such a rigor.

Limitation of existing methods. Prior work on predictable (m, K) guarantees [5]–[10] has focused on reducing the pessimism of schedulability analysis under traditional task-level fixed-priority scheduling, such as Rate Monotonic (RM) [4], by making strong assumptions on task timing behavior, e.g., fixed phasing (initial release offset) and fixed period with no release jitter. However, we believe that such assumptions limit their applicability to recent IoT and CPS applications that require flexibility and adaptability. The high complexity of the existing analysis also makes it difficult to use runtime admission control, which is required by systems running in a changing environment.

Moreover, we find that task-level fixed-priority scheduling cannot take full advantage of m permitted deadline misses in K consecutive job executions. As an example, let us consider a uniprocessor system with two periodic tasks. Task 1 has $(m, K) = (2, 4)$ with period of 11 and execution time of 6 time units. Task 2 has $(4, 7)$ with period of 7 and execution time of 4 units. Hence, Task 1 may miss up to 2 deadlines in 4 consecutive jobs; Task 2 may miss up to 4 in 7 jobs. As the total utilization of the two tasks exceeds 1, they cannot meet their deadlines all the time. However, there *may* exist a feasible weakly-hard schedule as the minimum utilization demand to meet the weakly-hard constraints is only $(6/11) \cdot (2/4) + (4/7) \cdot (3/7) \approx 0.52$.



(a)



(b)

Fig. 2. Dynamic failures under conventional task-level fixed-priority scheduling. (a) Task 1 with low priority. (b) Task 1 with high priority.

Under task-level fixed-priority scheduling, only two priority assignments can be found for the above taskset: (1) low priority to Task 1 and high priority to Task 2, which is also obtainable by RM, and (2) high to Task 1 and low to Task 2. Fig. 2 illustrates task scheduling timeline with these two priority assignments. In both cases, the taskset falls into a *dynamic failure* [11], where a task experiences more than m deadline misses in a window of K jobs. Thus, we can conclude that despite the very low minimum utilization demand of this taskset, none of the task-level fixed-priority assignments yields a feasible schedule.

In this paper, we take a completely different approach that significantly improves the scheduling efficiency and flexibility of weakly-hard real-time tasks. We propose a new scheduling policy, called *job-class-level fixed-priority preemptive scheduling*, for periodic and sporadic tasks with (m, K) constraints. This scheduler supports arbitrary initial offsets and non-zero release jitters. The running time of the resulting schedulability analysis is much shorter than that of the latest work [7] that uses mixed integer linear programming. The key to our scheduler lies in the classification of the jobs of each task and the assignment of fixed priority to each class of jobs. We will show in Section IV that the proposed job-class-level scheduling can successfully schedule the above taskset example.

Contributions. This paper makes the following contributions:

- We propose a new job-class-level fixed-priority scheduler for sporadic real-time tasks with weakly-hard constraints. Specifically, our scheduler is based on the *meet-oriented classification* of jobs of tasks, which effectively reduces the worst-case temporal interference imposed on other weakly-hard tasks.
- We present a schedulability analysis framework for weakly-hard tasks under our scheduler. It consists of two steps: (1) analyzing the response time of a job with each job-class, which is done by an extension of the existing task-level analysis, and (2) finding possible job-class patterns of a task, which is solved by constructing a reachability tree based on the response time of individual job-classes.
- We find that the schedulability analysis of tasks with $\frac{m}{K} \geq 0.5$ can be tested by using a single sufficient condition, without exploring all possible execution patterns. In other words, the time complexity of analyzing such tasks is much smaller than that for tasks with $\frac{m}{K} < 0.5$. We show this property with analytical and empirical results.
- We propose a semi-partitioned task allocation scheme for multicore weakly-hard systems under the proposed scheduler. The allocation is done in a job-class granularity; hence, the jobs that belong to the same job-class execute on the same processor core. We demonstrate that the proposed scheme yields a significant benefit in schedulability compared to conventional task-level partitioned scheduling.
- We prove that our proposed job-class-level fixed priority scheduler is a generalization of task-level fixed-priority scheduling. We also show that our scheduling framework can be used to upper bound the number of consecutive deadline misses.
- Experimental results demonstrate that the proposed scheduler outperforms the prior work [7], [12] in both task schedulability and analysis running time. In addition, we have implemented our scheduler in the Linux kernel running on Raspberry Pi with acceptably-small runtime overhead.

This paper is an extended version of our prior work [1]. The new contributions include: (i) improved priority assignment to address the schedulability issues of tasks with $\frac{m}{K} < 0.5$ constraints, (ii) semi-partitioned scheduling to enable multicore weakly-hard systems, and (iii) new experimental results and discussions with the improved priority assignment and the multicore support.

Organization. The rest of the paper is organized as follows. Section II discusses prior work on weakly-hard systems. The task model and notation used in this paper are presented in Section III. We then propose job-class-level scheduling in Section IV, and present our schedulability analysis in Section V. In Section VI, our semi-partitioned scheduling in a multicore platform is presented. The evaluation of our work is given in Section VII. Section VIII concludes the paper and discusses future work.

II. Related Work

The notion of (m, K) constraints was first introduced by Hamdaoui and Ramanathan [11]. They focused on reducing the probability of timing violations with dynamic priority assignment, and showed its positive impact in simulation. However, no analytical bound on the number of deadline misses was given. Bernat et al. [5] formally defined weakly-hard real-time systems, and proposed the first work on the schedulability of periodic tasks with weakly-hard constraints under fixed-priority scheduling. Extensions of this schedulability work have been studied, such as for bi-modal execution [10], [13] and non-preemptive tasks [14]. The former ones, however, assume that the initial release offset of each task is statically fixed and exactly known ahead of time, which is not always possible especially in an open system. The latter assumes that all jobs have zero release jitter. Recent work [7] relaxed these assumptions but at the expense of high analysis complexity, e.g., taking more than 10 minutes for 20 tasks on an Intel Xeon 8-core processor, which makes it difficult to use for online admission control in embedded platforms. Goossens [15] presented an exact schedulability test for periodic tasks with zero jitter and offset under distance-based dynamic-priority scheduling. Ismail and Jawawi [16] presented an analysis technique for fixed-priority scheduling in a multicore platform with the same assumptions on initial offset and release jitter. These approaches are done by thoroughly enumerating all task schedules over multiple hyperperiods and checking if there is a repeated feasible schedule. Thus, they can be used for periodic tasks but not for sporadic tasks.

Weakly-hard constraints have also been studied to bound the temporal violations of overloaded systems [17]–[19]. They use typical worst-case analysis (TWCA), which assumes the exact arriving patterns of task instances with occasional overloads are given in the form of arrival curves. Variants of TWCA have been studied for CAN bus analysis [20], tasks with dependencies [6], tasks with varying execution time [21], budget assignment [8]. While these TWCA-based approaches made significant contributions to weakly-hard systems, the precise identification and description of task arrival patterns is much harder than the use of periodic [4] or sporadic task models [22], as discussed in [7].

Besides schedulability, preserving control stability has been studied in the context of weakly-hard systems. Blind and Allgöwer [23] used weakly-hard constraints to capture the failure of unstable feedback control systems in a deterministic way. In [24], a state-based methodoldy is presented to analyze the performance of a control application using weakly-hard constraints. Frehse et al. [25] analyzed the closed-loop properties of control software based on TWCA. In [13], periodic task instances are classified into mandatory and optional ones based on (m, K) constraints, and only the mandatory ones are guaranteed to complete in time. Gaid et al. [26] and Marti et al. [27] extended this work to consider optional instances and non-periodic execution, respectively. Huang et al. presented an approach to formally verify the safety of weakly-hard systems in [28], and further improved its applicability in [29]. The (m, k) model has been further investigated for control-schedule co-design [30]–[33].

The recent work of Lee and Shin [34] focused on bounding consecutive deadline misses of periodic tasks for cyber-physical systems. They classified each job of tasks based on the number of consecutive deadline misses happened just before its arrival, and associated this number with control stability. While this job-level classification has inspired our work, there are two major differences. First, our work focuses on the (m, K) model, which is a superset of the model used in [34]. Specifically, x consecutive deadline misses are a special case of the weakly-hard constraint $(x, x+1)$ in the (m, K) form and thus can be safely bounded by our work. Second, our work uses a meet-oriented classification which will be detailed in Section IV.

III. System Model

This paper considers a multicore system where all processor cores run at the same fixed clock frequency. The system executes a taskset consisting of N periodic or sporadic real-time tasks with constrained deadlines.

Task model. Each task τ_i is characterized as follows:

$$\tau_i := (C_i, D_i, T_i, (m_i, K_i))$$

- C_i: The worst-case execution time of each job of a task τ_i.
- D_i: The relative deadline of each job of τ_i ($D_i \leq T_i$).
- T_i: The minimum inter-arrival time between consecutive jobs of τ_i. If τ_i is a periodic task, T_i is the period of τ_i.
- (m_i, K_i): The weakly-hard constraints of τ_i ($m_i < K_i$). If τ_i is a hard real-time task, $m_i = 0$ and $K_i = 1$.

Each task τ_i can have a non-zero initial release offset o_i. A release jitter, $\mathcal{J}_i$ ($\mathcal{J}_i \leq D_i - C_i$), represents the maximum time interval between the requested activation and the real-released time of an instance of a task τ_i. The j-th job of a task τ_i is denoted as $J_{i,j}$.

Utilization. To represent the resource usage and the effective performance of a weakly-hard system, we define a set of utilization metrics below.

Def. 1. *The maximum utilization of a task τ_i, U_i^M, is the maximum amount of CPU resource that τ_i can utilize. It is defined as $U_i^M = \frac{C_i}{T_i}$, which is in fact the same as the common task utilization. The maximum total utilization is defined as* *the sum of the maximum utilization of all tasks, i.e.,* $U^M = \sum_{i=1}^{N} \frac{C_i}{T_i}$*, where* N *is the number of tasks.*

Note that the maximum utilization U_i^M is the value that τ_i can achieve when it always meets the deadline.

Def. 2. *The minimum utilization of a task* τ_i*,* U_i^m*, is the CPU resource used by* τ_i *when it experiences the maximum deadline misses allowed by its* (m_i, K_i) *constraint, i.e.,* $U_i^m = \frac{C_i}{T_i} \times \frac{K_i - m_i}{K_i}$*. The minimum total utilization is defined as* $U^m = \sum_{i=1}^{N} \frac{C_i}{T_i} \times \frac{K_i - m_i}{K_i}$.

Each task requires at least U_i^m of CPU resource to be schedulable w.r.t. the weakly-hard constraint.

Deadline-missed Jobs. If a job of a task misses its deadline, there are two approaches to handle this job: (i) letting it continue to execute beyond the deadline, and (ii) dropping (descheduling) it immediately. If the output of a job has some usefulness even after the deadline, the first approach can be considered better than the second one [35]. Otherwise, the second approach is more appealing as it can prevent the deadline-missed job from blocking its next job and unnecessarily wasting CPU cycles. Therefore, this paper uses the second approach and shows that it can be implemented on embedded platforms with small overhead. Note that job dropping has also been used in other real-time contexts, e.g., shared resource access [36], [37] and mixed-criticality systems [38], [39].

IV. Job-class-level Fixed-Priority Scheduling

Unlike task-level fixed-priority scheduling, our work classifies the jobs of each task into *job-classes* and assigns priorities to individual job-classes. With this approach, a task can have as many priority levels as the number of job-classes it has, and the priority of each job is determined by the priority of its corresponding job-class. For the ease of presentation, we will assume a uniprocessor system in this section. This assumption will be relaxed in Section VI by introducing our semi-partitioned multicore scheduling approach.

A. Meet-oriented job classification

Bernat et al. [5] discussed that weakly-hard constraints can be categorized into four types based on the following criteria: consecutive vs. any order, and met vs. missed deadlines. In line with this idea, one may consider the following four classification approaches for a job $J_{i,j}$ based on the execution results of its prior jobs: the number of previous deadlines (i) met, (ii) missed, (iii) consecutively met, and (iv) consecutively missed. In this paper, we specifically use a *meet-oriented classification* to define a job-class.

Def. 3. *A job-class* J_i^q *includes a job whose nearest previous jobs have consecutively met q deadlines, where q has the range of* $[0,\ K_i - m_i]$ *and* $m_i \geq 1$.

For ease of explanation, we are intentionally being vague about the meaning of "nearest" previous jobs at this point and will refine it later in this section using Def. 6. The superscript of J_i^q is referred to as the index of that job-class. Due to the range of job-class indices, any job that follows more than $K_i - m_i$ consecutively-met deadlines belongs to a job-class $J_i^{K_i - m_i}$. If $m_i = 0$, meaning that no deadline miss is allowed, i.e., a hard real-time task, the number of job-classes for that task is always one. Note that a job-class is determined by the number of *nearest* deadlines consecutively met. If a job follows two distinct intervals of q and q' consecutively-met deadlines and q is the more recent one, this job gets the job-class index of q. More precisely, given the k-th job of a task τ_i, $J_{i,k}$, its nearest previous jobs with q consecutively met deadlines are $J_{i,x\ldots y}$, where $x \leq y < k$ and there is no other job $J_{i,z} : y < z < k$ that has met the deadline.

Each job-class J_i^q is assigned with a fixed priority, which is denoted as π_i^q. Since the job-class index of a job indicates the number of consecutive deadlines met just before its arrival, the higher index likely means the less urgent the job is (w.r.t. weakly-hard constraints). Therefore, we propose that the priority of a job-class decreases monotonically as a job-class index increases. For instance, a task τ_1 with $(m_1, K_1) = (2, 4)$ can have three job-classes, J_1^0, J_1^1, and J_1^2, with their priorities of 6, 4, and 2, respectively. More details on job-class priority assignment will be given in Section IV-B.

To better represent a sequence of job execution results, we introduce the following two patterns: μ*-pattern* and $\mathcal{C}$*-pattern*.

Def. 4 ([5], [11]). *A* μ*-pattern represents a sequence of deadline met and missed jobs of a task. For example,* `MMmMmMM`*, where* `M` *and* `m` *represent a met and a missed deadline, respectively.*

Def. 5. *A* $\mathcal{C}$*-pattern represents job-class indices of a sequence of jobs released by a task with a weakly-hard constraint. For example,* 0120101*, where each digit means a job-class index used.*[3].

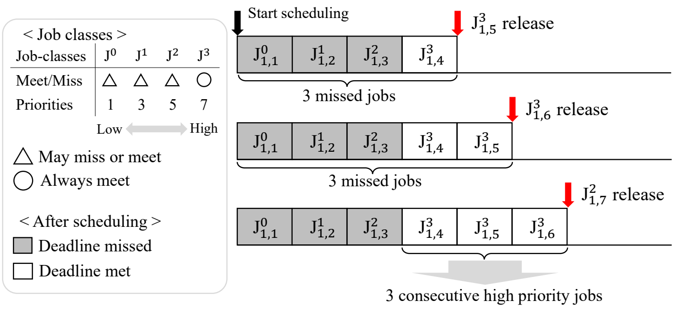


Fig. 3. Consecutive execution of high-priority jobs under *miss-oriented* job classification.

Benefits of meet-oriented classification. The rationale behind the meet-oriented job classification is that it can reduce the worst-case interference imposed on lower-priority jobs by modulating the execution of jobs with high priority. For comparison, let us consider the opposite approach where a job-class is defined by the number of deadline misses. In this *miss-oriented* approach, a job-class with a higher index means more deadlines missed previously and thus gets a higher priority. Fig. 3 shows an example of a task τ_1 with $(m_1, K_1) = (3, 6)$

[3]If there is a job-class index greater than 9, one can use a delimiter between each index, e.g., $0.1 \cdots 11.12.13$.

under the miss-oriented approach. On the left side of the figure, a circle indicates a job-class that always meets the deadline due to its high priority, and a triangle indicates a job-class that does not guarantee meeting the deadline. The task τ_1 has 4 job-classes and the highest-index job-class, J^3, has the highest priority.

The right side of Fig. 3 illustrates a possible scheduling result of the task τ_1. Assume that the first three jobs of τ_i, $J^0_{1,1}$, $J^1_{1,2}$ and $J^2_{1,3}$, miss the deadline due to the low priorities of their job-classes. Then, the 4th, 5th and 6th jobs of τ_1 get the highest job-class index, i.e., $J^3_{1,4}$, $J^3_{1,5}$ and $J^3_{1,6}$, because they have three missed deadlines in the window of $K_1 = 6$ prior jobs. This results in three consecutive execution of the highest-priority jobs of τ_1, thereby resulting in consecutive interference to the lower-priority jobs of other tasks. Under the miss-oriented classification, it is very hard (or may be impossible) to avoid such interference. On the other hand, in our *meet-oriented* approach, such consecutive execution of the highest-priority jobs is effectively prevented, because once the highest-priority job meets the deadline, its next job will be assigned a job-class with a lower priority. Hence, the time interval between highest-priority jobs becomes longer and other tasks likely experience less interference during their job execution.

Bounding consecutive deadline misses. The above Def. 3 does not specify a *distance* from the current job to the nearest previous deadline-met jobs. If we limit this distance to zero, only immediately-previous jobs will be checked. For example, for a job-class J^q_i, there will be no deadline miss allowed between the current job and the window of q prior jobs; if a job $J_{i,j}$ misses its deadline, the job-class index of its next job $J_{i,j+1}$ will be immediately demoted to zero, i.e., $J^0_{i,j+1}$. If we do not limit the distance, an unbounded number of consecutive deadline misses will be allowed at each job-class. For example, if a job of τ_i gets a job-class of J^q_i and its subsequent jobs continuously miss the deadline, they all will belong to the same job-class. Therefore, we define a *miss threshold* to limit the distance and bound the number of consecutive deadline misses at each job-class.

Def. 6. *A miss threshold w_i is the maximum number of consecutive deadline misses that a task τ_i can experience at any job-class J^q_i with $q > 0$. If a job of τ_i follows w_i consecutive deadline misses, this job is assigned the lowest-index job-class J^0_i. The value of w_i is given by:*

$$w_i = \max\left(\left\lfloor \frac{K_i}{K_i - m_i} \right\rfloor - 1, 1\right)$$

The reasoning behind this w_i value is to ensure that a task τ_i can have an enough number of jobs running with τ_i's highest-priority job-class, J^0_i. By the definition of the miss threshold, w_i cannot be smaller than 1, and as w_i goes larger, it takes more periods for τ_i to regain J^0_i. The way w_i is determined allows τ_i to run at least $K_i - m_i$ jobs with J^0_i (denominator in the equation) during K_i consecutive invocations (numerator) when the other m_i jobs all miss their deadlines.

Example. A task τ_i with a weakly-hard constraint $(m_i, K_i) = (5, 7)$ is assigned 3 job-classes and a miss threshold size of $w_i = 2$, based on Defs. 3 and 6. Suppose that the very first two jobs of τ_i have met their deadlines (τ_i's μ-pattern is `MM`). As a result, the job-class index of the 3rd job becomes 2, i.e., $J^2_{i,3}$. If $J^2_{i,3}$ misses the deadline (`MMm`), the 4th job continues to get $J^2_{i,4}$ as the number of consecutive misses is less than the miss threshold ($w_i = 2$). If $J^2_{i,4}$ also misses the deadline (`MMmm`), the task τ_i has reached the miss threshold and the 5th job of τ_i is assigned $J^0_{i,5}$. On the other hand, if $J^2_{i,4}$ meets the deadline (`MMmM`), the 5th job is assigned $J^1_{i,5}$ because the number of nearest consecutively-met deadlines is 1 (as given by Def. 3).

Relations to other scheduling approaches. Our job-class-level fixed-priority scheduling model is a generalization of the conventional task-level fixed-priority scheduling [4] and can represent the temporal constraints of the CFP scheduling model [34][4] that upper bounds the number of consecutive deadline misses of periodic tasks.

Lemma 1. *The job-class-level fixed-priority scheduling subsumes the task-level fixed-priority scheduling.*

Proof. If we assign the same priority to all jobs of τ_i (e.g., $\forall q : 0 < q \leq K_i - m_i, \pi^q_i = \pi^0_i$), the job-class-level scheduling can yield the same task schedule as the task-level fixed-priority scheduling. □

Lemma 2. *The task model of the job-class-level fixed-priority scheduling subsumes that of the CFP scheduling [34].*

Proof. The task model of the CFP scheduling uses a single parameter m' for each task, which means at most m' consecutive deadline misses are allowed for that task. If the (m, K) constraint of a task under the job-class-level scheduling is set to $(m', m'+1)$, it represents the maximum of m' deadline misses permitted in any $m'+1$ consecutive periods, which captures the case for m' consecutive deadline misses. Therefore, the job-class-level scheduling can represent any constraint imposed by the CFP scheduling model. □

B. Priority assignment to job-classes

Priority assignment can affect the overall performance of the job-class-level scheduler. An optimal priority assignment can be found by a brute-force method, but due to its extremely high computational complexity, it is not practically usable. In this paper, we propose two heuristic priority assignment methods: low-index job-class first with miss thresholds (LIF-w), and low-index job-class first with a priority holding mechanism (LIF-h). Both methods ensure that the priority of a job-class in each task decreases monotonically with increasing job-class index. We will compare the schedulability performance of the two proposed methods in Section VII.

The proposed methods, given in Alg. 1 (LIF-w) and Alg. 2 (LIF-h), are extensions of the deadline-monotonic (DM) priority assignment policy [40]. The algorithms first check the schedulability of a given taskset Γ under the task-level DM

[4]CFP stands for Cyber subsystem's state-level Fixed Priority (scheduling).

Algorithm 1 Job-class priority assignment (LIF-w)

```
Input: $\Gamma$: Taskset
1: $N \leftarrow |\Gamma|$
2: Sort $\tau_i$ in $\Gamma$ in ascending order of deadline
3: for all $\tau_i \in \Gamma$ do
4:     $l_i \leftarrow K_i - m_i + 1$        ▷ $l_i$: number of job-classes for $\tau_i$
5: end for
6: $prio \leftarrow \sum_{\tau_i \in \Gamma} l_i$        ▷ Priority to be assigned next
7: if $\Gamma$ is schedulable by DM then
8:     for all $\tau_i \in \Gamma$ do
9:         ▷ Assign the same priority to all job-classes of $\tau_i$
10:        for all $q \leftarrow 0$ to $l_i - 1$ do
11:            $\pi_i^q \leftarrow prio$
12:        end for
13:        $prio \leftarrow prio - 1$
14:    end for
15: else
16:    $L \leftarrow \max_{\tau_i \in \Gamma} l_i$
17:    for $q \leftarrow 0$ to $L - 1$ do
18:        if $q > 0$ then
19:            Sort $\tau_i \in \Gamma$ in ascending order of $w_i$ and deadline
20:        end if
21:        for all $\tau_i \in \Gamma$ do
22:            if $q < l_i$ then        ▷ Check if $q$ is a valid index
23:                $\pi_i^q \leftarrow prio$
24:                $prio \leftarrow prio - 1$
25:            end if
26:        end for
27:    end for
28: end if
```

assignment (line 7 of Alg. 1), which can be done by the conventional iterative response-time test [41]. If the taskset is schedulable by DM, the algorithms simply follow the task-level DM assignment. Hence, a task with a shorter deadline is assigned a higher priority and all job-classes of each task get the same priority.

Lemma 3. *The proposed job-class-level priority assignment algorithms, LIF-w and LIF-h, subsume the task-level DM priority assignment.*

Proof. Obvious as shown by lines 8-14 of Alg. 1. □

LIF-w. LIF-w assigns different priority to individual job-classes of tasks when the taskset is not schedulable by DM. By Def. 3, the lowest-index job-class of each task implies that there exists no previous job that has met the deadline; thus, the algorithm first assigns priority to the job-classes with the lowest index in ascending order of deadline. Then, for job-classes with higher indices, those with a lower miss threshold w are assigned higher priority because they can tolerate fewer consecutive deadline misses (see Def. 6). The main part of the LIF-w method consists of two-level iterations. The outer loop (line 17) iterates over job-class indices from 0 to L, where L is the maximum number of job-classes for each task. The inner loop (line 21) iterates over all tasks in ascending order of deadlines when $q = 0$ (sorted in line 2) and in ascending order of miss thresholds (w_i) with deadlines for tie-breaking when $q > 0$ (line 19). Hence, the job-class priorities assigned by the algorithm have the following properties: if $D_i < D_j$, $\pi_i^0 > \pi_j^0$, and if $w_i < w_j$, $\pi_i^q > \pi_j^q$ for $q > 0$.

Algorithm 2 Job-class priority assignment (LIF-h)

```
Input: $\Gamma$: Taskset
1: Assign priority to all job-classes by calling Alg. 1 (LIF-$w$).
2: if $\Gamma$ is schedulable then
3:     return
4: else
5:     $\forall i : idx_i \leftarrow 0$        ▷ $idx_i$ is a current job-class index of $\tau_i$.
6:     for all $\tau_i \in \Gamma$ do
7:         $l_i \leftarrow K_i - m_i + 1$        ▷ $l_i$: number of job-classes for $\tau_i$
8:         $h_i \leftarrow \lceil \frac{K_i - m_i}{m_i} \rceil$        ▷ $h_i$: priority holding value of $\tau_i$
9:         while $idx_i < l_i$ do
10:            $\pi_i^q \leftarrow \pi_i^{idx_i}$, $\forall q : idx_i \leq q < \min\{idx_i + h_i, l_i\}$
11:            $idx_i \leftarrow idx_i + h_i$
12:        end while
13:    end for
14: end if
```

Under the LIF-w priority assignment, a task cannot continue to use the same high priority for successive jobs. This is because, according to Def. 3, if a job has met its deadline, the priority of the next released job will be demoted by increasing the job-class index. In terms of schedulability, this leads to performance degradation for tasks with weakly-hard constraints of $m_i/K_i < 0.5$. As an example, let us consider a task τ_i with a weakly-hard constraint $(m_i, K_i) = (2, 7)$ where $m_i/K_i < 0.5$ and $w_i = 1$. It is assumed that only the lowest-index job-class J_i^0, i.e., the highest priority job-class, is guaranteed to meet the deadline. As no more than 2 deadlines can be missed in 7 periods, this task requires two or more successive jobs to execute with the highest-priority job-class. Based on Def. 3 and Def. 6, however, τ_i cannot satisfy this requirement. The μ-pattern of `MmMmMmM` is the sequence that τ_i can ensure under LIF-w, so the task is unschedulable.

LIF-h. To overcome the above limitation, LIF-h permits a certain number of low-index job-classes to share the priority of a high-index job-class. We first introduce a priority holding value, h_i, which is the number of successive job-classes holding the same priority within a task. From a schedulability perspective, h_i can be determined by the minimum number of consecutive jobs that are required to meet the deadline, in order for the task τ_i to satisfy the given weakly-hard constraint (m_i, K_i). Hence, we define h_i as follows:

$$h_i = \left\lceil \frac{K_i - m_i}{m_i} \right\rceil \tag{1}$$

The value of h_i is always 1 for a task with the weakly-hard constraint of $m_i/K_i \geq 0.5$, meaning that the task does not need consecutive high-priority jobs in order to be schedulable. If $m_i/K_i < 0.5$, h_i increases as the weakly-hard constraint permits fewer deadline misses (smaller m_i), thereby allowing more deadline-meeting jobs to execute with high priority.

By leveraging the value of h_i, LIF-h divides the job-classes of a single task into multiple groups, each of which consists of h_i job-classes, and then the first job-class of each group shares its priority with the others in the same group. Therefore, every h_i job-classes of a task τ_i have the same priority. Note that if the number job-classes of a task is not an integer multiple of h_i, the last group consists of job-classes less than h_i.

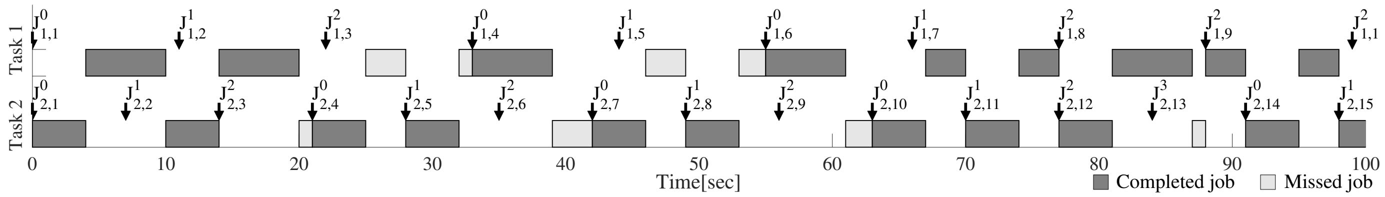


Fig. 4. Execution timeline of weakly-hard tasks under job-class-level scheduling.

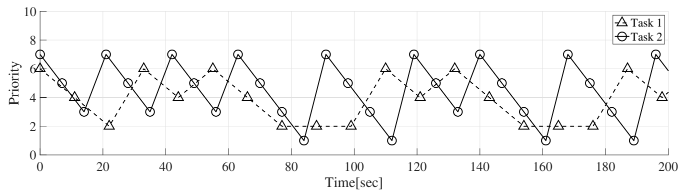


Fig. 5. Priority changes under job-class-level scheduling.

TABLE I
TASK SET I

	Specifications
Task 1	$\tau_1 : (C_1 = 6, T_1 = 11, m_1 = 2, K_1 = 4)$
Task 2	$\tau_2 : (C_2 = 4, T_2 = 7, m_2 = 4, K_2 = 7)$

	Priority
Task 1	$\pi_1^0 = 6, \pi_1^1 = 4, \pi_1^2 = 2$
Task 2	$\pi_2^0 = 7, \pi_2^1 = 5, \pi_2^2 = 3, \pi_2^3 = 1$

Alg. 2 describes the LIF-h priority assignment. As an extension of LIF-w, LIF-h assigns new priority only when the given taskset is unschedulable by LIF-w. In other words, by the time when the primary part of LIF-h begins (line 5), all job-classes of tasks already have priorities assigned by LIF-w. Under LIF-h, a group of h_i successive job-classes from the same task inherits the priority from the first job-class (π^{idx_i}) of that group (line 10). Then, idx_i is updated to indicate the first job-class of the next group (line 11). Hence, h_i successive job-classes starting from the lowest-index job-class of each task τ_i get the same priority.

Note that LIF-w and LIF-h are identical for a task with $w_i = 1$ and $h_i = 1$.

C. Example of job-class-level scheduling

Recall the example taskset of Fig. 2 which is unschedulable by any task-level fixed-priority scheduling, as discussed in Section I. Table I gives the detailed parameters of this taskset. However, the proposed job-class-level scheduling and priority assignment schemes satisfy the weakly-hard constraints of this taskset. Fig. 4 illustrates the execution timeline of the taskset under our scheduler, and Fig. 5 depicts the priority changes of the two tasks of this taskset. As can be seen, each task uses the priority levels of its all job-classes and the relative priority ordering of the tasks changes over time.

V. Schedulability Analysis

This section presents the schedulability analysis of weakly-hard tasks under job-class-level scheduling. Our analysis can be used for uniprocessor systems as well as semi-partitioned multicore systems which will be presented in the next section. The analysis consists of two parts: analyzing the response time of a job with each job-class, and finding job-class patterns that can possibly happen at runtime. We will describe each part and explain how the schedulability test is done.

A. Minimum job-class inter-arrival time

In order to analyze the worst-case response time (WCRT) of a job with a specific job-class, we need to upper bound the maximum interference imposed by the jobs of other tasks with higher-priority job-classes. Such interference can be captured by finding the minimum arrival time between any two jobs of the same job-class. Hence, we begin with analyzing this interval and call it the minimum job-class inter-arrival time.

We first analyze the minimum inter-arrival time of a job-class J_i^q whose index q is the highest index of a task τ_i, i.e., $q = K_i - m_i$.

Lemma 4. *The minimum inter-arrival time of a job-class J_i^q where $q = K_i - m_i$ is given by:*

$$\eta(J_i^q) = 1 \cdot T_i$$

Proof. If the WCRT of J_i^q is less than or equal to its relative deadline D_i, the job-class index of the next released job is always $q+1$. However, since q is the highest index of τ_i, the next released job maintains the current index even if the job meets its deadline. Thus, the minimum time for τ_i to regain $J_i^{K_i - m_i}$ is $1 \cdot T_i$. If the WCRT of J_i^q is greater than D_i, the job may or may not meet the deadline when it is scheduled at runtime. If the job meets the deadline, the minimum time to regain $J_i^{K_i-m_i}$ is the same as the case when the WCRT $\leq D_i$. If the job misses the deadline, the next job may get J_i^0, which causes longer time to regain J_i^q. Therefore, in the worst case, $\eta(J_i^q)$ is $1 \cdot T_i$. □

Lemma 5. *The minimum inter-arrival time of J_i^q where $q < K_i - m_i$ and the WCRT of J_i^q is greater than D_i is given by:*

$$\eta(J_i^q) = \begin{cases} (q+1) \cdot T_i & , \text{if } w_i = 1 \\ 1 \cdot T_i & , \text{if } w_i > 1 \end{cases}$$

Proof. The proof is done by contradiction. Assume that when $w_i = 1$, the minimum inter-arrival time of J_i^q is less than $(q+1) \cdot T_i$. Since the WCRT of J_i^q is greater than D_i, the job-class index q' of the next released job can be either 0 or $q+1$ at runtime. If $q' = 0$, by Def. 3, at least q subsequent jobs

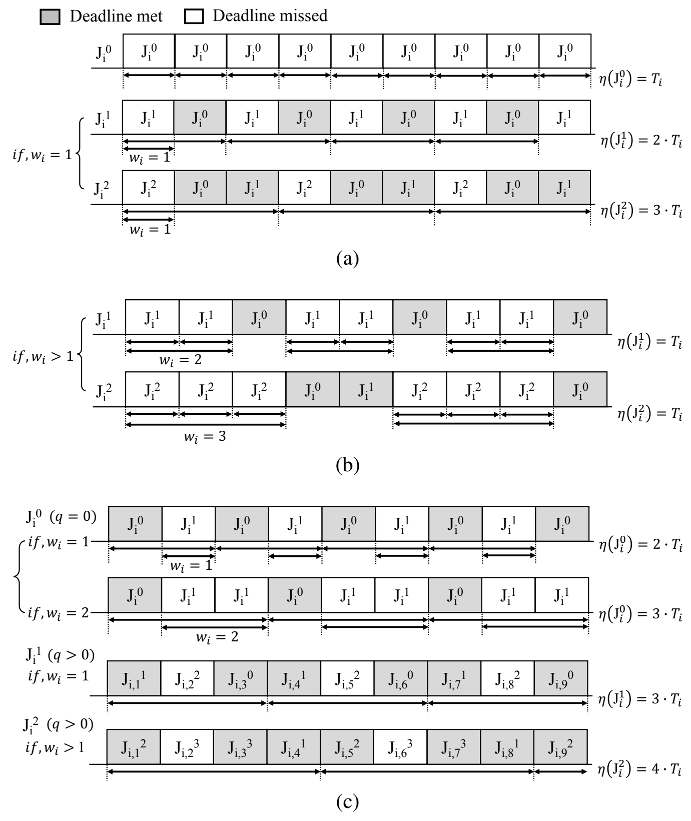


Fig. 6. The minimum time interval between any two jobs of the same job-class. (a) WCRT $> D_i$ and $w_i = 1$. (b) WCRT $> D_i$ and $w_i > 1$. (c) WCRT $\leq D_i$.

should meet their deadlines in order to get the job-class index of q again, giving $(q+1) \cdot T_i$ as the minimum inter-arrival time. If $q' = q + 1$, it requires at least $q + 1$ subsequent jobs (1 miss and q meets) to regain the job-class index q, resulting in $(q+2) \cdot T_i$. These contradict the assumption. Hence, the inter-arrival time of J_i^q is greater than or equal to $(q+1) \cdot T_i$ when $w_i = 1$. When $w_i > 1$, jobs can continue to have J_i^q as long as w_i permits, and thus the minimum inter-arrival time of J_i^q is $1 \cdot T_i$. The examples of these two cases, $w_i = 1$ and $w_i > 1$, are illustrated in Fig. 6a and 6b, respectively. □

Lemma 6. *The minimum inter-arrival time of J_i^q where $q < K_i - m_i$ and the WCRT of J_i^q is less than or equal to D_i is:*

$$\eta(J_i^q) = \begin{cases} (w_i + 1) \cdot T_i & \textit{, if } q = 0 \\ (q+2) \cdot T_i & \textit{, if } q > 0 \end{cases}$$

Proof. If $q = 0$, the proof is done by contradiction. Assume that the minimum inter-arrival time of $J_i^{q=0}$ is less than $(w_i + 1) \cdot T_i$. Since the job-class index q' of the next job is always $q + 1 = 1$ by Def. 3 ($\because$ WCRT of $J_i^q \leq D_i$), the inter-arrival time of J_i^0 is at least $2 \cdot T_i$. This contradicts the assumption because $w_i \geq 1$ by Def. 6. Therefore, the inter-arrival time of $J_i^{q=0}$ is greater than or equal to $(w_i + 1) \cdot T_i$. If $q > 0$, the minimum time for τ_i to get a job-class J_i^0 is $2 \cdot T_i$ because the next job $J_{i,k+1}$ always gets the job-class index of $q+1$ by Def. 3 while $q < K_i - m_i$ (the condition of this lemma), and only the second next job $J_{i,k+2}$ may be able to get the index 0. Then, at least $q \cdot T_i$ is required for τ_i to get the job-class index q. Therefore, the minimum inter-arrival time of J_i^q for $q > 0$ is $(q+2) \cdot T_i$. Fig. 6c shows the examples for both cases. □

B. Worst-case response time of job-classes

In order to capture the worst-case temporal interference from other tasks, we exploit the notion of the minimum job-class inter-arrival time given by Lemmas 4, 5 and 6.

Lemma 7. *The worst-case temporal interference imposed on a job-class J_i^q by the higher-priority jobs of another task τ_k during an arbitrary time window t, which starts with the release of J_i^q, is upper-bounded by:*

$$W_i^q(t, \tau_k) = \begin{cases} 0 & \textit{, if } \nexists p : \pi_i^q < \pi_k^p \\ \min\Big(\displaystyle\sum_{\substack{\forall p:\pi_i^q<\pi_k^p, \\ P(J_k^p)=P(J_i^q)}} \left\lceil \frac{t+\mathcal{J}_k}{\eta(J_k^p)} \right\rceil \cdot C_k, \left\lceil \frac{t+\mathcal{J}_k}{T_k} \right\rceil \cdot C_k \Big) & \textit{, o.w.} \end{cases} \tag{2}$$

where $\mathcal{J}_k$ is a release jitter of τ_k and $P(J_i^q)$ is a processor that J_i^q is assigned.

Proof. If τ_k does not have any job-class with a priority level higher π_i^q, it obviously will not cause any interference to J_i^q. This is captured by the first condition of Eq. (2). Otherwise, Eq. (2) computes the interference from the higher-priority jobs of τ_k by extending the conventional iterative response time test [41]. Instead of using a task-level period (T_k) in the ceiling function, we use the minimum inter-arrival time of any two jobs of a job-class ($\eta(J_k^p)$) because a job with the priority of J_k^p cannot repeat more often than $1/\eta(J_k^p)$. With a jitter, interference from a high-priority job is increased because a job can be released earlier by the amount of a jitter [42]. This is exactly captured by the first part of the min term. The amount of interference from τ_k can be also bounded by using its period T_k, which is the same as the task-level analysis [41]. Hence, W_i^q can be safely upper-bounded by the minimum of these two approaches. □

Theorem 1. *The worst-case response time of J_i^q, denoted by R_i^n, is bounded by the following recurrence:*

$$R_i^{q,n+1} \leftarrow C_i + \sum_{\tau_k \in \Gamma - \tau_i} W_i^q(R_i^{q,n}, \tau_k) \tag{3}$$

where Γ is the entire taskset. The recurrence starts with $R_i^{q,0} = C_i$ and terminates when $R_i^{q,n} + \mathcal{J}_i > D_i$ or $R_i^{q,n+1} = R_i^{q,n}$.

Proof. Obvious from Lemma 7. □

Lemma 8. *The job-class-level response time test for weakly-hard tasks given in Eq.* (3) *is a generalization of the task-level iterative response time test for hard real-time tasks [41].*

Proof. Any hard real-time task τ_k has only one job-class (J_k^0). Thus, the minimum inter-arrival of this job-class is

Algorithm 3 WCRT for all job-classes of a taskset Γ

```
Input: $\Gamma$: Taskset
 1: procedure WCRT($\Gamma$)
 2:     $\digamma \leftarrow$ all job-classes of a taskset $\Gamma$
 3:     ▷ $\pi_i^q$ is a priority of a job-class index $q$ of task $\tau_i$
 4:     ▷ $P(J_i^q)$ is a processor where $J_i^q$ is assigned
 5:     for all job-classes $\in \digamma$ in descending order of priority do
 6:         $i \leftarrow$ a task index of a job-class in $\digamma$
 7:         $q \leftarrow$ a job-class index of a task $\tau_i$
 8:         $R_i^q \leftarrow C_i$
 9:         while $R_i^{q,n+1} > R_i^{q,n}$ do
10:             $W_i^q \leftarrow 0$
11:             for $k = 1$ to $N$ do                    ▷ Check all tasks.
12:                 $v \leftarrow 0$
13:                 if $k \neq i$ then
14:                     for $p = 0$ to $K_k - m_k$ do
15:                         if $\pi_k^p > \pi_i^q$ and $P(J_k^p) = P(J_i^q)$ then
16:                             if WCRT of $J_k^p \leq D_k$ then
17:                                 if $p == 0$ then
18:                                     $\eta(J_k^p) \leftarrow (w_k + 1) \cdot T_k$
19:                                 else if $q > 0$ then
20:                                     $\eta(J_k^p) \leftarrow (p + 2) \cdot T_k$
21:                                 end if
22:                             else
23:                                 if $w_k == 1$ then
24:                                     $\eta(J_k^p) \leftarrow (p + 1) \cdot T_k$
25:                                 else if $w_k > 1$ then
26:                                     $\eta(J_k^p) \leftarrow 1 \cdot T_k$
27:                                 end if
28:                             end if
29:                             if $p == K_k - m_k$ then
30:                                 $\eta(J_k^p) \leftarrow 1 \cdot T_k$
31:                             end if
32:                             $v \leftarrow v + \left\lceil \frac{R_i^{q,n} + \mathcal{J}_k}{\eta(J_k^p)} \right\rceil \times C_k$
33:                         end if
34:                     end for
35:                 end if
36:                 $W_i^q \leftarrow W_i^q + min\Big(v, \left\lceil \frac{R_i^{q,n} + \mathcal{J}_k}{T_k} \right\rceil \times C_k\Big)$
37:             end for
38:             $R_i^{q,n+1} \leftarrow C_i + W_i^q$
39:             $n \leftarrow n + 1$
40:         end while
41:         WCRT of $J_i^q \leftarrow R_i^{q,n+1} + \mathcal{J}_i$
42:     end for
43:     WCRT of all job-classes in $\digamma$
44: end procedure
```

$\eta(J_k^p) = T_k$ and there is only priority level for τ_k. Then, the first part of the min term of Eq. (2) is reduced to $\lceil (t + \mathcal{J}_k)/(T_k) \rceil \times C_k$, which is the same as the second part, and expanding Eq. (3) with this reduced W_i^q gives the same analysis as the conventional response time test. ☐

Based on Eqs. (2) and (3), one can compute the WCRT of the job-classes of all tasks, in descending order of job-class priority. This is because the analysis needs the WCRT of higher-priority job-classes to get their inter-arrival time. The detailed procedure for doing this can be found in Alg. 3. All job-classes are sorted in descending order of their priorities in line 4 so that the $\eta(J_k^p)$ of any higher-priority job-classes assigned to the same processor as J_i^q can be considered by the WCRT calculation of the *inner* while loop (from lines 9 to 40). The worst-case interference W_i^q is found by Lemma 7 in line 36 and the WCRT of a job-class J_i^q is computed by Theorem 1 in line 41.

C. Schedulability test for tasks with weakly-hard constraints

After checking the WCRT of all job-classes, now we can test the schedulability of individual tasks w.r.t. weakly-hard constraints. Note that if a task τ_i has no job-class J_i^q with $R_i^q \leq D_i$, the task cannot be said schedulable in our job-class-level analysis framework.

Lemma 9 (Prerequisite for our schedulability analysis)**.** *For a task τ_i to be schedulable by our job-class-level schedulability analysis, the WCRT of the lowest-indexed job-class (J_i^0) should be less than or equal to its deadline D_i.*

Proof. Suppose that the WCRT of the lowest-indexed job-class J_i^0 is greater than its deadline. Then the WCRT of other job-classes (e.g., J_i^1 and J_i^2) of τ_i is always greater than the deadline because J_i^0 has the highest priority in τ_i. Thus, τ_i cannot be guaranteed to be schedulable by our analysis. ☐

If a task does not satisfy the above prerequisite, it is immediately considered unschedulable by our analysis. If it satisfies, we next check the m/K ratio of the task, which greatly reduces the time complexity of schedulability analysis.

Lemma 10. *A task τ_i is always schedulable if the ratio of m_i/K_i is greater than or equal to 0.5 and it satisfies the prerequisite given by Lemma 9.*

Proof. Recall the definition of a miss threshold w_i. If τ_i misses w_i deadlines consecutively, the next job of τ_i is assigned the lowest-index job-class J_i^0 whose WCRT is $\leq D_i$ (guaranteed by the prerequisite). This means that, even if τ_i's all other job-classes have WCRT $> D_i$, τ_i can have at least 1 deadline met every $w_i + 1$ periods. Based on this property, the proof can be done in two steps as follows.

Step 1. We prove that there are one or more occurrences of the $w_i + 1$ periods within the K_i window, by showing $\exists \alpha \in \mathbb{Z}^+$ that satisfies the following equation:

$$(w_i + 1) \cdot \alpha \leq K_i \tag{4}$$

By Def. 6, $w_i + 1$ can be substituted by $\left\lfloor \frac{K_i}{K_i - m_i} \right\rfloor$ as $m_i/K_i \geq 0.5$. With $m_i \leq K_i - 1$, the upper bound of the left-hand side is $\lfloor K_i \rfloor \cdot \alpha$. The inequality $\lfloor K_i \rfloor \cdot \alpha \leq K_i$ is always true for α, and thus it is proved.

Step 2. The inequality $(w_i + 1) \cdot \alpha \leq K_i$ means that there are at least α deadlines met in the K_i window. Hence, if we prove that α is greater than or eqaul to $K_i - m_i$ in the K_i window, the task is always schedulable as long as the condition of Lemma 9 is satisfied. Hence, we prove:

$$\frac{1}{w_i + 1} \geq \frac{K_i - m_i}{K_i} \tag{5}$$

Since $w_i = \left\lfloor \frac{K_i}{K_i - m_i} \right\rfloor - 1$ for $m_i/K_i \geq 0.5$ by Def. 6, Eq. (5) is rewritten as follows:

$$\frac{1}{\left\lfloor \frac{K_i}{K_i - m_i} \right\rfloor} \geq \frac{K_i - m_i}{K_i} \tag{6}$$

$$\Leftrightarrow \left\lfloor \frac{K_i}{K_i - m_i} \right\rfloor \le \frac{K_i}{K_i - m_i}$$

This inequality is always true as $m_i \le K_i - 1$, and thus completes the proof. □

When the ratio of m_i/K_i is less than 0.5, we check all possible μ-patterns that can happen at runtime. Thus, a reachability tree to be introduced next assumes that the ratio of $m_i/K_i < 0.5$, which also means $w_i = 1$. Moreover, since a single consecutive missed job is allowed, we find the exact upper-bound of the complexity of each tree, which is detailed in Section V-E

D. Reachability trees

A reachability tree consists of a *node*, which indicates a job-class, and a *branch*, which represents deadline missed or met of the node, as depicted in Fig. 7. Two types of nodes exist based on the WCRT. If the WCRT $\le D_i$, the node has a single meet branch, e.g., nodes 1 and 4 in Fig. 7. Otherwise, a node has two branches (miss and meet), e.g., nodes 2, 3, and 5. Thus, the proposed tree model is generated by the following two Lemmas.

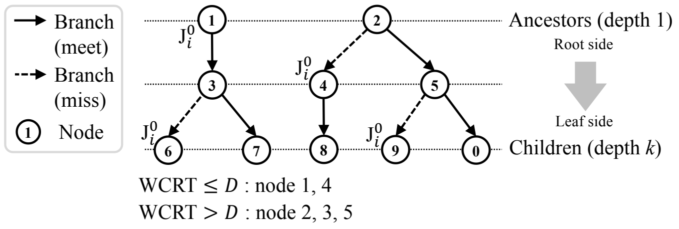


Fig. 7. A reachability tree model.

Lemma 11. *If a node is from its parent's miss branch, the node always generates a single meet branch.*

Proof. Since a miss threshold $w_i = 1$, the node from a miss branch always belongs to the lowest-indexed job-class J_i^0. By Lemma 9, the WCRT of the node is less than the deadline. Thus, it only generates a meet branch. □

Lemma 12. *If a node is from its parent's meet branch, this node generates two sub-branches, miss and meet, in the worst case.*

Proof. When a node is generated by a meet branch, two cases exist based on the WCRT of its parent node: its parent's WCRT $\le D_i$ and WCRT $> D_i$. In the former case, the node may generate one or two sub-branches based on its own WCRT, as depicted node 3 in Fig. 7. The node always generates two sub-branches when its parent's WCRT $> D_i$ because the WCRT of its parent node, which has a higher priority, is greater than the deadline (e.g., node 5 in Fig. 7). □

Note that the number of reachability trees of a task is equal to the number of job-classes of the task. This is because $w_i = 1$ and there are no other cases that a task can take as initial conditions. Each tree has the following properties.

1) There exist $K_i - m_i + 1$ trees for a task and the root node of each tree represents a different job-classes of the task.
2) Each tree has K_i depth in order to check the satisfiability of the (m_i, K_i) constraint of a task τ_i.
3) Each node has a μ-pattern, which indicates a series of deadline met or missed jobs from the root to its parent node.
4) The leaf nodes have $\mathcal{C}$-patterns that represent the indices of job-classes for the K_i execution window of a task.

Hence, the proposed reachability tree model of a task generates all possible job-class patterns that happens at runtime, as shown by the following lemma.

Lemma 13. *The reachability trees of a task τ_i represent all possible job-class patterns that the task can experience at its runtime for any execution window of K_i jobs.*

Proof. By the property 1) of the reachability tree, the trees cover all the starting job-classes that the task has. Each node creates all possible cases, i.e., deadline met and missed jobs, by Lemmas 11 and 12. Besides, since individual trees have K_i-depth as stated in property 2), our reachability trees represent all possible job-class patterns for K_i job executions. □

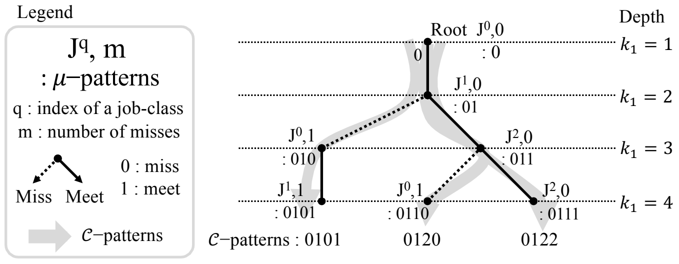


Fig. 8. Examples of a reachability tree.

Fig. 8 illustrates an example of a reachability tree from a taskset in Table I. In Fig. 4, we observe that the indices of the first four jobs of Task 1 are $J_{1,1}^0$, $J_{1,2}^1$, $J_{1,3}^2$, and $J_{1,4}^0$ respectively, which is depicted as a $\mathcal{C}$-pattern 0120 in Fig. 8.

Theorem 2. ***Schedulability.*** *A task is guaranteed to be schedulable if the μ-patterns at all leaf nodes in its reachability trees satisfy the weakly-hard constraint.*

Proof. The proof can be done by contradiction. Suppose that all μ-patterns of a task satisfy the constraints, but the task is unschedulable. Since the reachability trees of a task represent all possible job-class patterns by Lemma 13, this means that there exist other job-class patterns that make the task unschedulable. This, however, contradicts the Lemma 13, thus completing the proof. □

E. Complexity of a reachability tree

The proposed reachability tree model has a bounded computational complexity and is faster than other weakly-hard schedulability methods, which will be shown in the evaluation section. Moreover, as can be seen in Figs. 7 and 8, the number of nodes from the root to leaf nodes forms the Fibonacci sequence.

Theorem 3. *In a reachability tree, the upper-bound on the number of nodes follows the Fibonacci sequence, which starts from the root to the leaf node.*

Proof. Let us denote the number of nodes at depth i as f_i, e.g., $f_1 = 1$. In a reachability tree, there are two cases to consider for the root node at depth 1.

First, suppose that the root node has WCRT $\leq D$. At depth 2, $f_2 = 1$ because the root node generates a single meet sub-branch to depth 2. At depth 3, $f_3 = 2$ because, by Lemma 12, the node at depth 2 is from the meet branch and creates two sub-branches to depth 3. At depth 4, f_4 can be obtained considering the two nodes at depth 3: the one from the meet branch of depth 2 generates two sub-branches to depth 4 by Lemma 12 (analogous to the case for f_3), and the other from the miss branch generates one sub-branch by Lemma 11 (analogous to f_2). So $f_4 = f_3 + f_2 = 3$. At depth 5, $f_5 = f_4 + f_3 = 5$ because, among the three nodes of depth 4, two are from the meet and miss branches (the case considered when calculating f_4) and one is from the meet branch (the case considered for f_3). This pattern continues as the depth increases, and f_i is bounded by the sum of the number of its parents and grandparents, i.e., $f_{i+2} = f_{i+1} + f_i$, following the Fibonacci sequence.

Second, if the root node has WCRT $> D$, it generates two sub-branches to depth 2, i.e., $f_2 = 2$. The number of nodes at each subsequent depth follows the same pattern as the above, e.g., $f_3 = f_2 + f_1 = 3$, $f_4 = f_3 + f_2 = 5$, etc. Therefore, the Fibonacci sequence also holds for this case and the proof is done. □

Moreover, for a task, the total complexity of reachabiity trees can be bounded as follows.

Theorem 4. *The complexity of computing all the reachability trees of a task τ_i, O_i, is upper-bounded by*

$$O_i \leq (K_i - m_i + 1) \times \frac{\rho^{K_i+1} - (1-\rho)^{K_i+1}}{\sqrt{5}} \qquad (7)$$

where $\rho = \frac{1+\sqrt{5}}{2}$, which is the golden ratio, and $K_i - m_i + 1$ is the number job-classes.

Proof. By Theorem 3, each reachability tree follows the Fibonacci sequence, where the total number of nodes is bounded by $\frac{\rho^{K_i+1}-(1-\rho)^{K_i+1}}{\sqrt{5}}$, as already proved in [43]. Since one reachability tree is created for one distinct job-class, there are at most $K_i - m_i + 1$ trees for τ_i, and thus Eq. 7 holds. □

VI. Job-class-level Scheduling in Multicore Systems

The problem of task scheduling in multicore systems is typically solved by the following three strategies which are classified by how tasks are assigned to processor cores [44]–[46]: partitioned, semi-partitioned, and global scheduling. In this work, we propose a semi-partitioned task allocation scheme for a multicore platform using our job-class-level scheduler. Our scheme statically allocates individual job-classes of tasks to processor cores. It is semi-partitioned in the sense that: (i) a set of jobs belonging to the same job-class always executes on the same core and does not migrate to another core at runtime, and (ii) another set of jobs belonging to the different job-classes of the same task may execute on different cores.

Algorithm 4 Semi-partitioned weakly-hard task allocation

```
Input: $\Gamma$: Taskset, $P$: Number of CPUs
 1: function ASSIGNMENT($\Gamma$, $P$)
 2:     $\digamma \leftarrow$ all job-classes of a taskset $\Gamma$
 3:     $P \leftarrow$ set of all available processor cores
 4:     for each $J_i^q \in \digamma$ in descending order of priority do
 5:         alloc $\leftarrow$ false
 6:         for all $p \in P$ do
 7:             Allocate $J_i^q$ to $p$ and compute the response time $R_i^q$
 8:             if $R_i^q \leq D_i$ then
 9:                 alloc $\leftarrow$ true
10:                 break
11:             else
12:                 deallocate $J_i^q$ from $p$
13:             end if
14:         end for
15:         if alloc = false then
16:             $U_{min} \leftarrow \infty$
17:             for all $p \in P$ do
18:                 $U_p \leftarrow \sum_{J_j^{q'} \in p} \frac{C_j}{\eta(J_j^{q'})}$
19:                 if $U_p < U_{min}$ then
20:                     $U_{min} \leftarrow U_p$
21:                     Allocate $J_i^q$ to $p$
22:                 end if
23:             end for
24:         end if
25:     end for
26: end function
```

The proposed semi-partitioned allocation scheme has two major benefits. First, our uniprocessor schedulability analysis presented in Section V can be directly used on a per-core basis. Second, it distributes job-classes to processor cores in a way to schedule as many job-classes as possible based on their computed worst-case response time. We will compare the schedulability results of our proposed semi-partitioned scheme and the conventional partitioned scheduling approaches based on bin-packing heuristics, e.g., worst-fit decreasing (WFD), in Section VII.

Alg. 4 details the procedure of the proposed allocation scheme for all job-classes of a taskset Γ. First, all job-classes are sorted in descending order of their priorities in line 4. Each job-class J_i^q is assigned to a core where it is schedulable, i.e., the WCRT of J_i^q is less than or equal to D_i (lines 6 to 14). If a job-class cannot find a core where it is schedulable, it is assigned to a core with the lowest total job-class utilization U_p (lines 17 to 23). Here, the job-class utilization means the share of the CPU time required for the execution of a job-class. Thus, the utilization of a job-class J_i^q can be estimated by the WCET C_i divided by the minimum job-class arrival time $\eta(J_i^q)$ which is given in Section V-A. The total job-class utilization U_p is then computed by summing the utilization of all job-classes assigned to the processor core p (line 18). It is worth noting that the algorithm uses U_p as a proxy to measure the current load of the core.

VII. Evaluation

We first check the runtime overhead of the proposed job-class-level scheduler by using a prototype implementation in the Linux kernel. We then perform schedulability experiments to compare it with other existing approaches and to explore its performance characteristics in various scenarios.

A. Implementation cost

We have implemented the proposed scheduler in the Linux kernel v4.9.80 running on a Raspberry Pi 3 equipped with a quad-core ARM Cortex-A53 processor. The implementation largely consists of two parts: updating task priority and handling deadline-missed jobs. To update task priority, the scheduler first keeps track of individual job executions and updates the μ-pattern. It then determines the job-class index of a newly released job based on the μ-pattern, and finally sets the task's priority according this job-class. If a job misses the deadline, the scheduler drops this job immediately by stopping its execution, in accordance to our system mode, and rolls the task's state back to the previous clean state. Such a *rollback* is required for the correct operation of the next job as the previous job might have been dropped before releasing a lock or finishing memory writes.

In our implementation, we used a user-level checkpointing technique for the task rollback mechanism. This technique performs the following three steps: 1) creating a checkpoint of a task, 2) notifying a deadline miss from the kernel to the user space, and 3) recovering from the checkpoint. For step 1, a task calls `sigsetjmp` to store its status at the beginning of each job execution. For step 2, the kernel sends a signal to the task when its deadline is missed and the task implements the corresponding signal handler. For step 3, the task's signal handler calls `siglongjmp` to restore the status. If the task has other resources to be restored, such as lock releases, relevant code can be added to the signal handler before calling `siglongjmp`.

Overhead measurement. We measured the runtime overhead of our scheduler implementation. To minimize measurement errors, dynamic frequency scaling was disabled and the processor was configured to use its maximum clock frequency, 1.2 GHz. The overhead was measured by running a taskset consisting of five tasks with periods of 20 ms to 40 ms, and a total of 118,569 jobs were generated during 10 minutes of running time.

TABLE II
Runtime overhead [μs]

Type		Mean	Max	Min	99%th
Updating μ-pattern		0.3002	1.1460	0.1040	0.6250
Updating job-class index		1.5035	11.8750	0.5210	2.5000
Changing task priority		4.7633	28.9580	3.0210	11.3020
Rollback	Checkpointing	1.9413	9.3230	1.2500	3.2290
	Recovery	6.1257	24.8430	0.4680	8.3146

Table II reports the measurement results. In the rollback mechanism, checkpointing is the time to execute `sigsetjmp` and recovery is the time from the kernel sending a signal to the completion of the user-level signal handler. Changing priority and recovery are the two most costly operations. The sum of all entries in each column indicates the total amount of overhead imposed on each job invocation. Since the maximum total overhead per job is observed to be much less than 100 μs, we conclude that the runtime overhead of our scheduler is acceptably small on commodity embedded platforms like Raspberry Pi.

B. Uniprocessor experiments

This subsection is organized in two parts. The first part presents a comparative evaluation with the two other weakly-hard scheduling schemes [7], [12], and the second part examines the detailed behavior of our job-class-level scheduler.

Taskset generation. We use 1,000 randomly-generated weakly-hard tasksets for each experimental setting, e.g., each point on the x-axis of figures. For each taskset, task utilization is obtained by the UUniFast algorithm [47]. Task period is chosen randomly in $[10, 1000]$ ms. Task deadline is set equal to the period, i.e., $T_i = D_i$. Unless otherwise mentioned, release jitter is set to 0. Motivated by a recent study [48] that empirically discovered reasonable weakly-hard constraints from a practical application, the K value is selected from the set of $\{5, 10, 15\}$.

Comparison of schedulability tests. We compare our work with the two other existing approaches. The first one is the offset-free weakly-hard schedulability analysis for fixed-priority scheduling [7]. Although it uses a different method from ours to handle deadline-missed jobs, i.e., jobs continue to execute even after the deadline, we chose this work because it is the latest study on weakly-hard scheduling. The second one is the Red-Task-Only version of the skip-over algorithm [12], and it was chosen as it drops deadline-missed jobs, same as our work. Note that, although [34] has inspired our work, we do not consider it in the evaluation because it is not developed for weakly-hard systems and cannot handle tasks with (m, K) constraints. In summary, the following three methods are compared:

- **JCLS**: the proposed reachability tree analysis under Job-Class-Level Scheduler with the LIF-h priority assignment (our work).
- **WSA**: the Weakly-hard Schedulability Analysis for offset-free periodic tasks [7].
- **RTO-RM**: the Red-Task-Only approach for periodic tasks with RM priority assignment [12].

We used the source code of WSA provided in [49] by the authors of [7] and our own implementation of RTO-RM. For our experimental conditions to be consistent with [7], all tasks in the same taskset are set to use a common (m, K) constraint. This is because the currently available WSA source code does not support testing a taskset with various (m_i, K_i) constraints, but we will examine such cases for JCLS in the later part of this subsection. The weakly-hard constraint K for each taskset is set to 10 and m is chosen randomly in the range of $[1, 9]$. Following these rules, we generated 1,000 tasksets with 20 tasks each at each level of the total maximum utilization U^M.

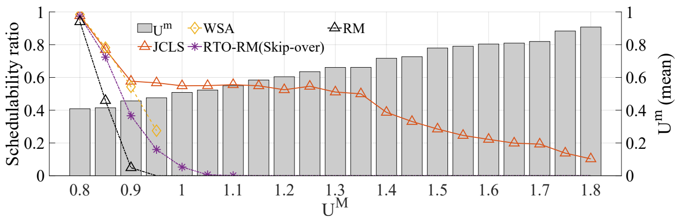


Fig. 9. Schedulability under JCLS, WSA, and RTO-RM.

Fig. 9 shows the ratio of weakly-hard schedulable tasksets under the three approaches. The schedulability results under hard RM scheduling is also shown for comparison purpose. At $U^M = 0.95$, JCLS schedules 56% of tasksets while WSA and RTO-RM schedule only 26% and 16% of tasksets, respectively. WSA and RTO-RM do not dominate each other. When $U^M < 1$, WSA outperforms RTO-RM, but when $U^M \geq 1$, WSA cannot find any schedulable taskset while RTO-RM still schedules some tasksets. JCLS shows much higher schedulability ratios than WSA and RTO-RM, e.g., even at $U^M = 1.8$, JCLS schedules 11% of tasksets. This result demonstrates that our JCLS scheduler utilizes the CPU resource more efficiently than the other two prior approaches and the benefit is significant especially when the system is overloaded.

Comparison of analysis running time. In this experiment, we evaluate the analysis running time of JCLS and WSA, which is the time to determine the schedulability of a given taskset. It is obviously affected by the number of tasks in the taskset. We thus consider three cases, where the number of tasks per taskset is 10, 30, and 50, respectively, and generate 1,000 tasksets for each case. The weakly-hard constraint K is set to 10 and m is randomly selected from $[1, 9]$. The analysis running time of JCLS is measured on Raspberry Pi 3 running at 1.2 GHz. On the other hand, WSA is measured on an Intel Core-i7 system running at 2.3 GHz because the CPLEX Optimizer required by the WSA program could not be installed on Raspberry Pi.

TABLE III
ANALYSIS RUNNING TIME OF JCLS AND WSA [SEC]

Number of tasks	Approach	Mean	Max
10	JCLS	0.0010	0.0046
	WSA	0.2739	114.2892
30	JCLS	0.0112	0.0432
	WSA	25.7284	1800.5996
50	JCLS	0.0331	0.1463
	WSA	78.5982	3002.5189

Table III shows the mean and maximum running time of JCLS and WSA. Although JCLS is measured on a much resource-constrained platform, its analysis time is significantly shorter than that of WSA. We observed that the analysis time of JCLS becomes even shorter when it runs on the same x86 platform. Therefore, we conclude that our schedulability analysis using reachability trees is much faster than WSA and is applicable to runtime admission control.

Various (m_i, K_i) constraints. We now use various (m_i, K_i) constraints for tasks in each taskset. Since the current implementation of WSA does not support this, we will limit our focus to JCLS. Furthermore, we conduct performance evaluation of JCLS using the two proposed priority assignment methods, i.e., LIF-w and LIF-h, for all subsequent experiments.

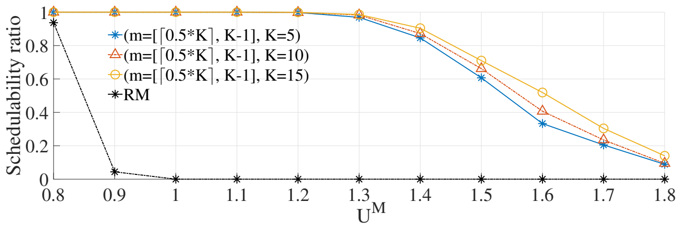


Fig. 10. Schedulability results with different K_i values.

We first evaluate the impact of the K_i parameter on weakly-hard schedulability in Fig. 10. Each generated taskset has 20 tasks. Three different K_i values are considered under JCLS: 5, 10, and 15. The range of m_i used is $[\lceil 0.5 \times K_i \rceil, K_i - 1]$, and each task's m_i is randomly chosen in this range. Hence, the average ratio of m_i/K_i is similar in all the three K_i cases. Since this condition gives the priority holding value of $h_i = 1$, the results from LIF-w and LIF-h are idential and thus shown using the same line. As can be seen in the figure, the ratio of schedulable tasksets slightly increases with K_i under JCLS. This is due to that a larger K_i can give more chances for tasks to satisfy their weakly-hard constraints.

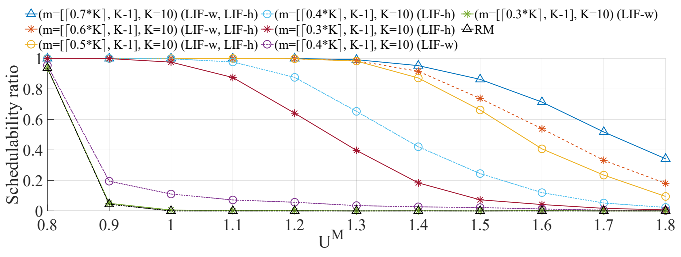


Fig. 11. Schedulability results with different m_i/K_i ratios.

We then investigate in Fig. 11 the impact of the m_i/K_i ratio under JCLS, by using different ranges of m_i for a fixed K_i value. The schedulability of JCLS decreases as m_i/K_i reduces. Specifically, for tasks with $m_i/K_i < 0.5$, we observe that the schedulability of JCLS with LIF-w drops drastically. On the other hand, JCLS with LIF-h gives much higher schedulability than LIF-w, because LIF-h allows more job-classes of such tasks to execute with high priority.

Bimodal tasksets with $m_i/K_i < 0.5$ tasks. In order to better understand the schedulability characteristics of JCLS for tasks with $m_i/K_i < 0.5$, we use bimodal tasksets consisting of light and heavy tasks. The utilization ranges for light and heavy tasks are $[0.01, 0.15]$ and $[0.2, 0.4]$, respectively. Either heavy tasks or light tasks are assigned $m_i/K_i < 0.5$ and the other $m_i/K_i \geq 0.5$ depending on experimental settings. For each taskset generation, light and heavy tasks are generated according to their given percentages until the taskset's total maximum utilization exceeds the target U^M, and then the

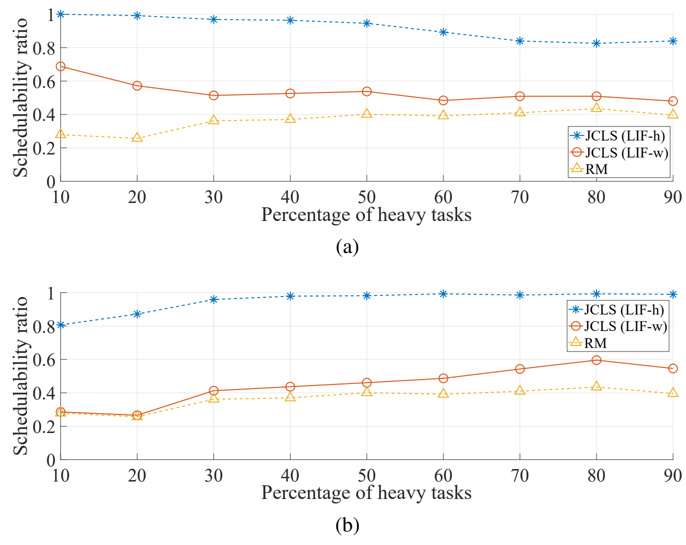


Fig. 12. Schedulability results with bimodal tasksets. (a) $m_i/K_i < 0.5$ for heavy tasks. (b) $m_i/K_i < 0.5$ for light tasks.

last task's utilization is reduced to meet U^M. The (m_i, K_i) constraints are set to $(4, 10)$ for the $m_i/K_i < 0.5$ case and $(9, 10)$ for the other case. The target maximum total utilization U^M is set to 0.9. Fig. 12 shows the schedulability results of bimodal tasksets under JCLS (LIF-w and LIF-h) and RM as the percentage of heavy tasks increases. As expected, we can observe that the difference between JCLS and RM reduces as the percentage of tasks with $m_i/K_i < 0.5$ increases, but in both heavy- and light-task cases, JCLS with LIF-h yields better results than JCLS with LIF-w and RM.

Varying m_i for a fixed (m_i, K_i) constraint. We also conduct experiments with bimodal tasksets by varying the m_i parameter for a fixed (m_i, K_i) constraint. Fig. 13 shows the results of this experiment. The percentages of light tasks and heavy tasks are 80% and 20%, respectively, and the m_i parameter of heavy tasks is varied, as shown in the legend of Fig. 13a and on the x-axis of Fig. 13b and 13c. Other task parameters remain the same as before. Similar to the trends observed in previous experiments, the schedulability decreases as m_i gets smaller. LIF-h outperforms LIF-w for tasks with the weakly-hard constraint of $m_i/K_i < 0.5$. In particular, for tasksets with m_i is 4 and 2 at $U^M = 0.95$, LIF-h improves schedulability ratio over LIF-w by 59% and 89% points, respectively.

Release jitters. One of the advantages of the proposed analysis is that it can analyze tasks with non-zero release jitters. In this experiment, we check the schedulability characteristics of JCLS in the presence of jitters that are proportional to the period of each task, e.g., 1% and 5% of T_i. Tasksets are generated in the same way as in Fig. 10, so there is no difference between LIF-w and LIF-h. As can be seen in Fig. 14, schedulability decreases slightly as the amount of jitter increases but there is no drastic reduction. This result demonstrates that our approach can be applied to realistic applications where release jitters exist.

Consecutive deadline misses. As discussed in Section IV, our work can safely bound the number of consecutively missed deadlines, m_i, by modeling $K_i = m_i + 1$ in the (m_i, K_i)

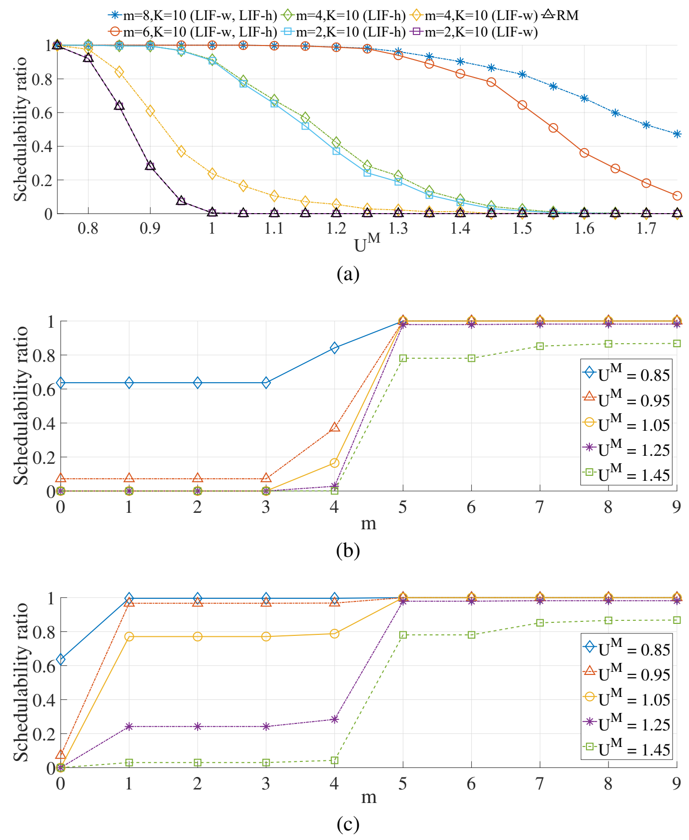


Fig. 13. Schedulability results with different m_i values. (a) Results as U^M increases. (b) Results of JCLS LIF-w as m increases. (c) Results of JCLS LIF-h as m increases.

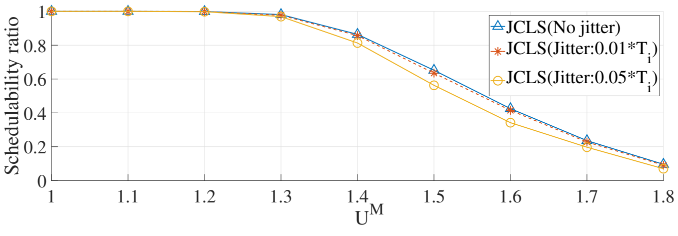


Fig. 14. Schedulability results with release jitters.

form. Tasksets are generated in the same way as in Fig. 10, except for the (m_i, K_i) parameters. Fig. 15 shows that the schedulability of tasksets slightly increases with a larger K_i value. This trend is similar to that in Fig. 10, but one difference here is that the ratio of m_i/K_i reduces as K_i increase, which helps improve schedulability.

C. Multicore experiments

We now evaluate the schedulability of JCLS in multicore systems. In this subsection, the following three task assignment methods are compared under the JCLS framework:

- **WFD-U**: a partitioning method based on worst-fit-decreasing (WFD) using the utilization (U_i) of each task.
- **WFD-U^m**: a partitioning method based WFD using the minimum utilization (U_i^m) of each task.
- **SPM-J**: a semi-partitioning method based on the WCRT of each job-class (Alg. 4).

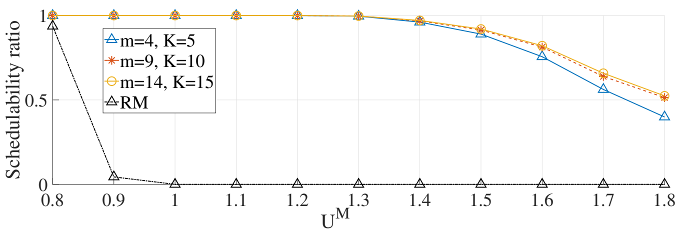


Fig. 15. Schedulability results for consecutive deadline misses

Basically, all jobs of each task execute on the same processor under the two partitioned methods (WFD-$\boldsymbol{U}$ and WFD-$\boldsymbol{U^m}$) while SPM-J assigns job-classes of each task to different processors as described in Alg. 4. It is worth noting that WFD is chosen for load balancing across processor cores, as done in the literature [50].

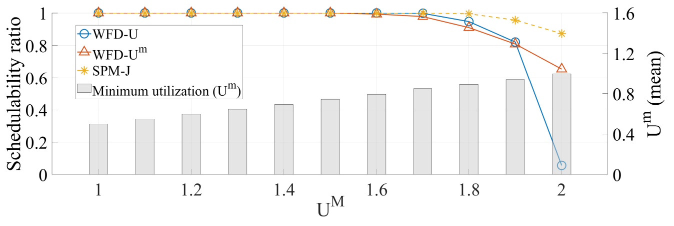


Fig. 16. Schedulability in multicore systems (2 CPUs).

Fig. 16 shows the schedulability results under WFD-U, WFD-U^m, and SPM-J when 2 processors are used. We generated tasksets over the range of the total maximum utilization U^M from 1 to 2 and other task parameters remain the same as before. As can be seen, SPM-J outperforms the others. WFD-U and WFD-U^m do not dominate each other. When $U^M < 2$, WFD-U has slightly higher schedulability ratio than WFD-U^m. When $U^M = 2$, however, we observe that the schedulability of WFD-U drops drastically. This is because there exist tasks that cannot be assigned to any processor due to the pessimism of U_i in representing the actual resource demand of weakly-hard tasks.

Number of processors. The results of schedulability ratio according to the given number of processors are shown in Fig. 18. We generated $1,000$ tasksets with 30 tasks with the maximum utilization $U^M = 8$. The number of processors is chosen as an integer over the range $[6, 12]$. As expected, we can observe that the schedulability ratio increases in all three methods as the number of processors increases. SPM-J dominates the others for any number of processors. In particular, when the number of processors is 8, SPM-J schedules 88% tasksets while WFD-U and WFD-U^m schedule only 0% and 5% of tasksets, respectively.

Schedulability of bimodal tasksets in multicore systems. Lastly, we use bimodal tasksets consisting of light and heavy tasks. We generated bimodal tasksets in the same way as done for the previous experiment of Fig. 12, except that the target maximum total utilization U^M is set to 8. The number of processors is chosen from the set of $\{8, 10\}$.

Fig. 17 shows the schedulability results depending on task assignment methods. Similar to the trends observed in the previous experiments, the schedulability ratio decreases as the percentage of tasks with $m_i/K_i < 0.5$ increases. WFD-U and WFD-U^m do not dominate each other. We can observe that the schedulability of WFD-U is overall higher than WFD-U^m in the heavy-task case (Fig. 17a and 17b). On the other hand, WFD-U^m yields better results than WFD-U in the light-task case (Fig. 17c and 17d). Interestingly, WFD-U^m does not exhibit uniform behavior as shown in Fig. 17a and 17b. This is because the minimum utilization U_i^m used by WFD-U^m is not a stable metric compared to U_i which is widely used for conventional task allocation approaches. However, SPM-J consistently yields higher schedulability than the others in both light- and heavy-task cases. These results demonstrate the effectiveness of our approach for multicore weakly-hard real-time systems.

VIII. Conclusion

In this paper, we propose a job-class-level fixed-priority scheduling and a schedulability analysis framework for weakly-hard real-time systems. Our scheduler employs a meet-oriented classification of jobs of tasks and supports the scheduling of periodic and sporadic tasks with arbitrary initial offsets and release jitters. Besides, we present two heuristic job-level priority assignment methods that are practically usable. The schedulability analysis framework for our proposed scheduler consists of two steps: analysis of the worst case response time for individual job-classes, and finding all possible scheduling patterns using the reachability trees. This framework is directly used for task scheduling in multicore platforms by the proposed semi-partitioned scheduling approach. Our scheduler has been implemented in the Linux kernel on Raspberry Pi with acceptably small runtime overhead. Evaluation results have demonstrated that the proposed scheduler and its schedulability analysis outperforms prior work with respect to the taskset schedulability and the analytical complexity. It has been also shown that tasksets with the maximum utilization higher than 1 is schedulable under our scheduler. The schedulability characteristics of two proposed priority assignment methods are explored through experiments of tasksets with various (m_i, K_i) constraints. Furthermore, we have observed that the proposed semi-partitioned scheme yields significant benefit over conventional approaches in multicore scheduling. For future work, we are interested in investigating the impact of shared memory contention in order to further enhance the practical usability of multicore weakly-hard real-time systems.

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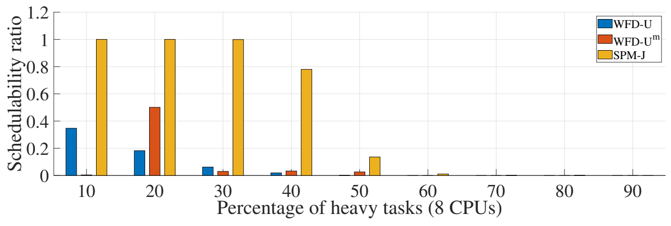
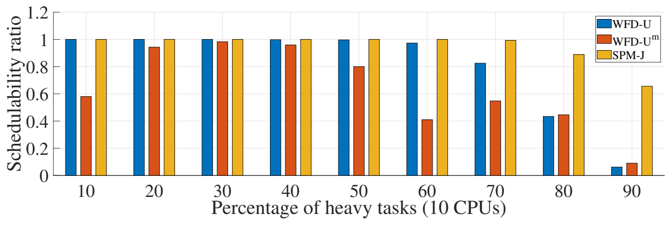


(a)



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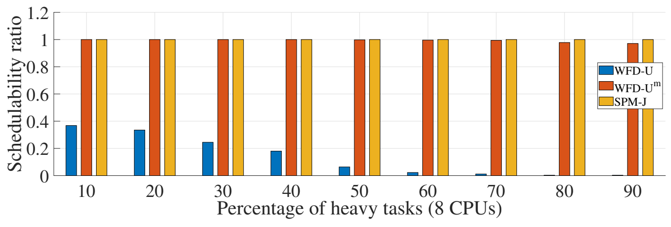
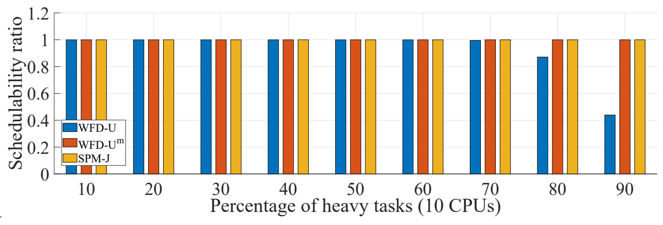


(c)



(d)

Fig. 17. Schedulability results with bimodal tasksets in multicore systems. (a) $m_i/K_i < 0.5$ for heavy tasks (8 CPUs). (b) $m_i/K_i < 0.5$ for heavy tasks (10 CPUs). (c) $m_i/K_i < 0.5$ for light tasks (8 CPUs). (d) $m_i/K_i < 0.5$ for light tasks (10 CPUs).

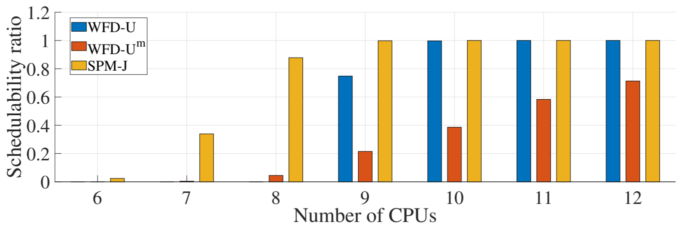


Fig. 18. Schedulability results of tasksets with $U^M = 8$ in multicore systems.

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