

# A Stochastic Programming Framework for Optimal Storage Bidding in Energy and Reserve Markets

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**Abstract**—This paper focuses on a scenario where a group of independently-operated investor-owned storage units seek to offer both energy and reserve in the day-ahead as well as the hour-ahead markets. We are particularly interested in the case when a significant portion of the power generated in the grid is from wind and other intermittent renewable energy sources. In this regard, we formulate a stochastic programming framework to choose optimal energy and reserve bids for the storage units. Our design takes into account the fluctuating nature of the market prices due to the randomness in the renewable power generation availability. We show that the formulated stochastic program can be converted into a convex optimization problem and therefore it can be solved efficiently. Our simulation results show that our design can assure profitability of the private investment on storage units. In particular, our design results in much higher profit compared to a similar but deterministic design that simply uses the expected values of the price parameters.

**Keywords:** Independent storage systems, energy and reserve markets, wind power integration, stochastic optimization

## I. INTRODUCTION

Due to their *intermittency* and *inter-temporal variations*, the integration of renewable energy sources is proven to be very challenging [1], [2]. A recent study in [3] has shown that significant wind power curtailment may become inevitable if more wind generation capacities are installed without improving the existing infrastructure or using energy storage. Other studies, e.g., in [4], [5], have similarly suggested that energy storage systems (ESS) have a great potential to help better integrate renewable energy resources, in particular large-scale wind farms. However, it is still not clear *how* an ESS, which is likely to be owned and operated by a private investor, can participate in a deregulated electricity market.

The existing literature on integrating energy storage systems into smart grid is diverse. One thread of research, e.g. in [6]–[8], has addressed fulfilling objectives such as increasing the overall power system reliability, reducing carbon emissions, and minimizing the total cost of power generation in the system. While seeking these *social* objectives, the papers along this line of research do *not* see the storage units as independent entities and they rather assume that the operation of energy storage systems is governed by a centralized controller. Another thread of research, e.g., in [9]–[12], control the operation of a storage unit when it is combined with a wind farm. They essentially assume that it will be the owners of a wind farm who will pay and invest in adding storage units to a power

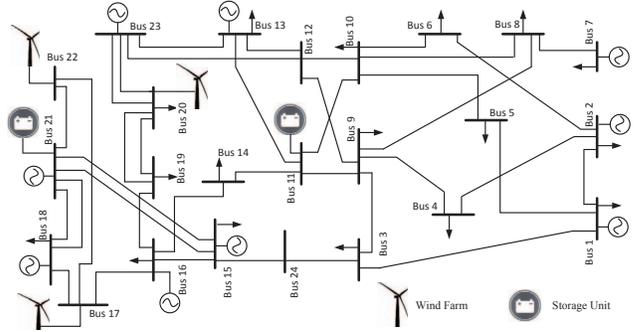


Fig. 1. The IEEE 24 bus test system with independent storage units at buses 11 and 21. There are also three wind farms at buses 17, 20, and 22.

grid. However, this assumption may not always hold and it can limit the opportunities to attract investment to build new energy storage systems. Finally, there are some papers, such as [12]–[16], that aim to select optimal strategies for certain storage technologies, e.g., pumped hydro storage units, to bid in the electricity market. However, most of them do not address the following two key issues. First, they do not account for the uncertainties in the market prices which can be a major decision factor if the amount of renewable power generation in the system is significant. Second, they do not consider the opportunities for the energy storage systems to participate not only in energy markets but also in reserve markets.

In this paper we would like to answer the following question: *How can an energy storage unit that, is owned by a private investor and is operated independently, bid in both energy and reserve markets to maximize its profit, when there exists significant wind power penetration in the power grid?* An example for such system is shown in Fig.1. We can see that the storage units may or may not be co-located with the renewable or traditional generators. To reach the optimal operation of the storage unit, we propose a stochastic optimization-based bidding mechanism for both energy and reserve in both day-ahead and hour-ahead markets. Since the exact utilization of the reserve bids is not decided by the storage unit, as it is rather decided by the market, optimal charging and discharging planning is particularly challenging to offer reserve capacities. Another key challenge is to formulate the bidding optimization problem as a convex program in order to make it tractable and therefore a good option for practical implementations. Our contributions in this paper can be summarized as follows.

- We formulate a new stochastic optimization-based bidding mechanism for *independent* storage systems in the day-ahead and the hour-ahead *energy and reserve* markets. Our design operates the charging and discharging cycles of batteries such that meeting the future reserve commitments are guaranteed, regardless of the uncertainties that are present in the decision making process.
- In our proposed energy and reserve market participation framework, the power grid does *not* treat independent storage units differently from other energy and reserve market participants. Therefore, our design can be used to encourage large-scale integration of energy storage resources without the need for restructuring the market.
- Our computer simulation show that our proposed stochastic optimization-based energy and reserve bidding mechanism is beneficial to independent storage units as it assures profit gain as a result of their investment. Moreover, our design results in much higher profit compared to a similar but deterministic design that simply uses the expected values of the price and other system parameters.

The rest of this paper is organized as follows. The system model and the formulation of our proposed bidding mechanism based on stochastic optimization are explained in Section II. Simulation results are presented in Section III. The concluding remarks and future work are discussed in Section IV.

## II. OPTIMAL ENERGY AND RESERVE BIDDING

Consider a power grid, e.g., as in Fig. 1, with several traditional and renewable power generators as well as multiple independent energy storage systems. We assume that some generators as well as some storage units can bid and participate in the deregulated electricity market. As pointed out in Section I, our key assumption is that the storage units are *not* treated any differently from the generators that participate in the energy or reserve markets. Since the energy storage units are owned and operated by private entities, they seek to maximize their own profit. In presence of high wind power penetration in the system, the independent storage units need to particularly take into account the stochastic nature of wind power injection and its impact on fluctuations in the energy and reserve market prices when they plan their bids. Next, we formulate a bidding optimization problem in form of a stochastic program. We assume that the storage unit operates as a price taker and its operation does not have impact on market prices, due to its typically lower size compared to traditional generators

Let  $P_h$  denote the power delivered or consumed during hour  $h$  by the storage unit, for each  $h = 1, \dots, 24$ . A positive  $P_h$  indicates charging while a negative  $P_h$  indicates discharging. Also, let  $R_h \geq 0$  denote the reserve commitment of the storage unit in hour  $h$  of the next day. Before we see more details on the problem formulation, it is worth to emphasize that the storage unit's bid in the day-ahead market has direct impact on the storage unit's future profit in the hour-ahead market. This is because the commitments in the day-ahead market will put some constraints in the charging and discharging cycles of the storage unit when it later participates in the hour-ahead markets. An example for the charging and discharging cycles

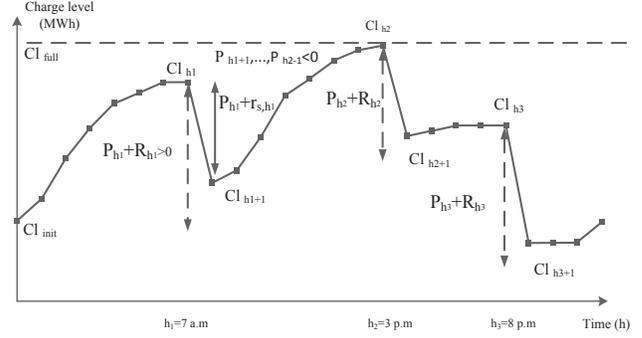


Fig. 2. An example for the charging and discharging cycles of an independent storage unit when it participates in energy and reserve markets.

for an independent storage unit in the day-ahead energy and reserve markets is shown in Fig. 2, where the storage unit has committed to offer energy and reserve at three hours:  $h_1 = 7:00$  AM,  $h_2 = 3:00$  PM, and  $h_3 = 8:00$  PM. In each case, the total state of charge of the batteries should reach a level  $Cl_h \geq P_h + R_h$  for all committed hours  $h \in \{h_1, h_2, h_3\}$ .

The storage unit aims to bid in the day-ahead market (DAM) such that the profit of its participation in the day-ahead market *plus* the expected value of its profit in the next 24 hour-ahead markets (HAM) are maximized. This can be mathematically formulated as the following optimization problem:

$$\begin{aligned}
 \max_{\mathbf{P}, \mathbf{R}} \quad & \sum_{h=1}^{24} (P_h \cdot CP_h + R_h \cdot CR_h) \\
 & + \mathbb{E} \{ HAM(\mathbf{P}, \mathbf{R}; \tilde{c}\mathbf{p}, \tilde{c}\mathbf{r}, \tilde{\mathbf{r}}) \} \\
 \text{s.t.} \quad & \sum_{t=1}^h (P_t + R_t) \geq Cl_{init} - Cl_{full} \quad \forall h = 1, \dots, 24 \\
 & \sum_{t=1}^h (P_t + R_t) \leq Cl_{init} \quad \forall h = 1, \dots, 24 \\
 & R_h \geq 0 \quad \forall h = 1, \dots, 24 \\
 & DCh_{max} \leq P_h + R_h \leq Ch_{max} \quad \forall h = 1, \dots, 24,
 \end{aligned} \tag{1}$$

where  $CP_h$  and  $CR_h$  are the price values for energy and reserve in the day-ahead market at each hour  $h$ . The rest of the notations in (1) are defined as follows. Parameters  $Cl_{full}$  and  $Cl_{init}$  denote the full charging capacity and the initial charge level of the storage unit, respectively. Parameter  $Ch_{max}$  denotes the maximum charging rate and  $DCh_{max}$  denotes the maximum discharging rate for the storage unit. Notation  $\mathbb{E}$  is mathematical expectation and  $HAM$  denotes the aggregate profit of the storage unit in the next 24 hour-ahead markets. Note that, the expected value of the profit in the hour-ahead market, i.e., the second term in (1), depends on not only the choices of  $\mathbf{P}$  and  $\mathbf{R}$ , but also the price of power in the hour-ahead market  $\tilde{c}\mathbf{p}$ , the price of reserve in the hour-ahead market  $\tilde{c}\mathbf{r}$ , the actual reserve utilization in hour-ahead market  $\tilde{\mathbf{r}}$ , and of course the fluctuations in the amount of wind power generated. The third constraint in (1) assures that the reserve bid is non-negative. Finally, the last constraint in (1) assures that the charging and discharging rates of the storage unit are feasible.

Using the definition of mathematical expectation, we can rewrite the second term in (1) as a weighted summation of

the aggregate hour-ahead profit, denoted by  $HAM$  at *many* but *finite* scenarios, where the weight for each scenario is the probability for that scenario. That is, we have,

$$\mathbb{E} \{HAM(\mathbf{P}, \mathbf{R}; \bar{\mathbf{c}}\mathbf{p}, \bar{\mathbf{c}}\mathbf{r}, \bar{\mathbf{r}})\} = \sum_{k=1}^K \gamma_k HAM_k, \quad (2)$$

where  $K$  denotes the total number of scenarios,  $HAM_k$  denotes the aggregate hour-ahead profit when scenario  $k$  occurs, and  $\gamma_k$  denotes the probability that scenario  $k$  occurs. Note that, we have  $\sum_{k=1}^K \gamma_k = 1$ . It is worth to clarify here that one of the main causes for profit uncertainty is the fluctuations in available wind power. Therefore, in our system model, each scenario is derived as a realization of available wind power generation at different wind generation locations, given their probability distribution functions that are assumed to be available, e.g., by using the wind forecasting techniques in [17]–[19]. For each scenario  $k$ , the corresponding aggregate hour-ahead profit can be calculated as follows:

$$\begin{aligned} HAM_k(\mathbf{P}, \mathbf{R}; \mathbf{c}\mathbf{p}_k, \mathbf{c}\mathbf{r}_k, \mathbf{r}_k) = \\ \max_{\mathbf{p}_k} \sum_{h=1}^{24} (p_{k,h} \cdot cp_{k,h} + r_{k,h} \cdot cr_{k,h}) \\ \text{s.t.} \quad \sum_{t=1}^h p_{k,t} \leq \sum_{t=1}^{h-1} (R_t - r_{k,t}) \quad \forall h = 1, \dots, 24, \end{aligned} \quad (3)$$

where  $\mathbf{p}_k$  is the adjustment to the power draw or power injection of the storage unit in the hour-ahead market for  $h = 1, \dots, 24$ , under scenario  $k$ . Here,  $\mathbf{c}\mathbf{p}_k$ ,  $\mathbf{c}\mathbf{r}_k$ , and  $\mathbf{r}_k$  are the actual realizations of the stochastic parameters  $\bar{\mathbf{c}}\mathbf{p}$ ,  $\bar{\mathbf{c}}\mathbf{r}$  and  $\bar{\mathbf{r}}$  when scenario  $k$  occurs. We note that they are all set by the grid operator. The constraint in (7) indicates that the total generation bid up to hour  $h$  of the hour-ahead market has to be limited to the extra charge available to the storage unit at hour  $h$ . Such extra charge is calculated as the sum of the hourly difference between the reserve bid in the day-ahead market, i.e.,  $R_h$ , and the reserve utilization in the hour-ahead market, i.e.,  $r_{k,h}$ , for all previous hours  $t = 1, \dots, h-1$ . Note that, the actual reserve utilization  $r_{k,h}$  may *not* always be as high as the committed reserve, as the grid may *not* need to utilize the entire reserve power offered by the storage unit. Therefore, it is possible for the storage unit to sell *unused reserve power* at any hour in the hour-ahead energy market in a *later hour*. In other words, in the hour-ahead market, the storage unit makes *corrective decisions* to maximize the utilization of the extra charge which is available due to different reserve utilizations caused by different wind availability scenarios.

While problem (1) is solved in the day-ahead market, it needs to obtain the optimal solution of problem (2) which is going to be solved later during the upcoming 24 hour-ahead markets, given that scenario  $k$  occurs. However, parameters  $cp_{k,h}$ ,  $cr_{k,h}$  and  $r_{k,h}$  are *not* known to the storage unit at the time of making decisions in the day-ahead market. The situation is particularly complicated for parameter  $r_{k,h}$ , as shown in Fig.3. On one hand, at each hour  $h$ , parameter  $r_{k,h}$  depends on the wind power availability in each scenario  $k$  because wind power availability would have impact on the grid's need for reserve power. However, such availability is

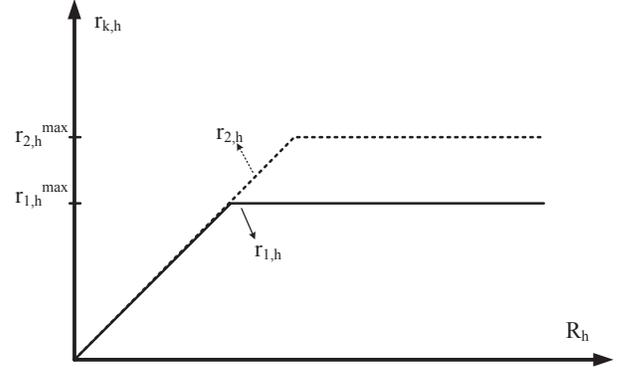


Fig. 3. The exact utilization of the storage unit in the reserve market at hour  $h$  depends on two factors: First, the storage unit's committed reserve amount  $R_h$ . Second, the grid's need  $r_{k,h}^{max}$  under stochastic scenario  $k$ . Two examples for the value of  $r_{k,h}$  as a function of  $R_h$  and  $r_{k,h}^{max}$  are shown in this figure. They match the mathematical formulation in (6).

not known ahead of time. To model this issue mathematically, let  $r_{h,k}^{max}$  denote the actual reserve power that the grid will need at hour  $h$  if scenario  $k$  occurs. It is required that:

$$r_{k,h} \leq r_{k,h}^{max} \quad h = 1, \dots, 24. \quad (4)$$

On the other hand, parameter  $r_{k,h}$  also depends on the storage unit's reserve commitment for each hour  $h$  based on its bid in the day-ahead market. Therefore, it is further required that

$$r_{k,h} \leq R_h \quad h = 1, \dots, 24. \quad (5)$$

From (4) and (5), at each hour  $h = 1, \dots, 24$ , and under each scenario  $k = 1, \dots, K$ , it is required that we have:

$$r_{k,h} = \min\{r_{k,h}^{max}, R_h\}. \quad (6)$$

Replacing (6) in the hour-ahead problem (7), it becomes:

$$\begin{aligned} HAM_k = \max_{\mathbf{p}_k} \sum_{h=1}^{24} (p_{k,h} \cdot cp_{k,h} + \min\{r_{k,h}^{max}, R_h\} \cdot cr_{k,h}) \\ \text{s.t.} \quad \forall h=1, \dots, 24 \quad \sum_{t=1}^h p_{k,t} \leq \sum_{t=1}^{h-1} (R_t - \min\{r_{k,t}^{max}, R_t\}). \end{aligned} \quad (7)$$

We can see that it is extremely difficult to obtain a closed-form solution for the above optimization problem such that we can replace the solution in the objective function (1) in order to solve problem (1). Therefore, next, we make a practical yet simplifying assumption that the storage unit *immediately* sells any excessive power available at each hour in case the entire committed reversed power is not utilized. That is, at each hour  $h$  and for each scenario  $k$ , we assume that:

$$p_{k,h} = R_h - r_{k,h}. \quad (8)$$

From (8), the inequality constraints in (7) always hold as equality for each  $h = 1, \dots, 24$ . From (7) and (8), we can

now rewrite the hour-ahead profit under scenario  $k$  as follows:

$$\begin{aligned}
HAM_k &= \sum_{h=1}^{24} (R_h - r_{k,h}) \cdot cp_{k,h} + r_{k,h} \cdot cr_{k,h} \\
&= \sum_{h=1}^{24} R_h \cdot cp_{k,h} + (cr_{k,h} - cp_{k,h}) \cdot r_{k,h} \\
&= \sum_{h=1}^{24} R_h \cdot cp_{k,h} + (cr_{k,h} - cp_{k,h}) \cdot \min\{r_{k,h}^{max}, R_h\}.
\end{aligned} \tag{9}$$

From (2) and (9), we can rewrite problem (1) as follows:

$$\begin{aligned}
\max_{\mathbf{P}, \mathbf{R}} \quad & \sum_{h=1}^{24} (P_h \cdot CP_h + R_h \cdot CR_h) \\
& + \sum_{k=1}^K \gamma_k \sum_{h=1}^{24} \left( R_h \cdot cp_{k,h} + \right. \\
& \left. (cr_{k,h} - cp_{k,h}) \cdot \min\{r_{k,h}^{max}, R_h\} \right) \\
\text{s.t.} \quad & \sum_{t=1}^h (P_t + R_t) \geq Cl_{init} - Cl_{full} \\
& \sum_{t=1}^h (P_t + R_t) \leq Cl_{init} \\
& R_h \geq 0 \\
& Dch_{max} \leq P_h + R_h \leq Ch_{max}.
\end{aligned} \tag{10}$$

Since min is a convex function and the rest of the objective function and constraints are all linear, problem (10) is a convex optimization problem as long as we have  $cr_{k,h} - cp_{k,h} \geq 0$ , for all  $k = 1, \dots, K$  and for all  $h = 1, \dots, 24$ . Interestingly, this condition holds in real markets as otherwise the generation units would choose to sell their entire capacity in the energy market and would not offer any capacity in the reserve market. If this condition holds, then problem (9) can also be written as a linear program. To show how, next, we introduce an auxiliary variable  $v_{k,h}$  and rewrite problem (10) as:

$$\begin{aligned}
\max_{\mathbf{P}, \mathbf{R}, \mathbf{v}} \quad & \sum_{h=1}^{24} (P_h \cdot CP_h + R_h \cdot CR_h) + \\
& \sum_{k=1}^K \gamma_k \sum_{h=1}^{24} \left( R_h \cdot cp_{k,h} + v_{k,h} \cdot (cr_{k,h} - cp_{k,h}) \right) \\
\text{s.t.} \quad & v_{k,h} \leq r_{k,h}^{max} \quad \forall k = 1, \dots, K \\
& v_{k,h} \leq R_h \quad \forall k = 1, \dots, K \\
& v_{k,h} \geq 0 \quad \forall k = 1, \dots, K \\
& \sum_{t=1}^h (P_t + R_t) \geq Cl_{init} - Cl_{full} \\
& \sum_{t=1}^h (P_t + R_t) \leq Cl_{init} \\
& R_h \geq 0 \\
& Dch_{max} \leq P_h + R_h \leq Ch_{max},
\end{aligned} \tag{11}$$

where  $\mathbf{v}$  is a  $24K \times 1$  vector of all auxiliary variables  $v_{k,h}$  for all  $k = 1, \dots, K$  and  $h = 1, \dots, 24$ . While problems (10) and (11) are *not* exactly the same, yet they are *equivalent*, i.e., they both lead to the same optimal solution [20, Chapter 4]. Therefore, solving one problem readily gives the solution for the other problem. Linear program (11) can be solved efficiently using the interior point method [20].

Before we end this section, we note that in order to solve the linear program in (11) we need to know the values of parameters  $r_{k,h}^{max}$ ,  $cp_{k,h}$ , and  $cr_{k,h}$  as well as  $CP_h$  and  $CR_h$ . To obtain these parameters, we can use an efficient off-line calculation by using a standard *stochastic unit commitment* (SUC), which is explained in detail in the Appendix. First, we solve the SUC problem. Since the storage unit is assumed to be price-taker, we then calculate  $cp_{k,h}$  from the Lagrange multipliers of the hour-ahead market constraints in the SUC problem. To calculate  $cr_{k,h}$ , we assume that it is proportional to  $cp_{k,h}$ . Next, we obtain  $CP_h$  directly from the definition of locational marginal price (LMP) and based on comparing the SUC's optimal objectives *with* and *without* having an additional unit of load at each bus [21]. After that, we obtain  $CR_h$  by using the regulation market clearing price (RMCP), which is obtained based on the calculation of the opportunity costs for generation units [22]. In this regard, we also assume that the independent system operator (ISO), uniformly utilizes all available units which are deployed for reserve service. Therefore, parameter  $r_{k,h}^{max}$  is obtained by dividing the total reserve utilization in each scenario by the total number of units that offer reserve. Note that, all these parameters are obtained based on historical data on previous market operations, i.e., by following the standard procedure in solving SUCs. The obtained solutions are then placed in some look-up tables which are later used every day that problem (11) is to be solved by the independent storage unit.

### III. NUMERICAL RESULTS

To evaluate the proposed approach in an illustrative example, the IEEE 24-bus standard test system is considered [23] which has a total of 2850 MW maximum consumption in any hour. We modified this test system by replacing the hydro units with three wind farms with 150, 70, and 30 wind turbines at buses 22, 17, and 11, respectively as shown in Fig. 1. Each wind turbine is assumed to have a maximum generation capacity of 1.5 MW. Therefore, the wind penetration is a rather moderate value of 13 percent. The wind speed across these three wind sites is assumed to be the same, due to relative proximity. Two 4.5 MW storage units are assumed to be connected to buses 21 and 11. The maximum discharge capacity of the storage units is 2 MW and the maximum charging capacity is one MW. The initial charge level is set to 1.5 MW. The price values for the two storage units are the same, since there was no congestion in the transmission lines in this study. The price values for the day-ahead and the hour-ahead markets are obtained from the standard SUC analysis that we explained in section III. The price curves for the day-ahead energy market, and the expected values of the prices in the hour-ahead energy markets, at bus 21, are shown in Fig.

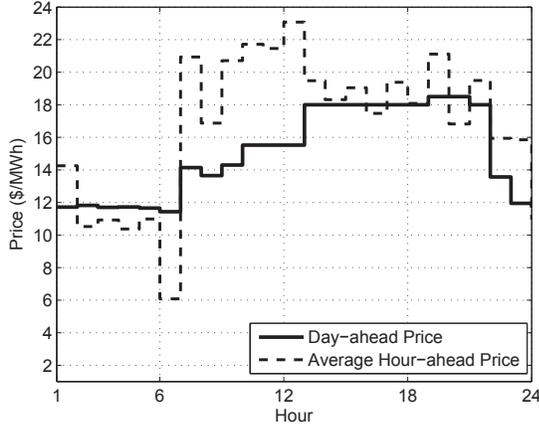


Fig. 4. Day-ahead and average hour-ahead market prices at bus 21.

III. For the hour-ahead price curve, the expected value at each hour is calculated based on different scenarios and depending on the scenario, the price may go up to \$45 /MWh.

The price curves for the day-ahead market and for different scenarios of the hour-ahead market are used to set the storage bid for purchasing or selling of energy and reserve services in the day-ahead market. The storage unit decision is obtained based on two methods. In the first approach, which is our proposed design, the bidding solution is obtained based on optimal solution of stochastic optimization problem (11). In the second approach, however, the storage schedule is obtained as the solution of a deterministic optimization problem which simply uses the expected values of the hour-ahead market prices in the next day. The state of the charge based on these two methods are shown in Fig. 5. We can see that they result in very different state-of-charge curves.

The daily profit obtained using our proposed stochastic optimization approach as well as the straightforward deterministic approach are shown in Fig. III when the storage capacity of the independent storage unit is assumed to vary from 2 MW to 50 MW. We can see that investment is profitable for both curves. However, our proposed approach can significantly increase the profit, in particular, as the size of the storage unit increases. When the storage size is as high as 50 MW, the merit of using our proposed stochastic method becomes particularly more evident as it leads to about 18% higher profit compared to the case when the deterministic approach is being used. Last but not least, we can notice that the storage profit curves in Fig. III do not increase linearly as the size of storage grows. These curves are rather concave. This is due to the fact that the maximum reserve utilization limit, may eventually prevent the storage unit to benefit from increasing its storage capacity. Therefore, the curves such as those in Fig. III can be used for *optimal capacity planning* of storage units. We plan to investigate this important issue in our future research.

#### IV. CONCLUSIONS

Integration of storage systems in the power system is a key component of the future smart grids. In this paper a novel approach is proposed for investor owned storage units

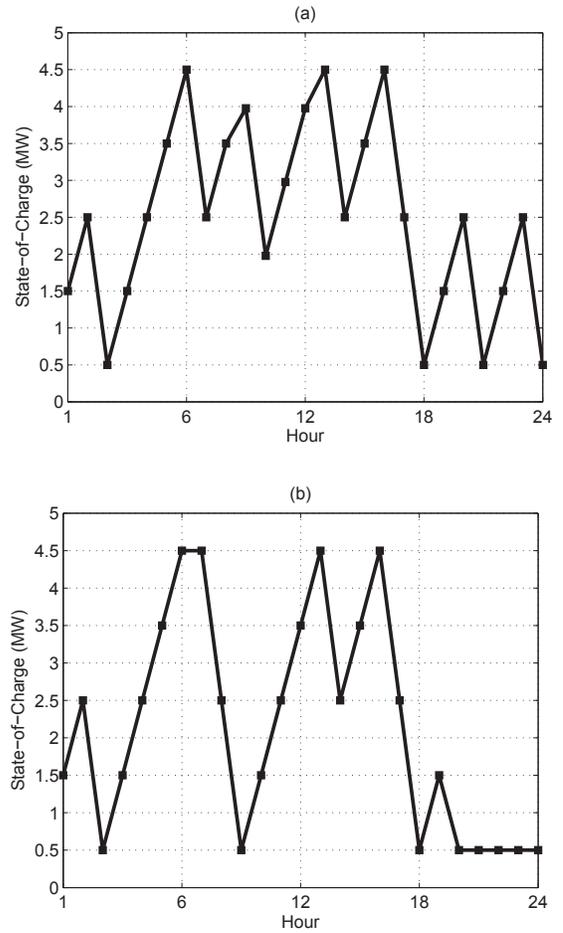


Fig. 5. State-of-charge for the independent storage unit. (a) Based on our proposed stochastic optimization planning. (b) Based on deterministic optimization planning using on expected value of price parameters.

to optimally bid for both energy and reserve, in the day-ahead and hour-ahead markets when significant fluctuation exists in the market prices due to wind generation variations. We developed a stochastic programming framework for storage optimal bid in the energy and reserve markets and formulated a tractable convex optimization problem to obtain the solution of the stochastic decision method. We showed that accounting for the unpredictable feature of market prices due to wind power fluctuations, can improve the decisions made by the storage unit, hence increase the profit. We showed that this approach, can bring about profits for the storage unit compared to conventional deterministic approaches.

#### APPENDIX

First, we explain the new set of notations that we need in order to formulate and solve the standard stochastic unit commitment problem.  $C(\cdot)$  denotes the cost of a particular service.  $Com_{i,h}$  denotes commitment of  $i$ th unit in hour  $h$ .  $P_{i,h}$  denotes generation of  $i$ th unit in hour  $h$ .  $R_{i,h}$  denotes reserve commitment of  $i$ th unit in hour  $h$ .  $P_i^+$  denotes maximum capacity of  $i$ th unit.  $P_i^-$  denotes minimum capacity of  $i$ th unit.  $Ramp^+$  denotes maximum ramp up of  $i$ th unit.  $Ramp^-$  denotes maximum ramp down of  $i$ th unit.  $G_f$  denotes subset of

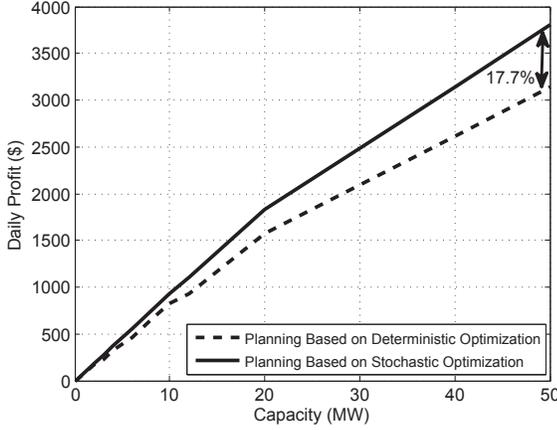


Fig. 6. The daily profit of the storage unit with the stochastic decision method and the deterministic method for the increasing storage capacities

fast generators.  $p_{i,k,h}$  denotes generation of  $i$ th fast unit in the hour ahead market for hour  $h$  and scenario  $k$ .  $r_{i,k,h}$  denotes reserve usage of  $i$ th unit in the hour ahead market for hour  $h$  and scenario  $k$ .  $NL_{k,h}$  denotes the total *net* load in hour  $h$  of scenario  $k$  which is the actual demand minus the wind power generation. Given these notations and parameters, we can now formulate the standard SUC as follows, in which the realization scenarios are considered in order to minimize the the expected value of the unit commitment cost in the system:

$$\begin{aligned}
 \min_{Com_i, P_i, R_i, p_{i,k,h}, r_{i,k,h}} \quad & \sum_h \sum_i C_{Com_i} \cdot Com_{i,h} + C_{P_i} \cdot P_{i,h} + C_{R_i} \cdot R_{i,h} \\
 & + \sum_k \gamma_k \sum_h \sum_i (c_{p_i} \cdot p_{i,k,h} + c_{r_i} \cdot r_{i,k,h}) \\
 \text{s.t.} \quad & P_{i,h} + R_{i,h} \leq P_i^+ \quad \forall i, h \\
 & P_{i,h} + R_{i,h} \geq P_i^- \quad \forall i, h \\
 & P_{i,h}, R_{i,h} \geq 0 \quad \forall i, h \\
 & P_{i,h} - P_{i,h-1} \leq Ramp_i^+ \quad \forall i, h \\
 & P_{i,h-1} - P_{i,h} \leq Ramp_i^- \quad \forall i, h \\
 & \sum_i P_{i,h} + p_{i,k,h} + r_{i,k,h} = NL_{k,h} \quad \forall h, k \\
 & 0 \leq r_{i,k,h} \leq R_{i,h} \quad \forall i, k, h \\
 & 0 \leq p_{i,k,h} \leq p_i^+ \quad \forall i \in G_f, k, h \\
 & p_{i,k,h} - p_{i,k,h-1} \leq Ramp_i^+ \quad \forall i \in G_f, k, h \\
 & p_{i,k,h-1} - p_{i,k,h} \leq Ramp_i^- \quad \forall i \in G_f, k, h
 \end{aligned}$$

The above SUC is a convex optimization problem and can be solved efficiently using algorithm such as Interior Point Method (IPM) which are available in CVX or MOSEK software. Given the solution and the Lagrange multipliers corresponding to each constraint, the needed system parameters are calculated accordingly, as we have already explained at the end of Section III. Note that the above problem is solved in centralized fashion. We also note that, since the storage units are assumed to have no impact on prices, this optimization problem does not include any variable from the storage units.

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